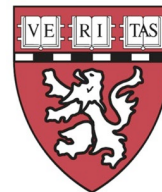


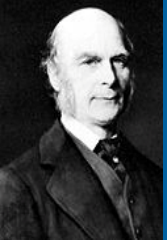
LD Score: theory

Benjamin Neale, Ph.D.

Analytic and Translational Genetics Unit, MGH

Stanley Center for Psychiatric Research & Program in Medical and
Population Genetics, Broad Institute





Francis Galton

Twin and family studies



RATE OF REGRESSION IN HEREDITARY STATURE.
Fig. (a.)

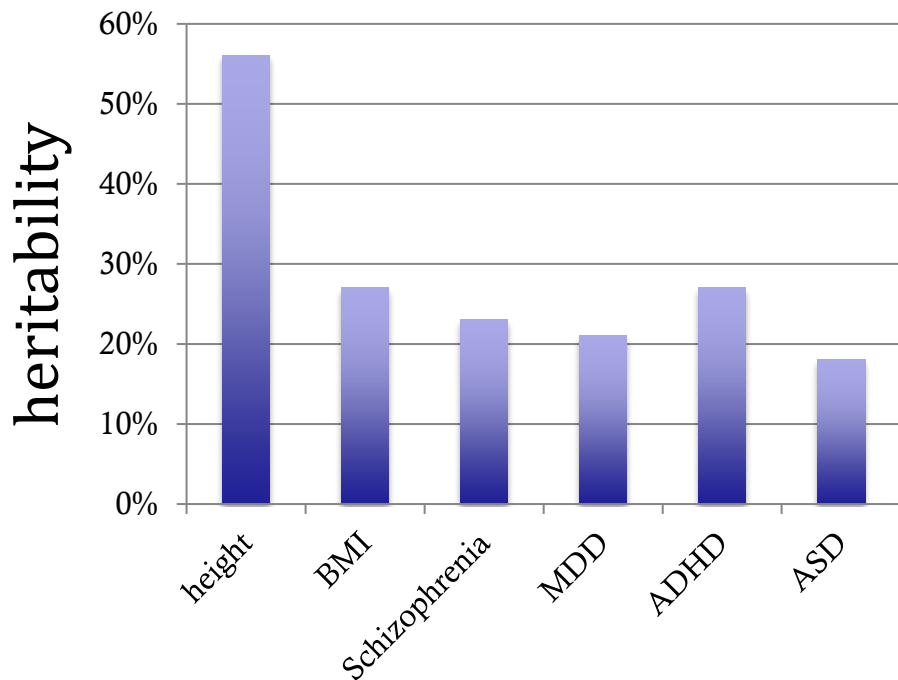
• Relatives are more similar

Meta-analysis of the heritability of human traits based on fifty years of twin studies

Tinca J C Polderman^{1,10}, Beben Benyamin^{2,10}, Christiaan A de Leeuw^{1,3}, Patrick F Sullivan⁴⁻⁶,
Arjen van Bochoven⁷, Peter M Visscher^{2,8,11} & Danielle Posthuma^{1,9,11}

Average estimate of heritability 49%
69% of twin studies support a purely additive genetic model

GREML/GCTA



- Use estimated genetic similarity

REPORT

GCTA: A Tool for Genome-wide Complex Trait Analysis

Jian Yang,^{1,*} S. Hong Lee,¹ Michael E. Goddard,^{2,3} and Peter M. Visscher¹

nature
genetics

ANALYSIS

Common SNPs explain a large proportion of the heritability for human height

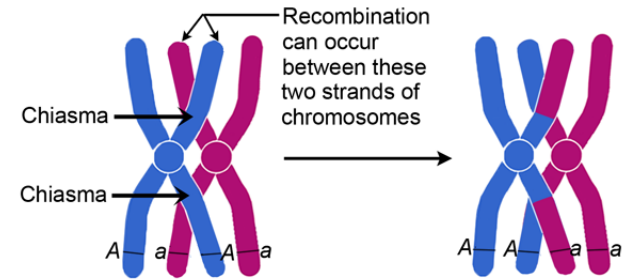
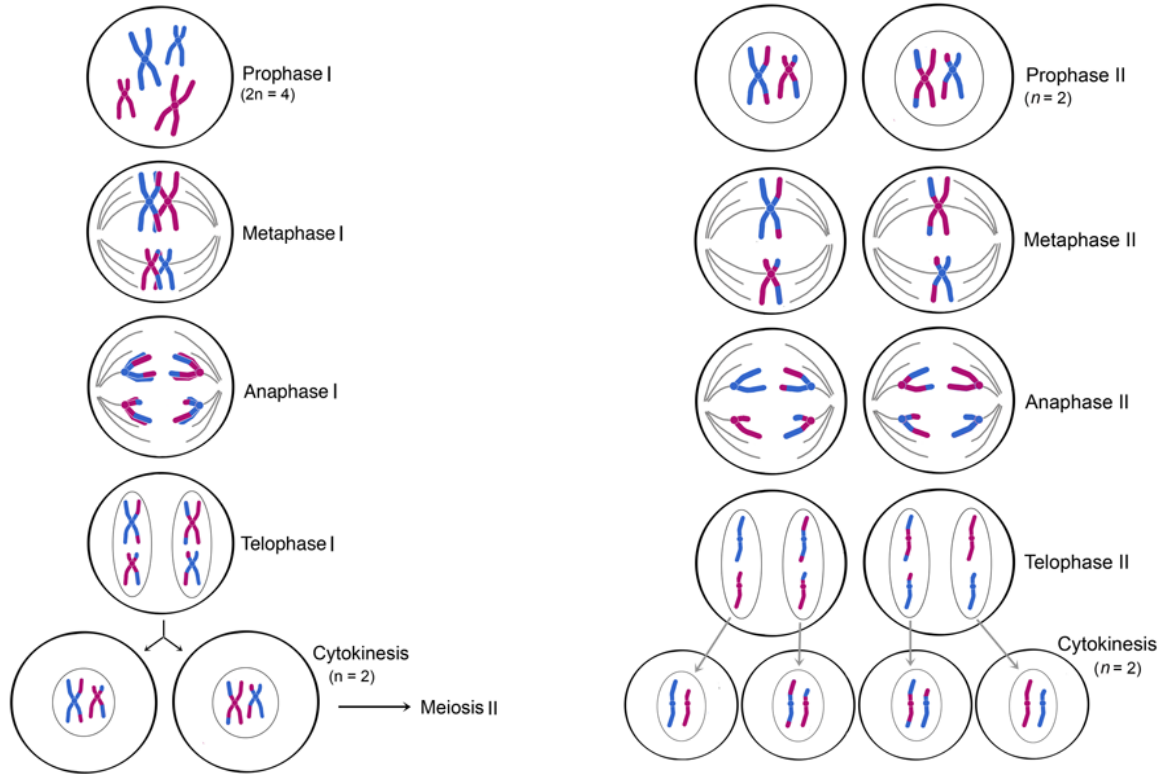
Jian Yang¹, Beben Benjamin¹, Brian P McEvoy¹, Scott Gordon¹, Anjali K Henders¹, Dale R Nyholt¹, Pamela A Madden¹, Andrew C Heath², Nicholas G Martin¹, Grant W Montgomery¹, Michael E Goddard² & Peter M Visscher¹

ARTICLE

Estimating Missing Heritability for Disease from Genome-wide Association Studies

Sang Hong Lee,¹ Naomi R. Wray,¹ Michael E. Goddard,^{2,3} and Peter M. Visscher^{1,*}

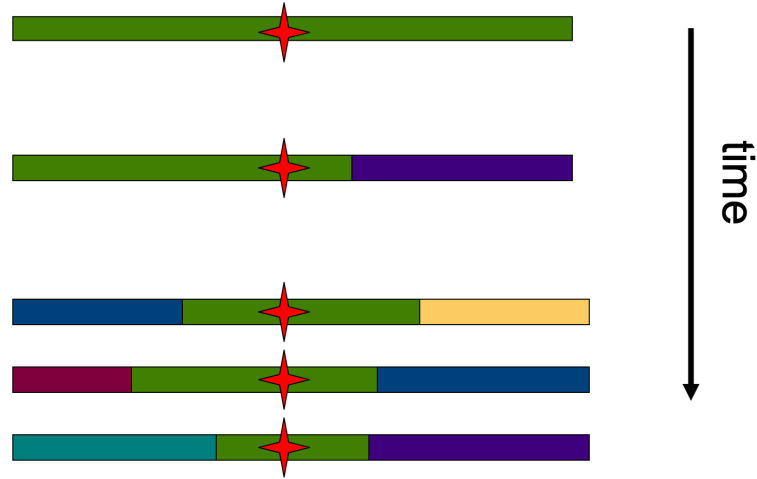
What happens to our genomes when making sperm and egg cells?



What happens when a new mutation arises?



Recombination mixes up the haplotype structure



Estimators of LD



$$D_{AB} = p_{AB} - p_A p_B$$

$$D' = \frac{D}{D_{\max}}$$

where

$$D_{\max} = \begin{cases} \max\{-p_A p_B, -(1-p_A)(1-p_B)\} & \text{when } D < 0 \\ \min\{p_A(1-p_B), (1-p_A)p_B\} & \text{when } D > 0 \end{cases}$$

Two SNPs A and B
Genotypes: AA, Aa, aa
 BB, Bb, bb

$$r^2 = \frac{D^2}{p_A(1-p_A)p_B(1-p_B)}.$$

[nature](#) > [nature reviews genetics](#) > [review articles](#) > [article](#)

Review Article | [Published: June 2008](#)

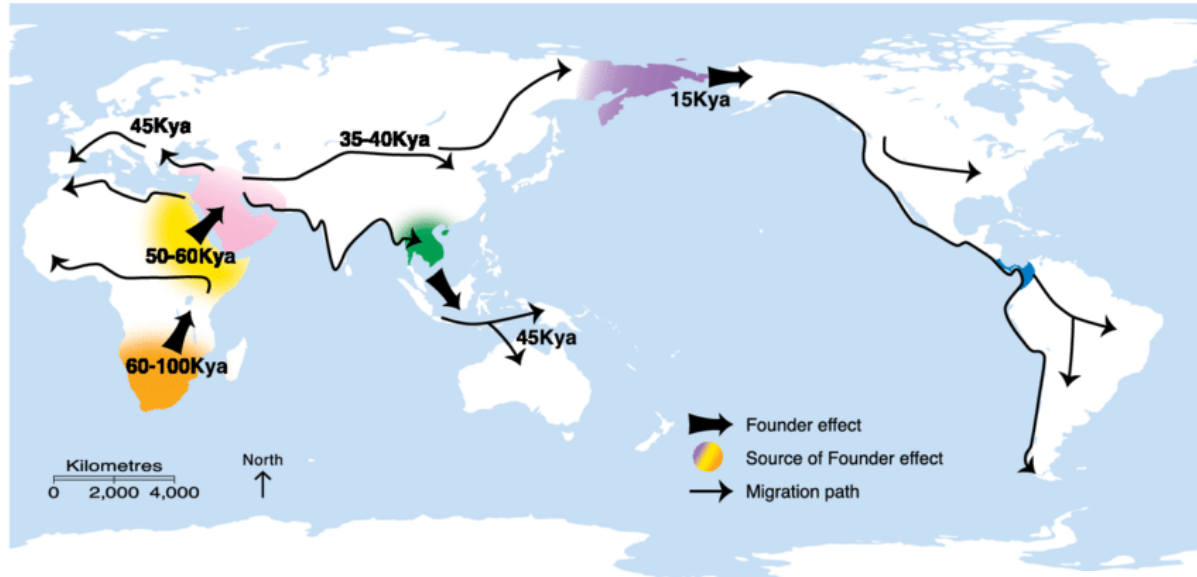
Linkage disequilibrium – understanding the evolutionary past and mapping the medical future

[Montgomery Slatkin](#)

[Nature Reviews Genetics](#) 9, 477–485 (2008) | [Cite this article](#)

54k Accesses | 816 Citations | 13 Altmetric | [Metrics](#)

Genetic ancestry



PERSPECTIVE | BIOLOGICAL SCIENCES |



The great human expansion

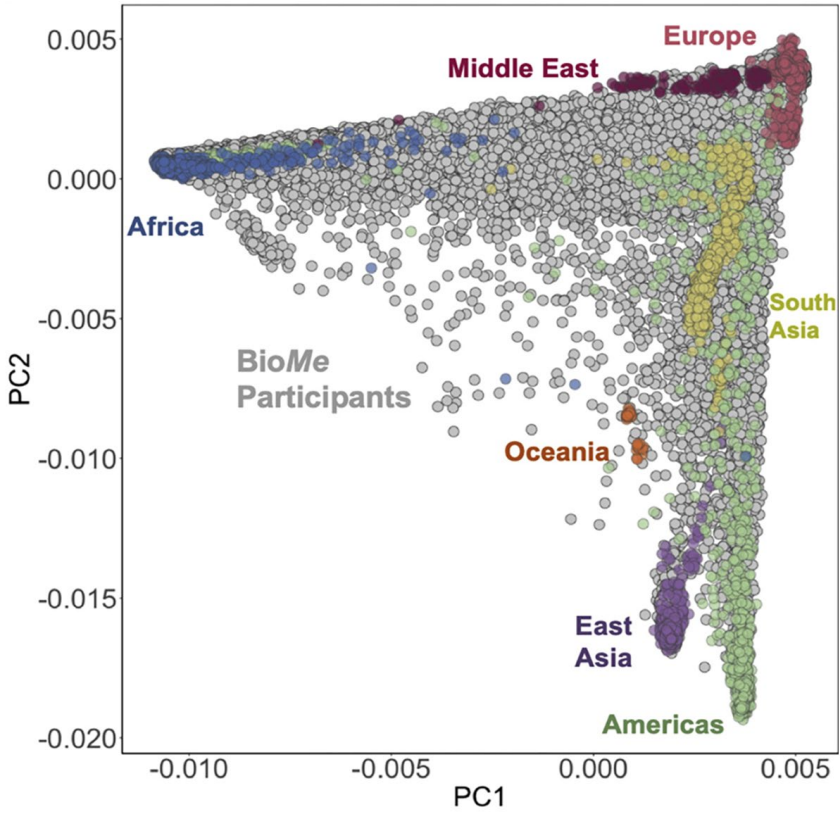
Brenna M. Henn, L. L. Cavalli-Sforza, and Marcus W. Feldman [✉](#) [Authors Info & Affiliations](#)

Edited by C. Owen Lovejoy, Kent State University, Kent, OH, and approved September 25, 2012 (received for review July 19, 2012)

October 17, 2012 | 109 (44) 17758-17764 | <https://doi.org/10.1073/pnas.1212380109>

Henn et al. (2012)

Principal component analysis on genetic data estimates structure



🔒 | POLICY FORUM | GENETICS AND SOCIETY



Getting genetic ancestry right for science and society

We must embrace a multidimensional, continuous view of ancestry and move away from continental ancestry categories

ANNA C. F. LEWIS, SANTIAGO J. MOLINA, PAUL S. APPELBAUM, BEGE DAUDA, ANNA DI RIENZO, AGUSTIN FUENTES, STEPHANIE M. FULLERTON, NANIBAA' A. GARRISON,

NAYANIKHA GHOSH, [...] AND DANIELLE S. ALLEN

+10 authors

[Authors Info & Affiliations](#)

LD Score regression



With thanks



Brendan Bulik-Sullivan



Hilary Finucane



Po-Ru Loh



Mark Daly



Alkes Price

How does LD shape association?



LD Score regression distinguishes confounding from polygenicity in genome-wide association studies

Brendan K Bulik-Sullivan, Po-Ru Loh, Hilary K Finucane, Stephan Ripke, Jian Yang, Schizophrenia Working Group of the Psychiatric Genomics Consortium, Nick Patterson, Mark J Daly, Alkes L Price & Benjamin M Neale

[Affiliations](#) | [Contributions](#) | [Corresponding author](#)

Nature Genetics **47**, 291–295 (2015) | doi:10.1038/ng.3211
Received 07 March 2014 | Accepted 07 January 2015 | Published online 02 February 2015



How does LD shape association?

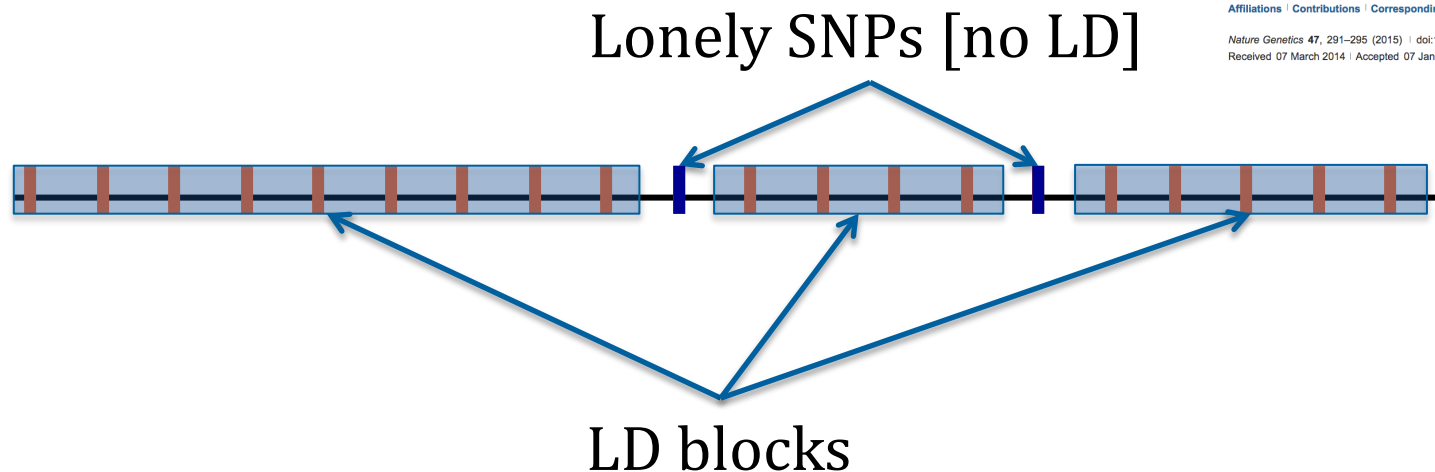


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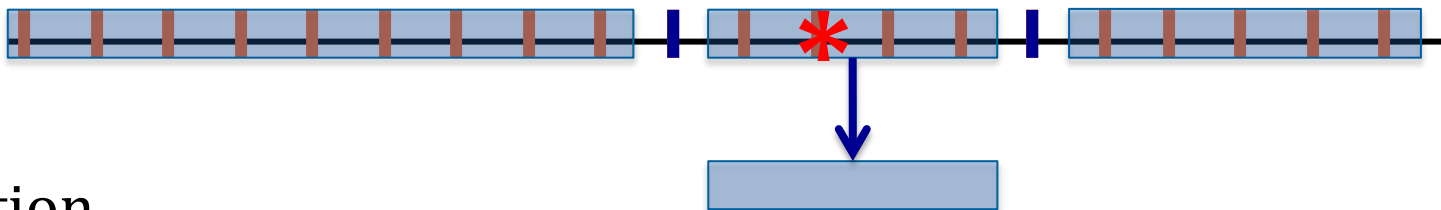
How does LD shape association?



┃ Lonely SNPs [no LD]

▭ LD blocks

* Causal variants



LD Score regression distinguishes confounding from polygenicity in genome-wide association studies

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All markers correlated with a causal variant show association

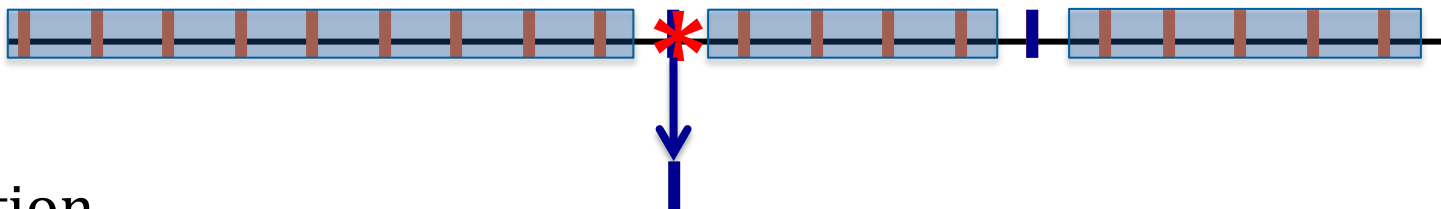
How does LD shape association?



■ Lonely SNPs [no LD]

■ LD blocks

* Causal variants



Association

Lonely SNPs only show association if they are causal

LD Score regression distinguishes confounding from polygenicity in genome-wide association studies

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What happens under polygenicity?



■ Lonely SNPs [no LD]

■ LD blocks

* Causal variants

LD Score regression distinguishes confounding from polygenicity in genome-wide association studies

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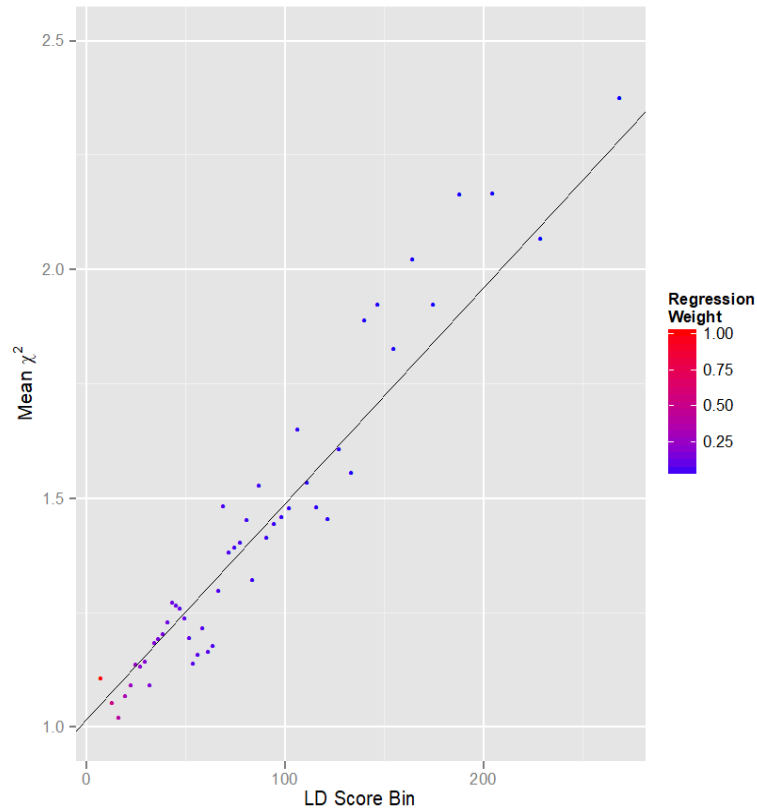
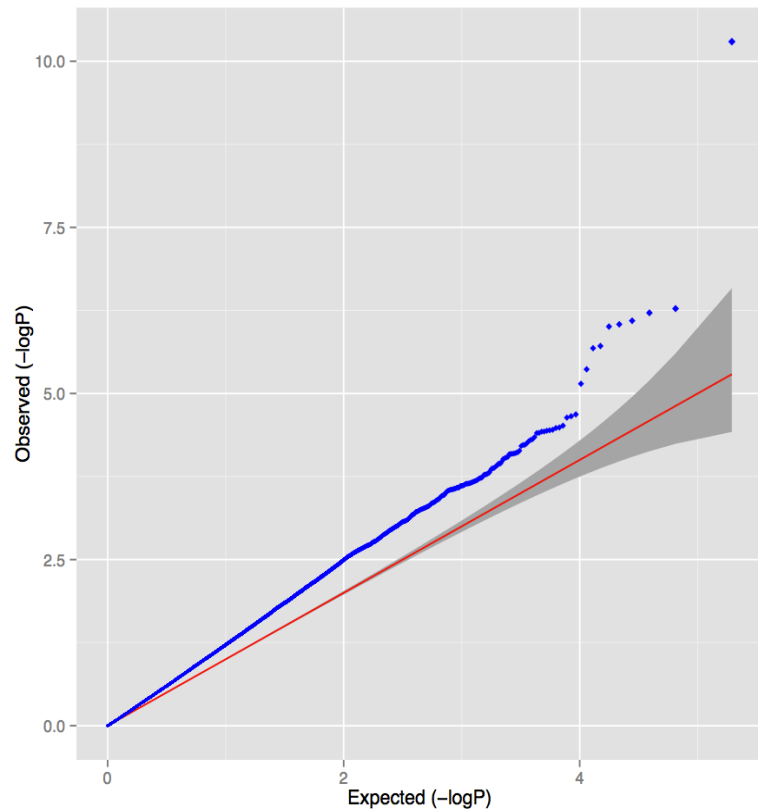


Assuming a uniform prior, we see SNPs with more LD friends showing more association

The more you tag, the more likely you are to tag a causal variant

Simulated polygenic architecture

Lambda = 1.30 LD score intercept = 1.02



What happens under stratification?



■ Lonely SNPs [no LD]

■ LD blocks

* Causal variants

LD Score regression distinguishes confounding from polygenicity in genome-wide association studies

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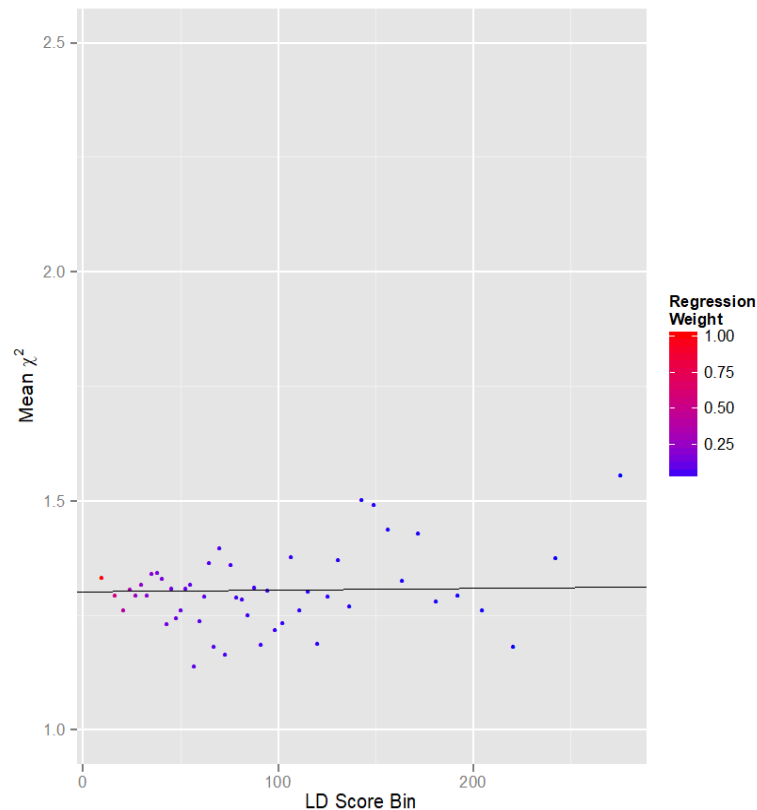
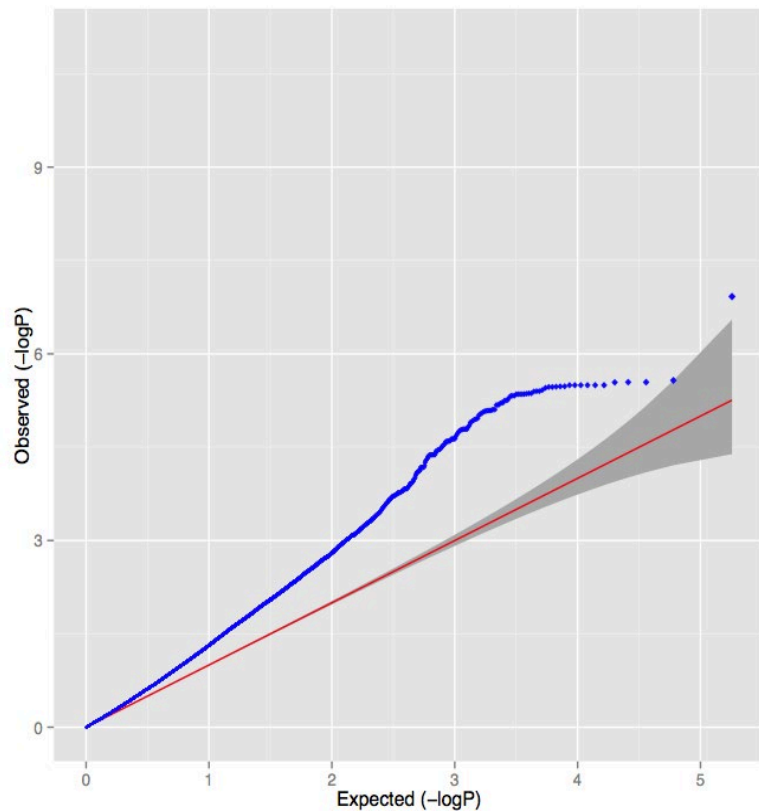
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Under pure drift we expect LD to have no relationship to differences in allele frequencies between populations

UK controls versus Sweden controls

$\Lambda = 1.30$ LD score intercept = 1.32



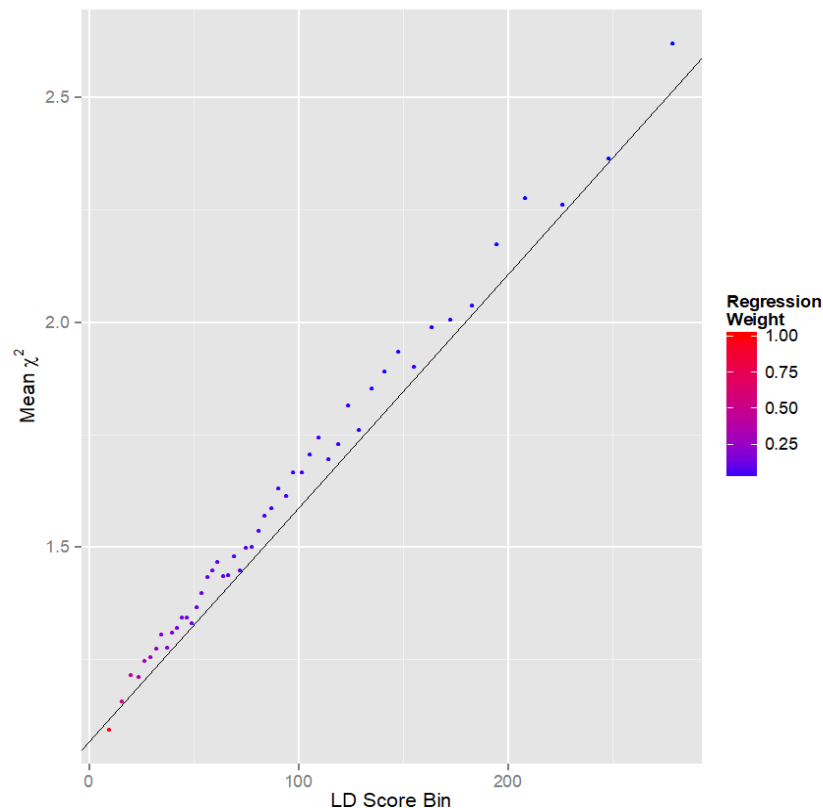
PGC Schizophrenia

Lambda = 1.48

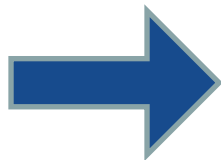
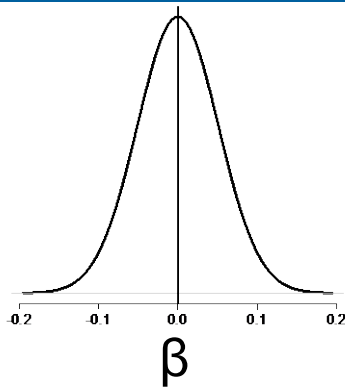
Intercept = 1.06

Slope p -value $< 10^{-300}$

Overwhelming majority of
inflation is consistent with
polygenic architecture



LD Score regression



Draw polygenic effects from $N(0, n/m^2)$, var =



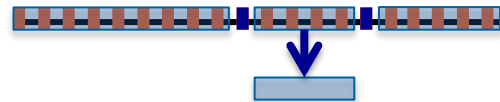
What is the $E[\chi^2]$ for variant j ?

$$E[\chi_j^2] = 1 + Na + \frac{h_g^2 N}{M} l_j$$

New estimator of heritability

where N =sample size, M =# of SNPs, a =inflation due to confounding, h_g^2 is heritability (total obs.) and l_j is the *LD Score*

$$l_j = \sum_{k \neq j} r_{jk}^2$$



What isn't in LD score?



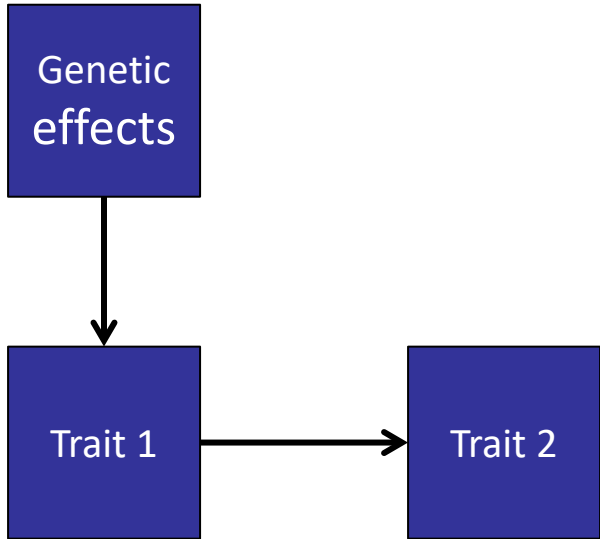
- Genetic variation that is not tagged well by common variation
- Heterogeneity of traits



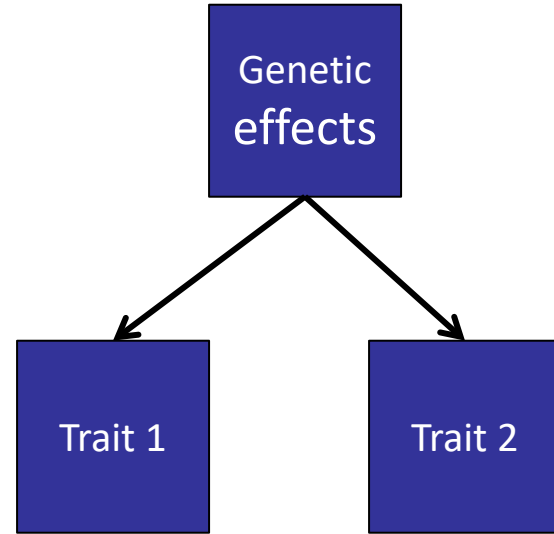
Genetic Correlation Method in:

An atlas of genetic correlations across human diseases
and traits

Potential sources of genetic correlation



Trait 1 exerts causal effect on Trait 2



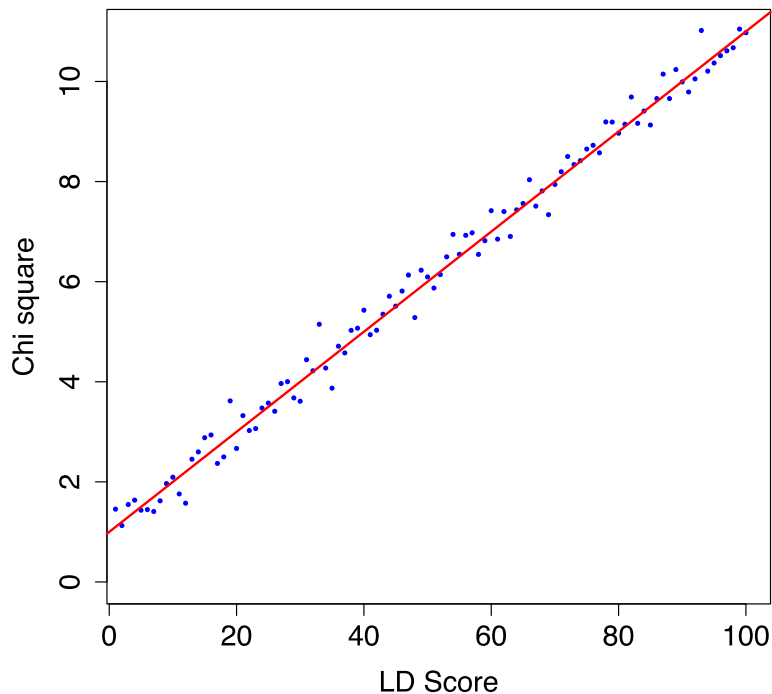
Genetic effects influence
Trait 1 and Trait 2

LD Score regression

Genetic correlation



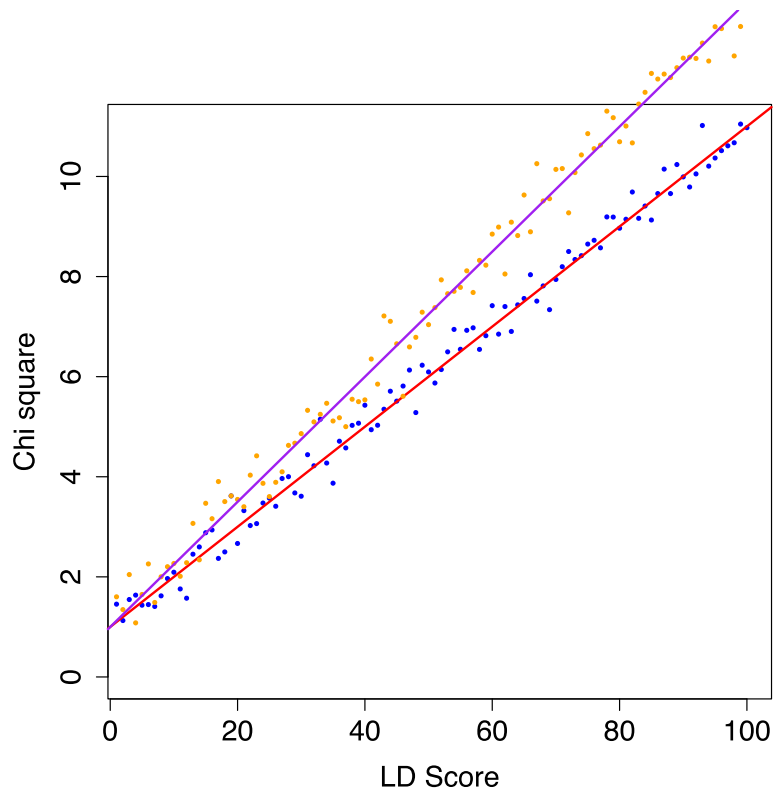
| Trait 1



Slope estimates heritability

LD Score regression

Genetic correlation

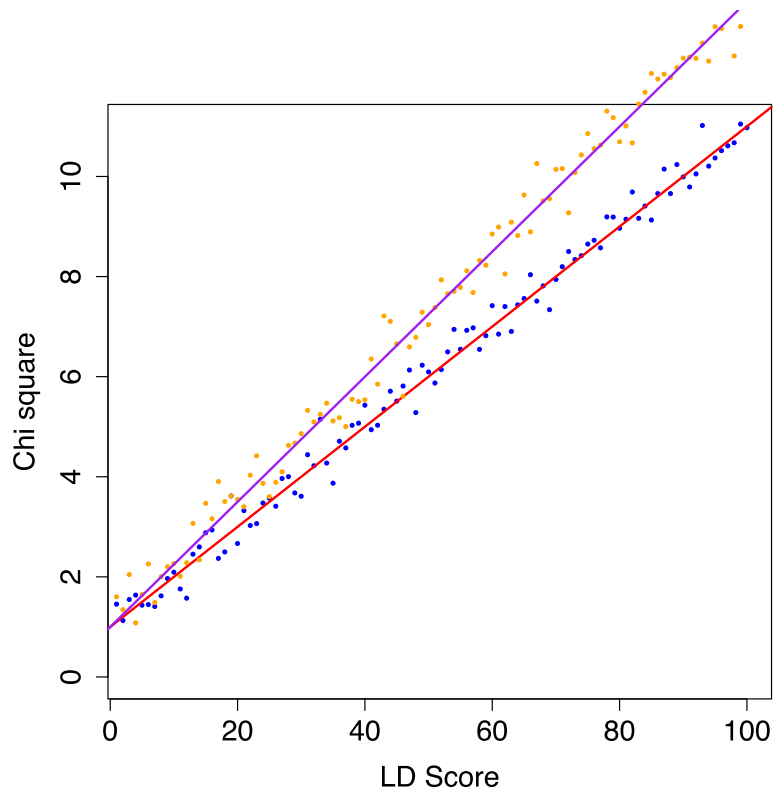


Trait 1
Trait 2

We can a second trait and
obtain two heritability
estimates

LD Score regression

Genetic correlation



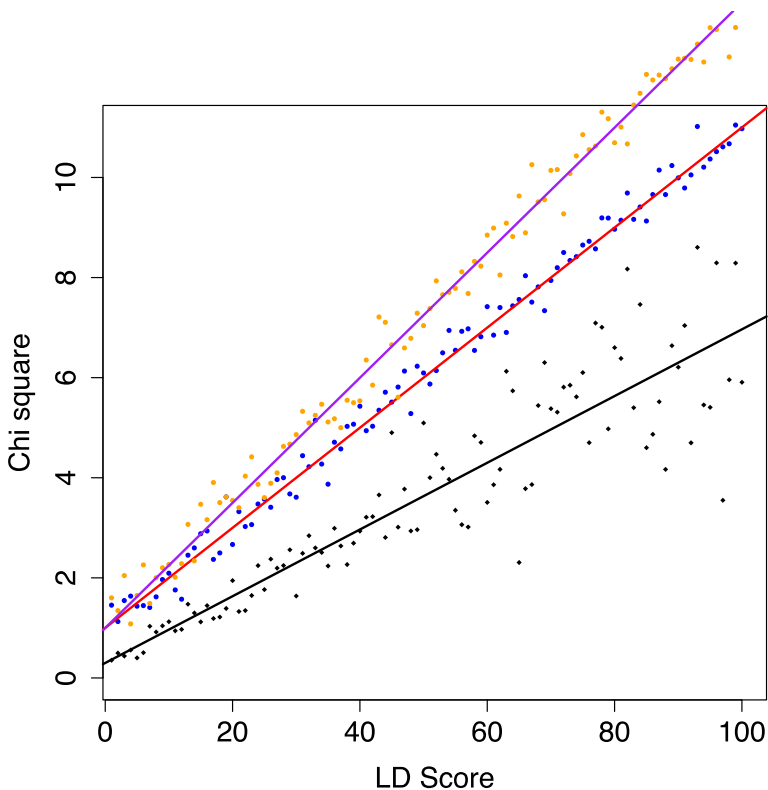
Trait 1
Trait 2

$$\mathbf{Z}^* \mathbf{Z} = \chi^2$$

So we can estimate genetic covariance from the product of the Z-scores

LD Score regression

Genetic correlation



Trait 1
Trait 2
 R_G

$$Z^*Z = \chi^2$$

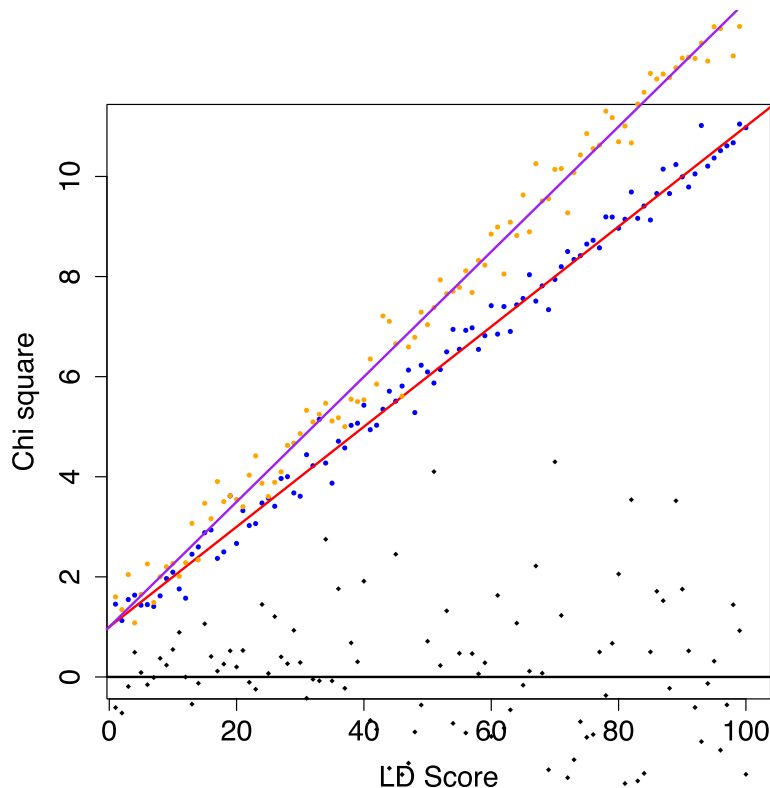
So we can estimate genetic covariance from the product of the Z-scores for the two traits

$$R_G = 0.5$$

LD Score regression

Genetic correlation

Trait 1
Trait 2
 R_G



Here $R_G = 0$

This approach is robust to sample overlap as all variants are equally inflated