

# Estimators & bias of CTDs and expectations of NTFD parameters

## ACE

Estimator

$$\hat{V}_A = 2(\hat{C}U_{m2} - \hat{C}U_{02})$$

$$\hat{V}_C = 2\hat{C}U_{02} - \hat{C}U_{m2}$$

New estimator if  $V_D = .05$

$$\hat{V}_A = 2((\hat{C}U_{m2} - .05) - (\hat{C}U_{02} - .0125))$$

$$\hat{V}_C = 2(\hat{C}U_{02} - .0125) - (\hat{C}U_{m2} - .05)$$

Bias

$$\hat{V}_A = V_A + \frac{3}{2}V_D$$

$$\hat{V}_C = V_C - \frac{1}{2}V_D$$

(.075 lower than our ACE est.)

(.025 higher than our ACE est.)

## ADE

Estimator

$$\hat{V}_A = 4\hat{C}U_{02} - \hat{C}U_{m2}$$

$$\hat{V}_D = 2\hat{C}U_{m2} - 4\hat{C}U_{02}$$

New estimators if  $V_C = .05$

$$\hat{V}_A = 4(\hat{C}U_{02} - .05) - (\hat{C}U_{m2} - .05)$$

$$\hat{V}_D = 2(\hat{C}U_{m2} - .05) - 4(\hat{C}U_{02} - .05)$$

Bias

$$\hat{V}_A = V_A + 3V_C$$

$$\hat{V}_D = V_D - 2V_C$$

(.15 lower than our ADE est.)

(.10 higher than our ADE est.)

In Direct Symmetric approach

In ADE,  $\hat{V}_C$  you would have gotten in ACE =  $-\frac{1}{2}\hat{V}_D$

In ACE,  $\hat{V}_D$  you would have gotten in ADE =  $-2\hat{V}_C$

So  $\hat{V}_D$  is an estimate of  $-2\hat{V}_C$  (so  $\hat{V}_C = -\frac{1}{2}\hat{V}_D$ )

$\hat{V}_C$  is an estimate of  $-\frac{1}{2}\hat{V}_D$  (so  $\hat{V}_D = -2\hat{V}_C$ )

$$1 = \frac{1}{2} + \frac{1}{2} + \frac{1}{2}\delta_m^2 = 1 + \delta_m^2$$

$$g = \frac{1}{2} + 2(\frac{1}{2}g^2) + a^2 20m \sim$$

$$x = 2(m\sigma_m^2) + 2(m\sigma^2 m\sigma_m^2) = 2m^2\sigma^2(1 + \sigma_m^2)$$

$$w = 2(\frac{1}{2}\delta m) + 2(\frac{1}{2}\delta m\sigma_m^2) = \delta m(1 + \sigma_m^2)$$

$$CV_{D2} = \frac{1}{4}d^2 + s^2 + f^2x + \frac{1}{2}a^2[g + \delta^2 m] + 2awf$$

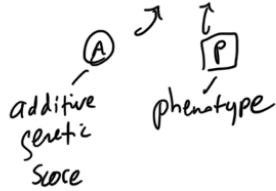
$$CV_{P0} = \frac{1}{2}a\delta + \frac{1}{2}a\delta m\sigma^2 + m\sigma^2 + m\sigma^2 m\sigma^2$$

$$CV_{P0} = \frac{1}{2}a\delta(1 + m\sigma^2) + m\sigma^2(1 + m\sigma^2)$$

$$CV_{P0} = (\frac{1}{2}a\delta + m\sigma^2)(1 + \sigma_m^2)$$

$$CV_{spouse} = \sigma^2 m \sigma^2$$

$$\delta = \text{cov}(A, P) = ga + wf$$



( $\delta$  is just a convenient "shortcut" to make the math look less messy - it is a sub-chain that shows up over and over)

$$V(P) = \sigma^2 = a^2g^2 + f^2x + 2awf + d^2 + s^2 + e^2$$

NOTE: we have to arbitrarily fix either  $m$  or  $f$  to be 1 for identification (this convention isn't an assumption - it's just needed to fit the model - kind of like deciding to fix latent variances to 1 or the path coefficients to 1). I typically set  $f=1$  & estimate  $m$ .

WTF!

(see Keller et al. 2009 in the Answers folder)