Prediction Invariance & Measurement Invariance

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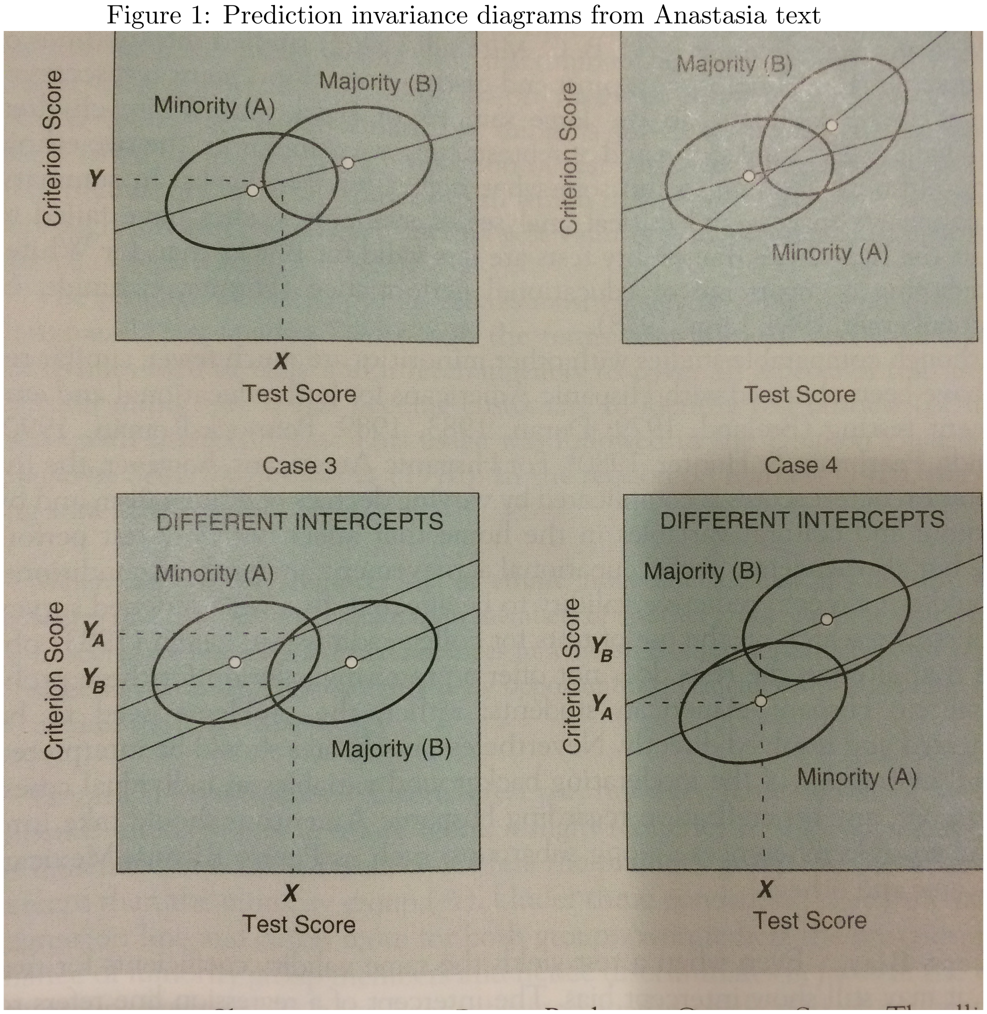
1 Prediction Invariance - Anastasia

Prediction invariance refers to the property of a test when it is applied to two groups. A prediction invariant test is one where the relationship between the test score X and the outcome Y is the same in both groups. In figure 1, taken from the Anastasia text, we see four possible scenarios. Only one scenario shows prediction invariance. Case 3 is

Figure 1: Prediction invariance diagrams from Anastasia text



2 Measurement Invariance - Reise, Widaman, & Pugh

2.1 The common factor model

The confirmatory factor model defined by the authors is a simple linear model of the m manifest variables X on the p latent **common factor** ξ plus the m **specific factors** δ. The values of m and p used in the article are $m = 5$ manifest variables (the items on the test) and $p = 1$ common factors. But, more generally,

$$X_m = \lambda_{mp}\xi_p + \delta_m \tag{1}$$

What this equation gives us is a model of how the latent variable ξ ("anxiety", for example) is related to the test score for each individual. That is, if we knew the 1st individual's anxiety level $\xi_{p,m=1}$, the slope λ_p, and the error $\delta_{m=1}$ we would know that individual's score on the test. Maybe more interesting, if we knew the score on the test $X_{m=1}$, the slope λ_p and the error $\delta_{m=1}$ we could derive the individual's value on the latent anxiety variable. Unfortunately, we cannot observe the latent variable ξ_p or δ_m, so it is impossible to derive an individual's score on the latent variable.

While we do not know an individual's value on the latent variables, it can be shown (don't ask me) that equation 1 implies a model for the variance-covariance matrix of the observed variables,

$$\text{Var}(X) = \Lambda \text{Var}(\xi)\Lambda' + \text{Var}(\delta) \tag{2}$$

or, using more efficient notation used by Reise et al.,

$$\Sigma = \Lambda\Phi\Lambda' + \Psi \tag{3}$$

If we break out this matrix equation for the Reise paper we would see the following matrices, showing the model that relates the variance-covariance matrix of the common factor(s), the factor loadings, and the specific factors, to the variance-covariance matrix of the manifest variables.

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} & \sigma_{14} & \sigma_{15} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} & \sigma_{24} & \sigma_{25} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} & \sigma_{34} & \sigma_{35} \\ \sigma_{41} & \sigma_{42} & \sigma_{43} & \sigma_{44} & \sigma_{45} \\ \sigma_{51} & \sigma_{52} & \sigma_{53} & \sigma_{54} & \sigma_{55} \end{bmatrix} = \begin{bmatrix} \lambda_{11} \\ \lambda_{21} \\ \lambda_{31} \\ \lambda_{41} \\ \lambda_{51} \end{bmatrix} \begin{bmatrix} \phi_{11} \end{bmatrix} \begin{bmatrix} \lambda_{11} & \lambda_{21} & \lambda_{31} & \lambda_{41} & \lambda_{51} \end{bmatrix} + \begin{bmatrix} \psi_{11} & & & & \\ & \psi_{22} & & & \\ & & \psi_{33} & & \\ & & & \psi_{44} & \\ & & & & \psi_{55} \end{bmatrix}$$

For example, to calculate the variance σ_{11} of manifest variable 1 we simply calculate $\lambda_{11} \times \phi_{11} \times \lambda_{11} + \psi_{11}$.

We can also represent these matrix relationships as a path diagram:

We can "trace" paths to calculate the variance of $X_1 = \sigma_{11}$. Start at X_1 and trace up λ_{11} against the arrow to ξ_1, then around the horn ϕ_{11} then back down λ_{11}. Multiply each path's parameters together and you have $\lambda_{11} \times \phi_{11} \times \lambda_{11}$. Then trace down to δ_1 around the horn ψ_{11} and back up. Add that to the previous term and you get $\lambda_{11}\phi_{11}\lambda_{11} + \psi_{11}$

That trace is for variances. To get covariances you just trace from one variable up to the common factor, around the variance horn, and back down to the other variable. Voila, that is the covariance between those two manifest variables. Using matrices to compute the covariance σ_{12} of X_1 and X_2 we would have $\lambda_{11}\phi_{11}\lambda_{21}$ or, equivalently, $\lambda_{21}\phi_{11}\lambda_{11}$.

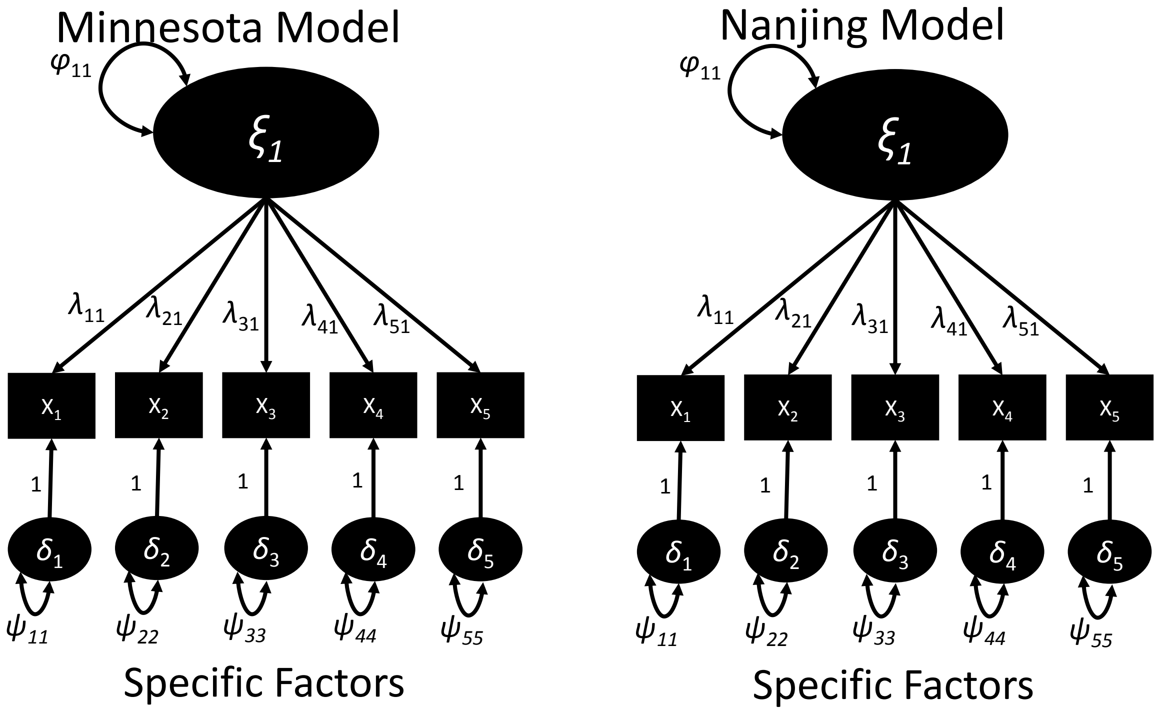
2.2 Measurement invariance

Measurement invariance is similar to the notion of prediction invariance, and can be defined as existing when "the conditional distribution of a manifest variable given a value of the latent variable, is the same over groups or subpopulations of interest."

There have been defined different extents of measurement invariance: following is stolen verbatim from Meredith & Teresi.

- **Weak factorial invariance** implies that the regression slopes are invariant across groups. Pattern invariance requires only invariant factor loadings λ.

Figure 2: Path representation of models fit in Riese et al.



- **Strong factorial invariance** implies that the conditional expectation of the response given the common and specific factors is invariant across groups. That is, that the loadings λ and factor means are invariant across groups.

- **Strict factorial invariance** implies that, in addition, the conditional variance of the response given the common and specific factors is invariant across groups. That is, that the factor means, loadings λ, and specific factors ψ are equivalent across groups.

2.3 Reise et al

- Model
- Degrees of freedom
- Baseline models—what are the differences? Why fit them?
- How does partial invariance, the invariance of even a single item, make comparisons between two groups possible?

3 Prediction invariance $\neq$ Measurement invariance

Recal from the Borsboom article that a prediction invariant test is not guaranteed to be measurement invariant, and vice versa.