

# **Heterogeneity**

**Sarah Medland & Hermine Maes**

**Mike, Liz, Brad, Dave, Baptiste, Rob, Mike, Jose, Daniel, Philip, Luis, Dan**

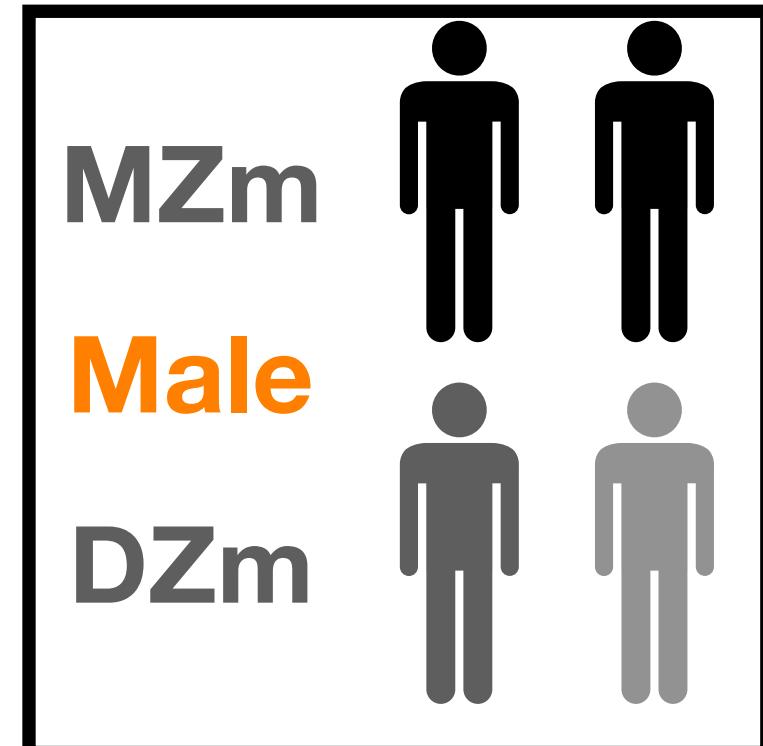
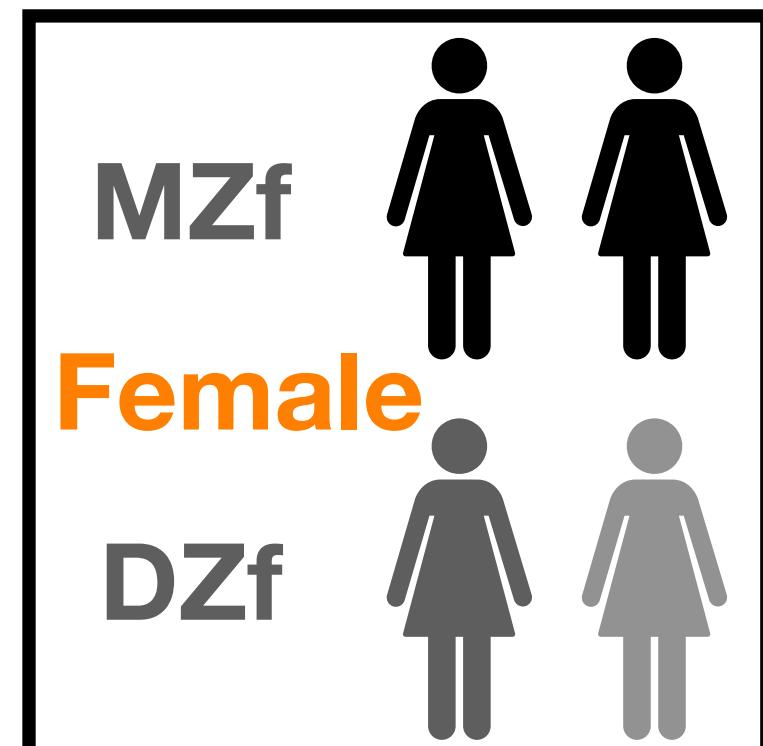
**Virtual Boulder Workshop 2022**

# Heterogeneity

**2-group 10 min**

oneADEvca

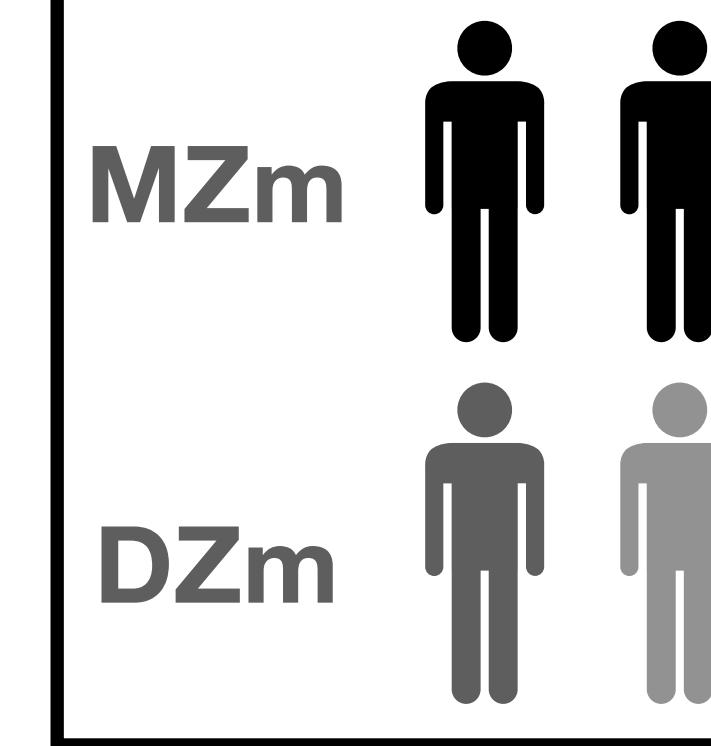
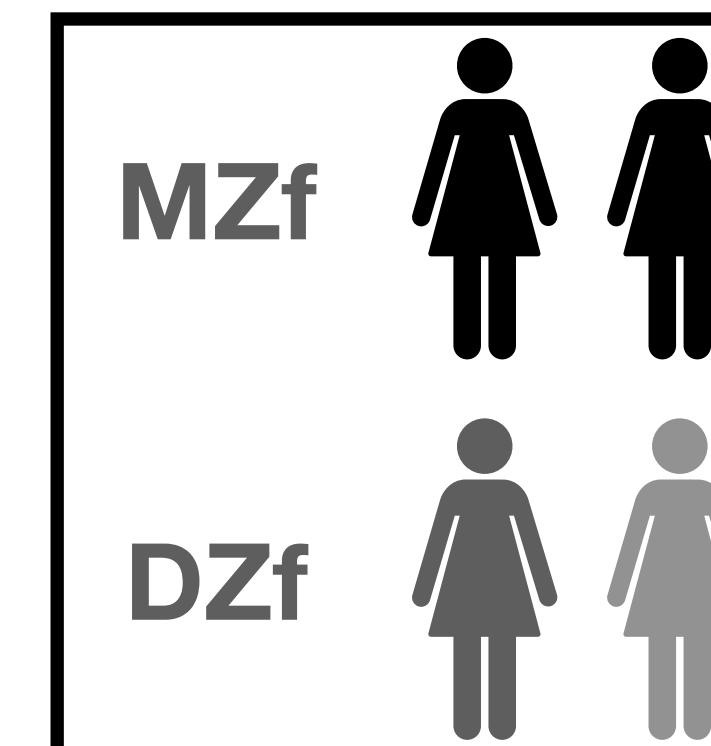
T1 T2



**4-group 15 min**

oneADE4vca

T1 T2



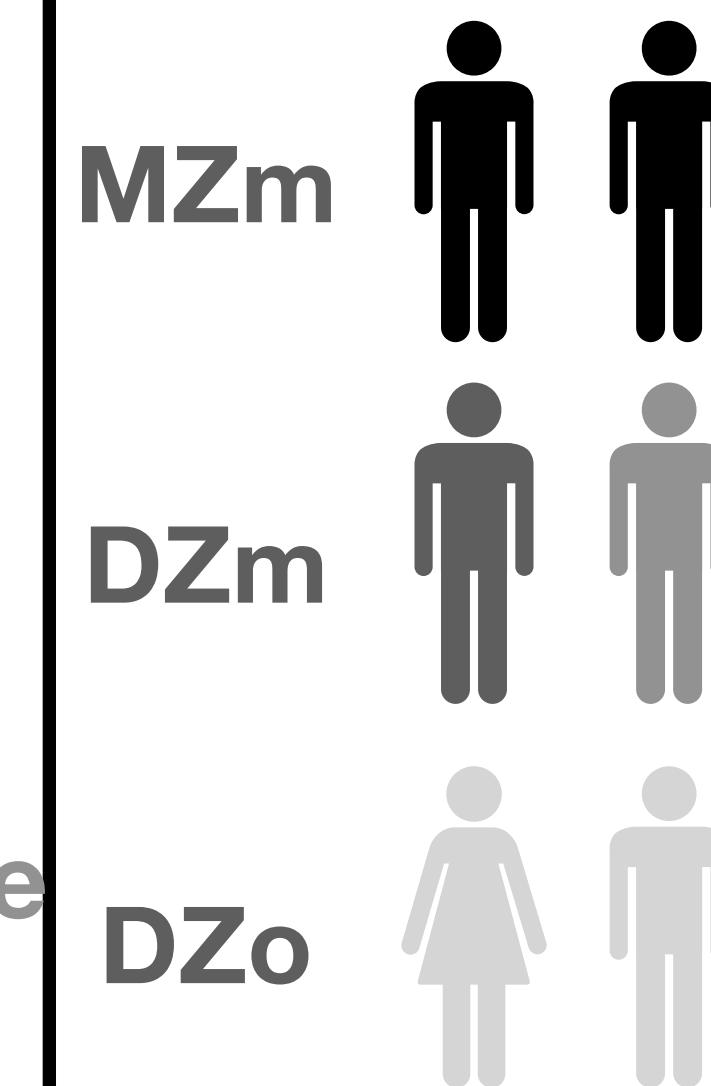
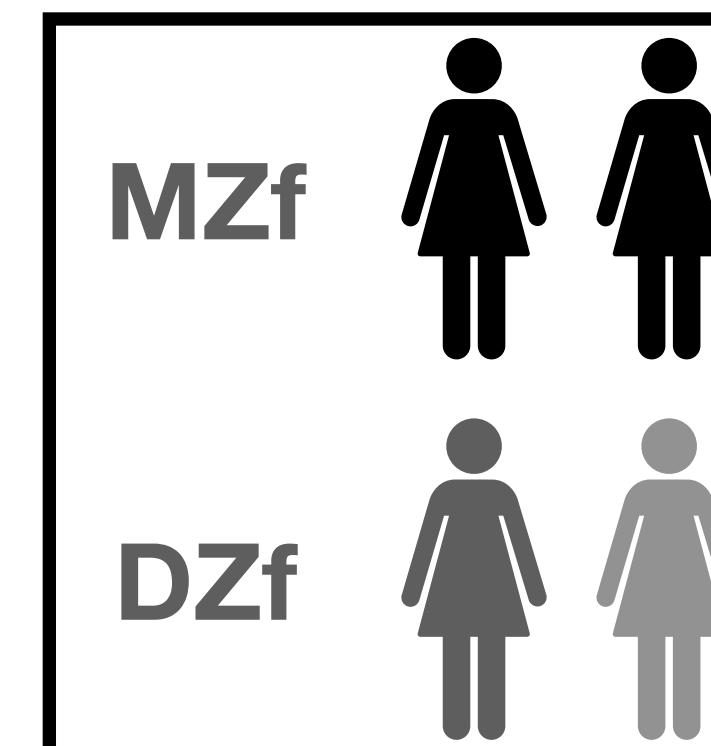
Same Sex

+Opposite Sex

**5-group 15 min**

oneADE5vca

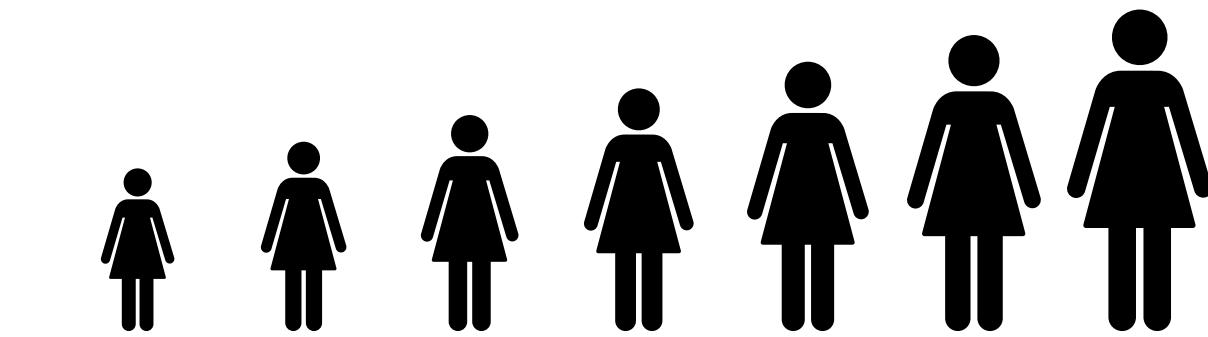
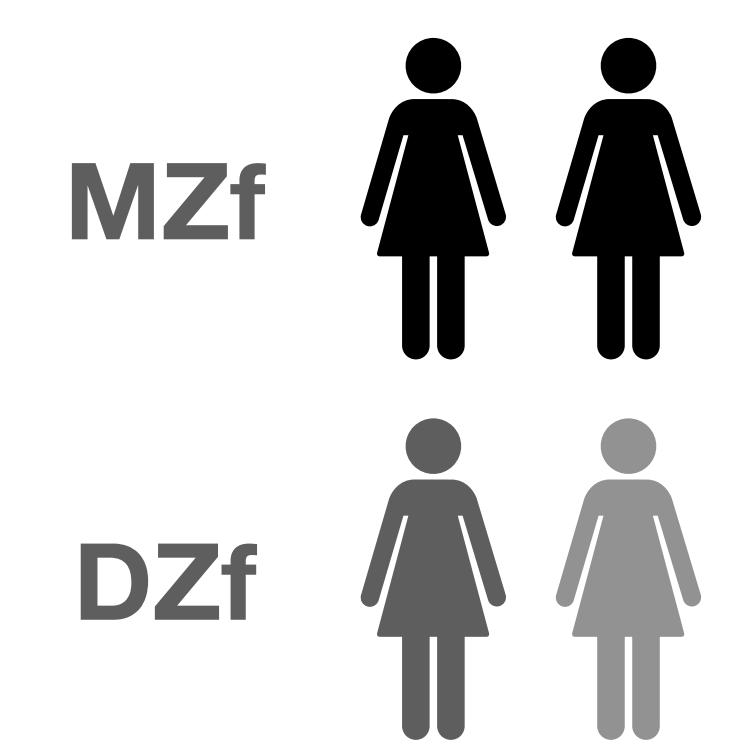
T1 T2



**interaction 20 min**

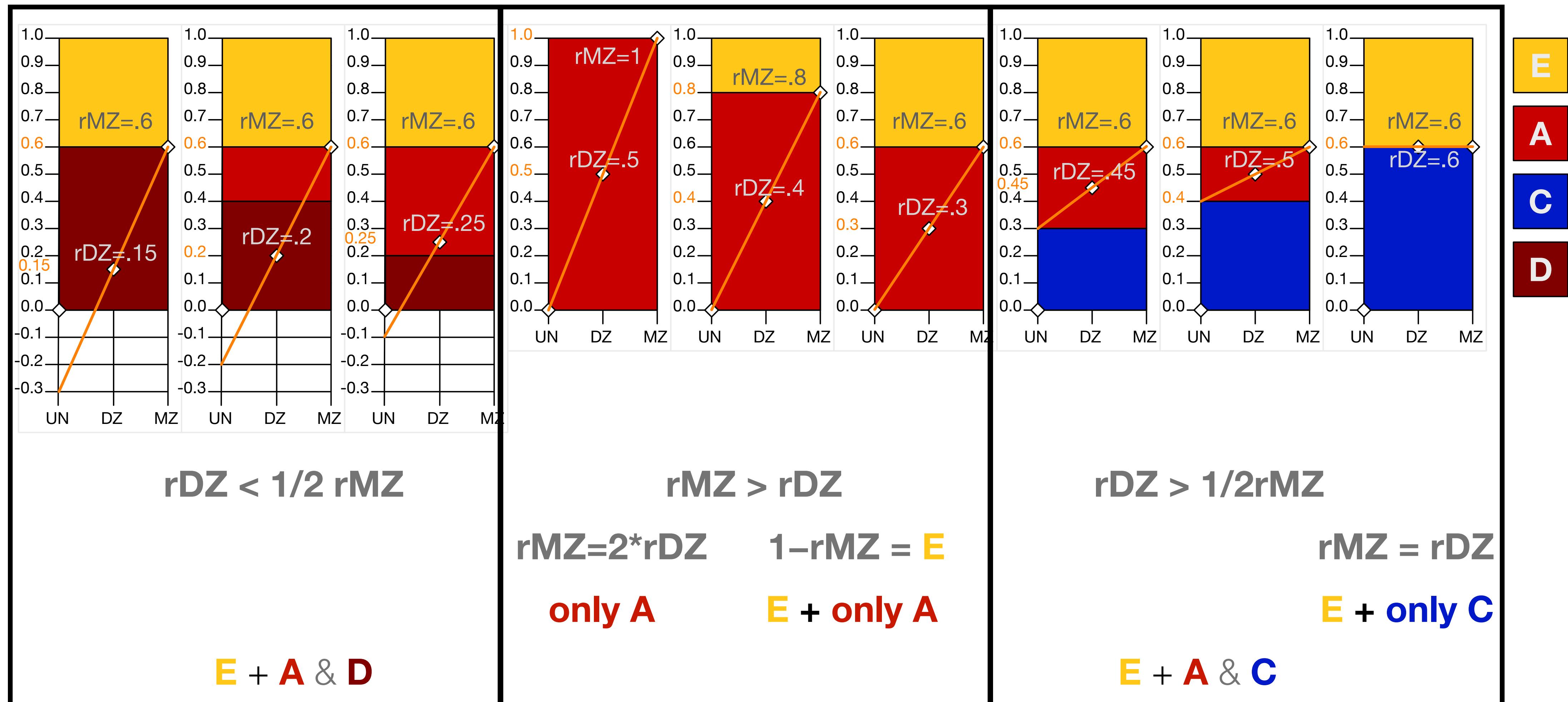
oneADEcal

T1 T2



Age 15 25 35 45 55 65 75

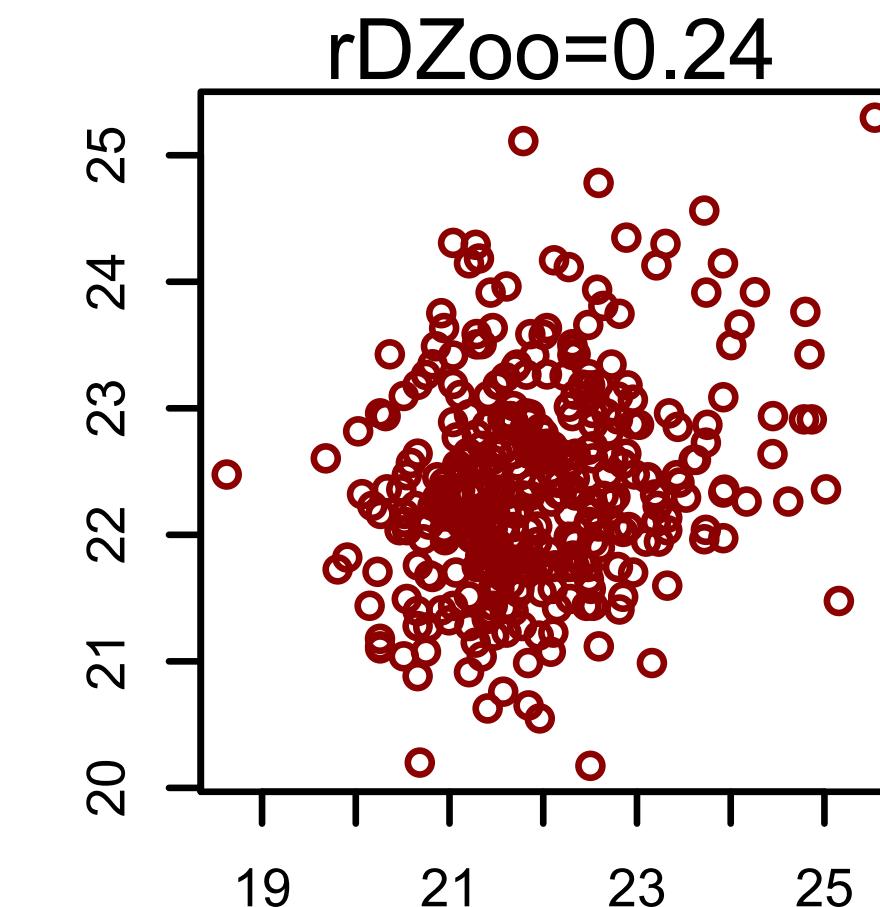
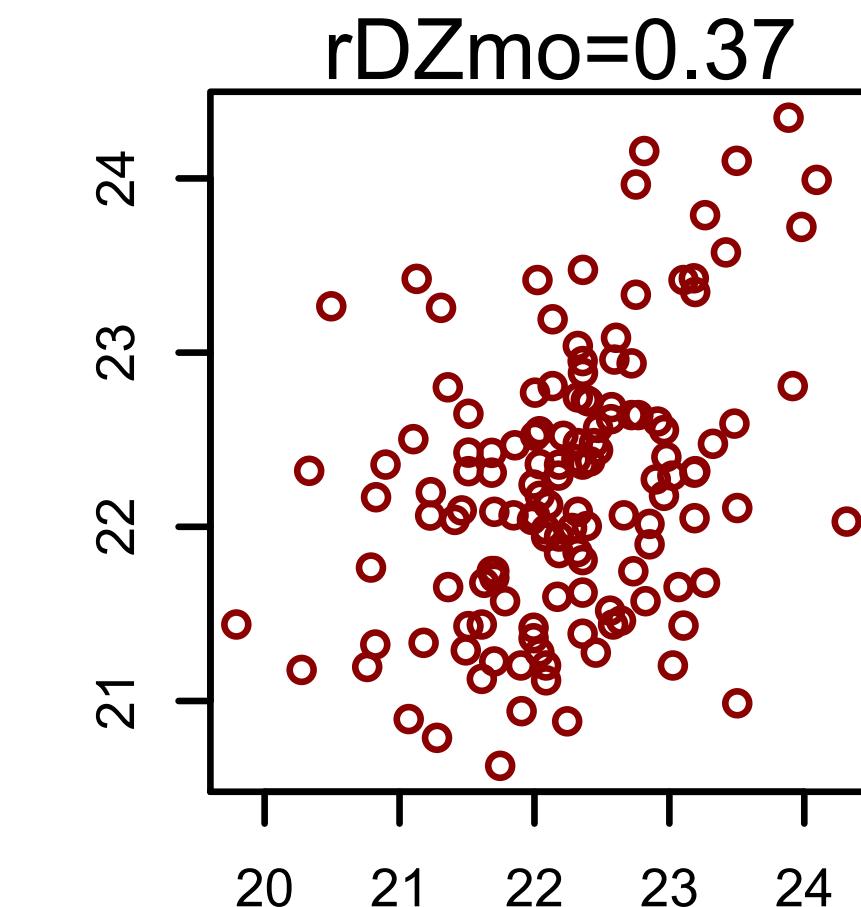
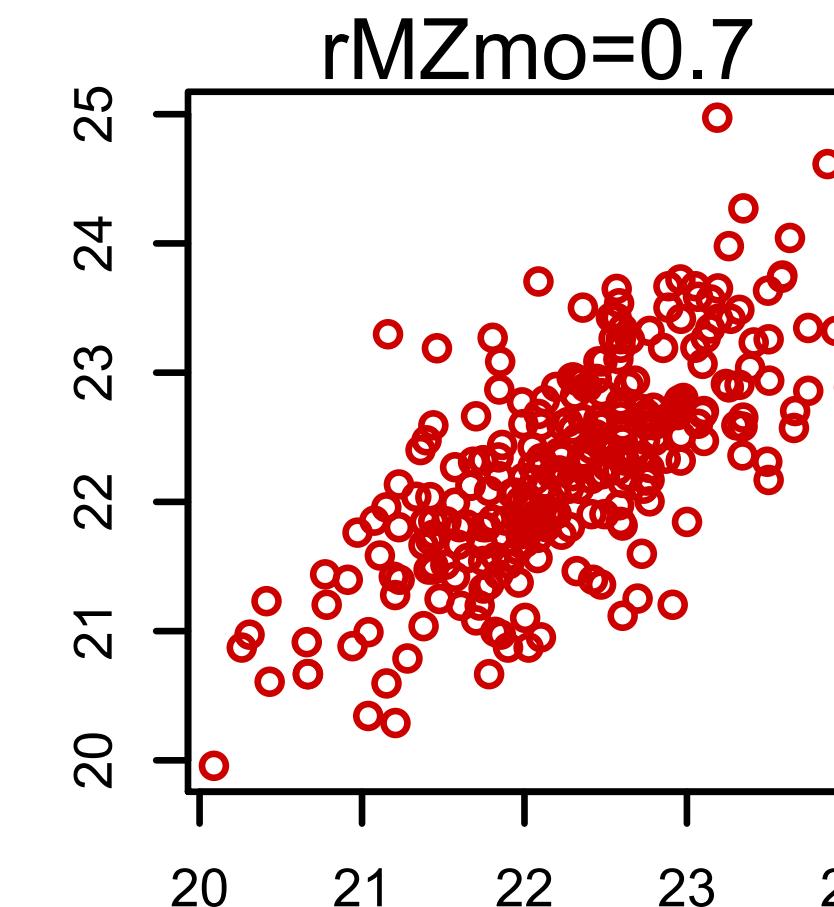
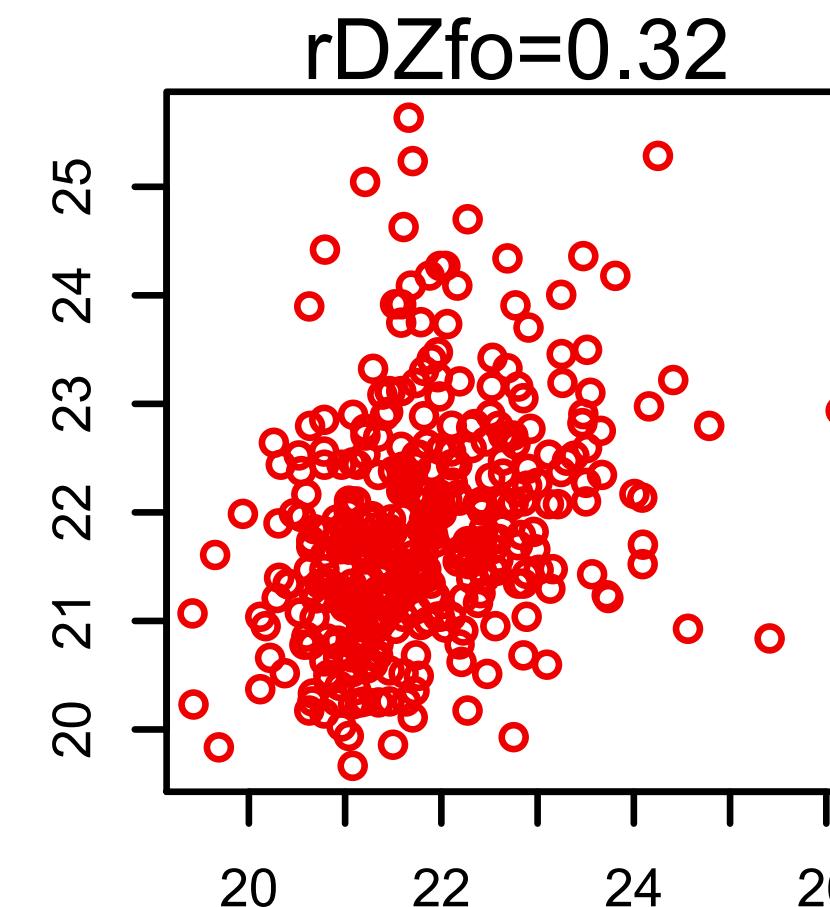
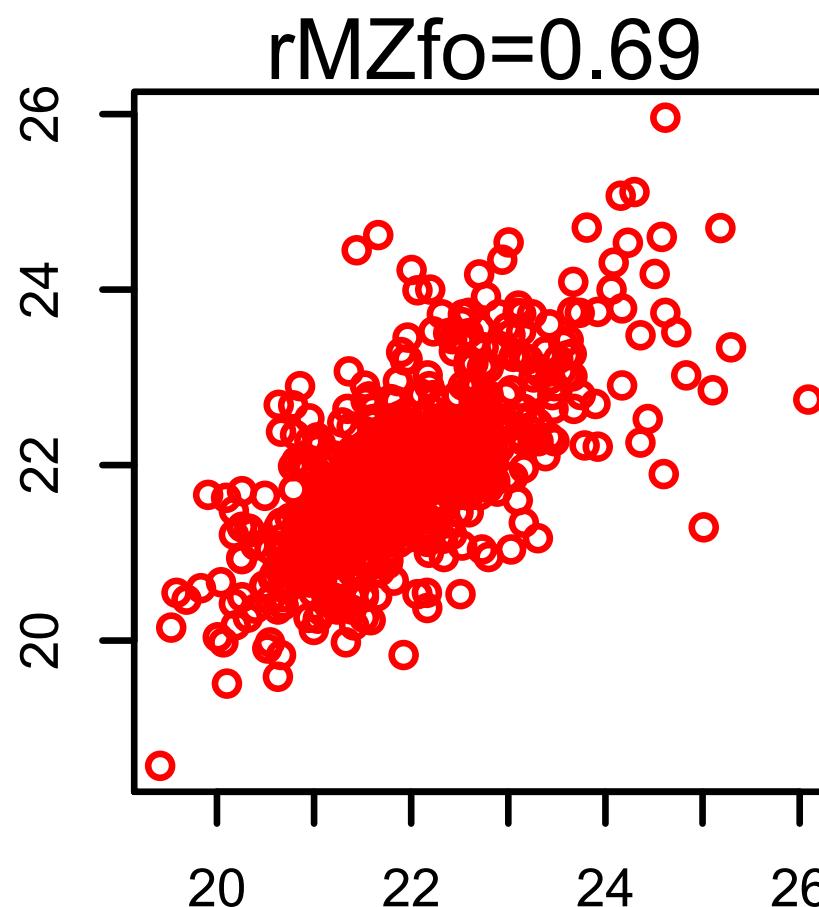
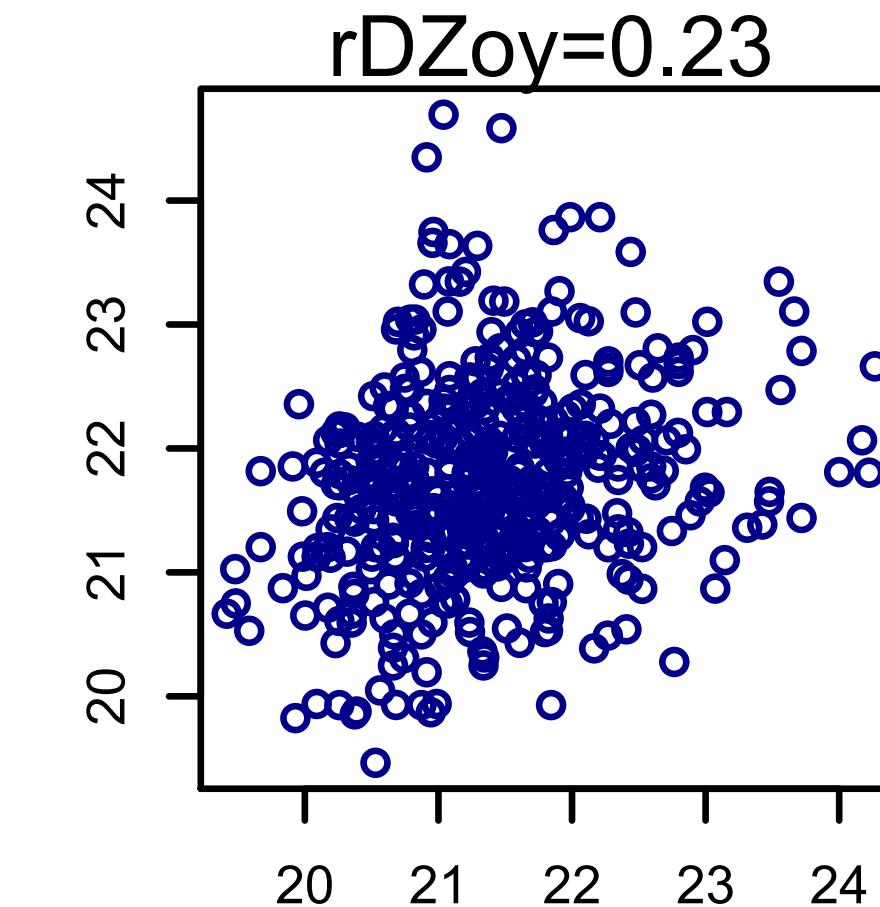
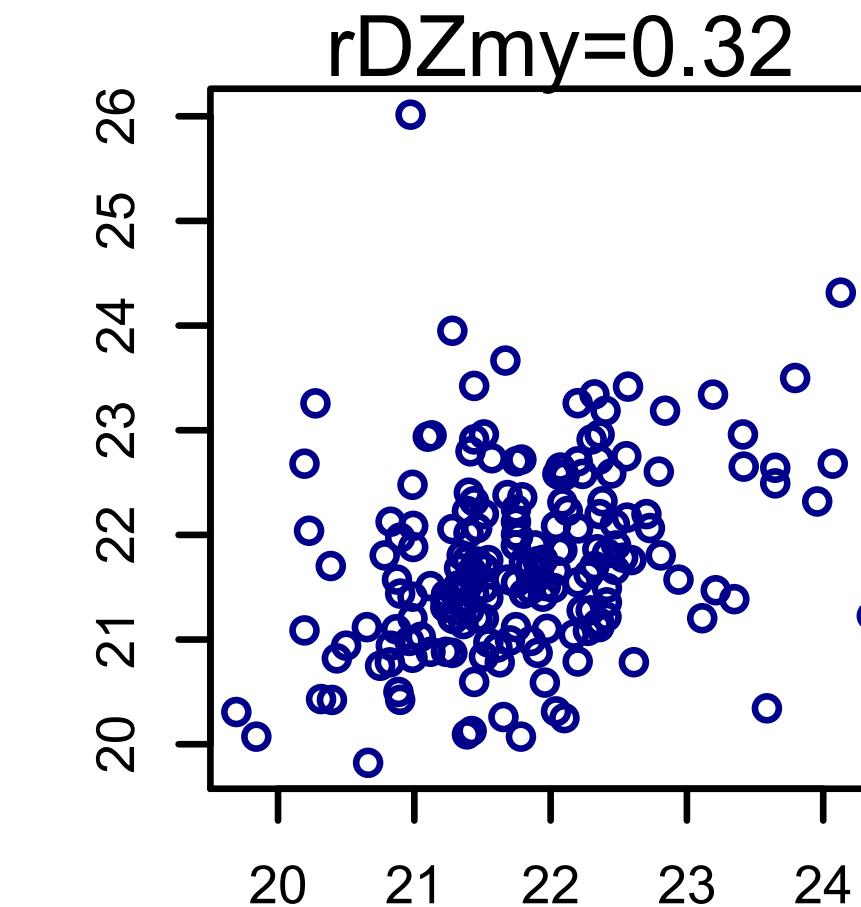
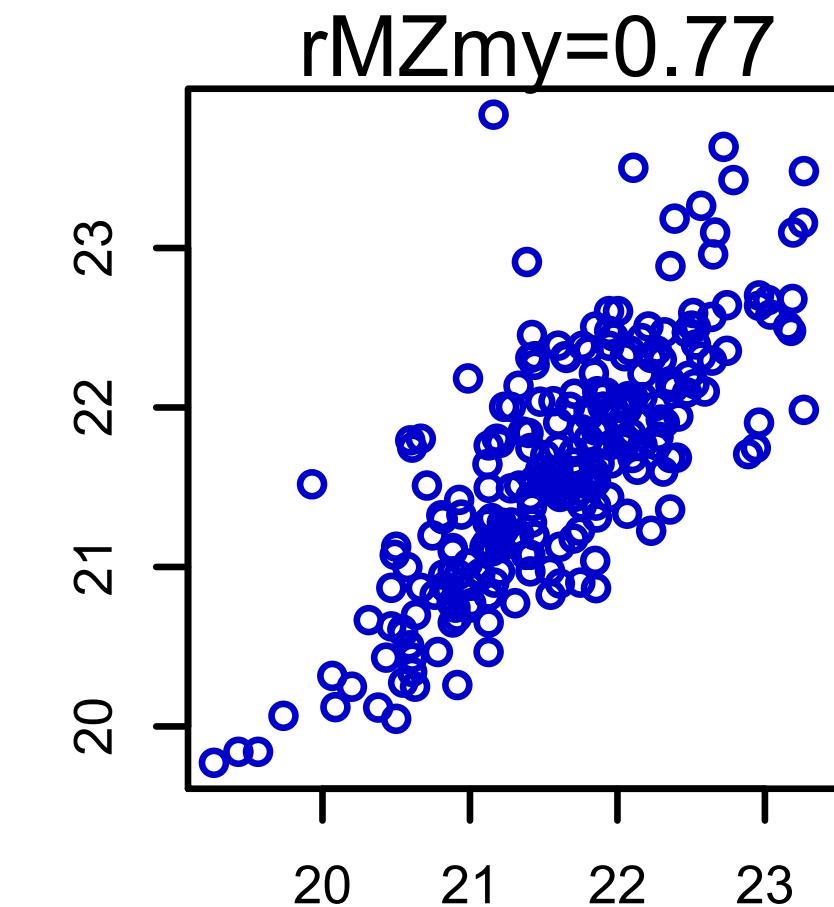
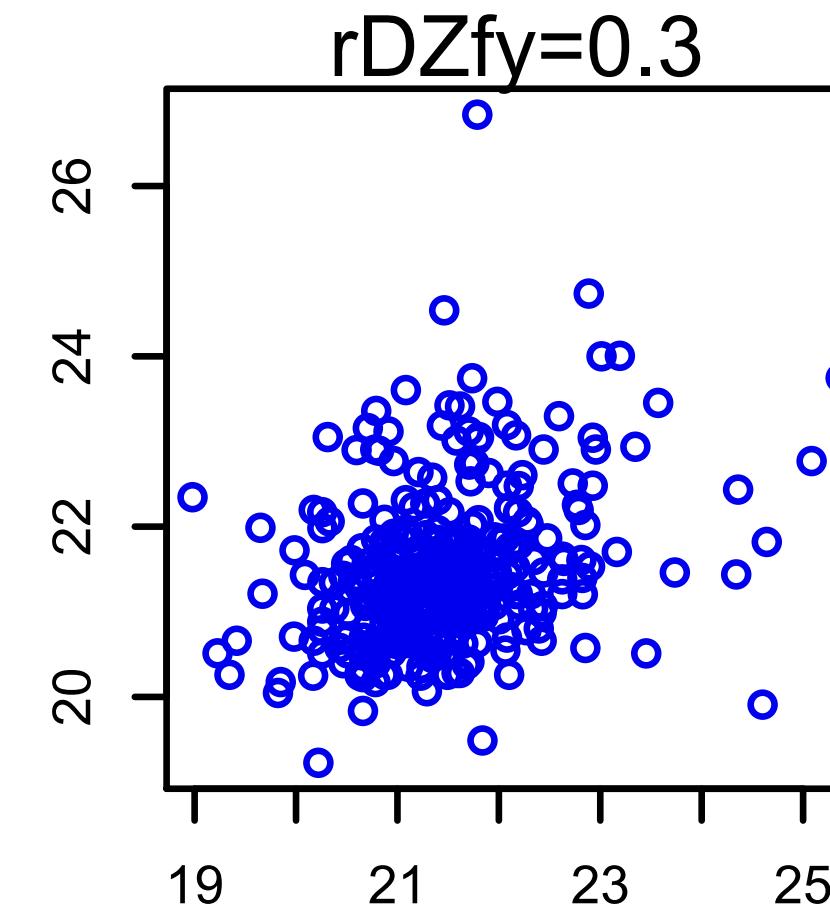
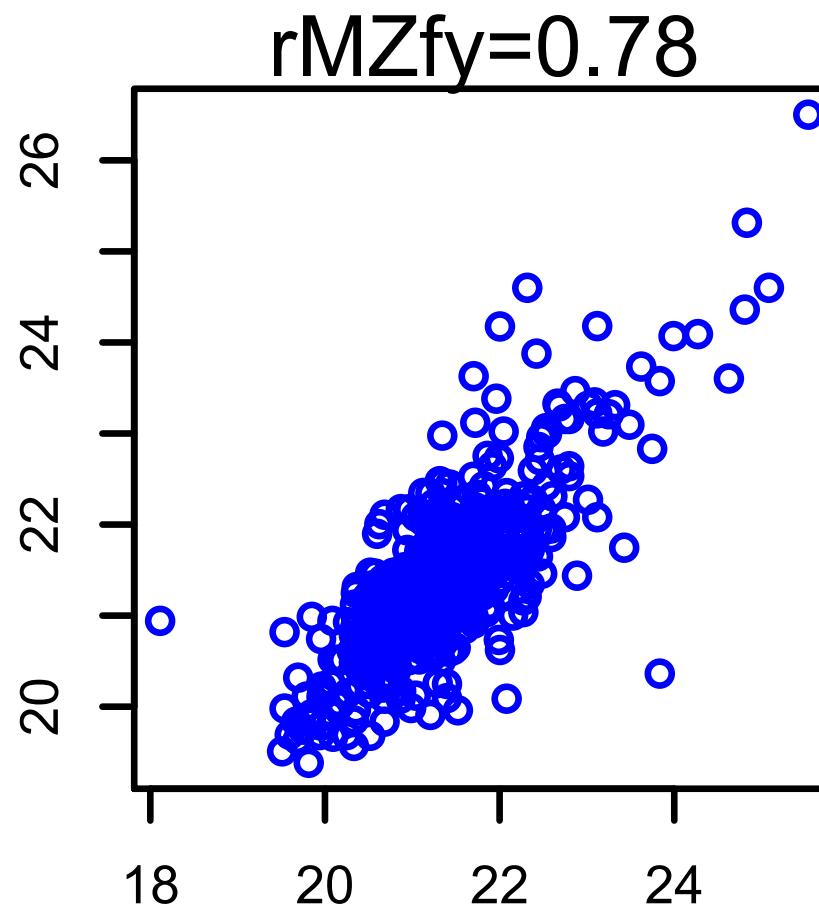
# From twin correlations to sources of variance



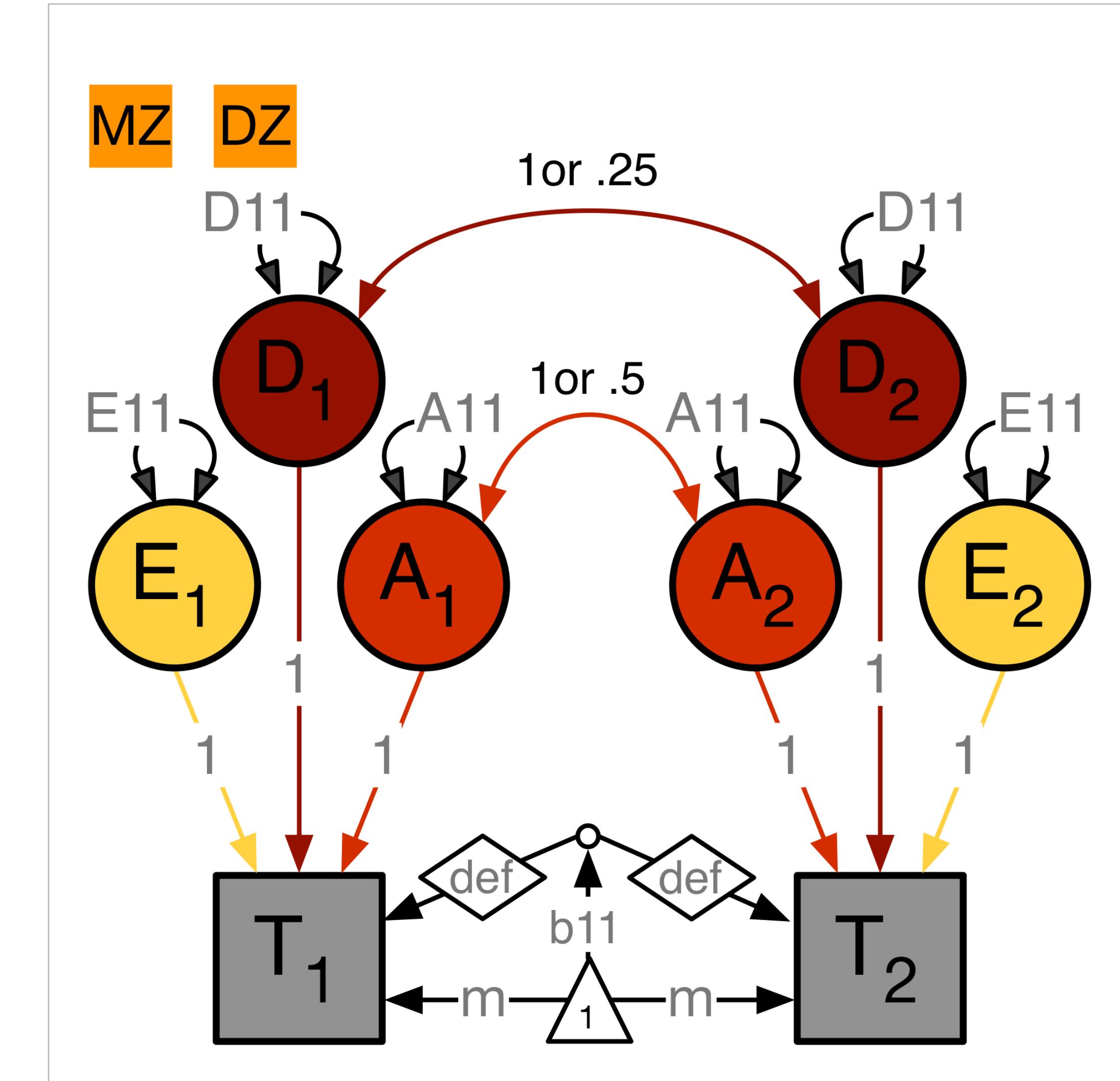
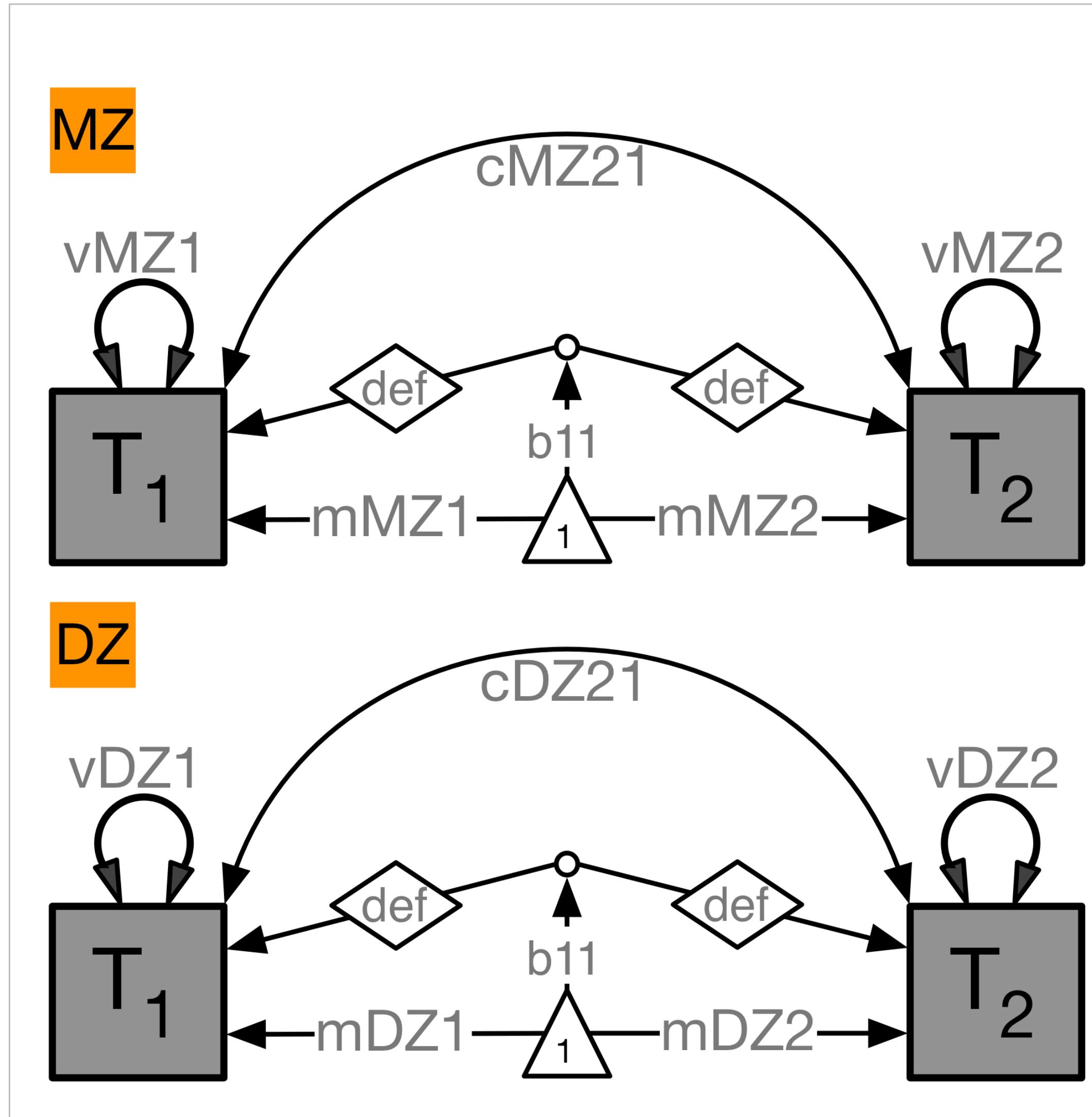
rMZ: monozygotic twin correlation; rDZ: dizygotic twin correlation;

A: additive genetic factors; E: unique environment; C: common environment; D: dominance

# Lindon: ‘Look at the bloody data’



# 2-group multi-group



# 2-group Females

Mx:oneSATc os=1775 ns=919 ep=11 co=0 df=1764 ll=4015.1185 cpu=0.1924 opt=NPSOL ver=2.19.5.1 stc=0

	b11	mMZ1	mMZ2	vMZ1	cMZ21	vMZ2	mDZ1	mDZ2	vDZ1	cDZ21	vDZ2
	2.7532	20.6888	20.6936	0.7214	0.5841	0.7843	20.7835	20.8082	0.7281	0.2415	0.8030

	base	comparison	ep	minus2LL	df	AIC	diffLL	diffdf	p	
1	oneSATc		<NA>	11	4015.1185	1764	4037.1185	NA	NA	NA
2	oneSATc	oneEM0c	9	4015.3493	1766	4033.3493	0.2308120	2	0.89100432	
3	oneSATc	oneEMV0c	7	4018.6107	1768	4032.6107	3.4922277	4	0.47906112	
4	oneSATc	oneEMVZc	5	4022.7885	1770	4032.7885	7.6700423	6	0.26328698	

	base	comparison	ep	minus2LL	df	AIC	diffLL	diffdf	p	
1	oneSATc		<NA>	11	4015.1185	1764	4037.1185	NA	NA	NA
2	oneSATc	oneADEvc	5	4022.7885	1770	4032.7885	7.6700423	6	0.26328698	

Mx:oneADEvc os=1775 ns=919 ep=5 co=0 df=1770 ll=4022.7885 cpu=0.324 opt=NPSOL ver=2.19.5.1 stc=0

	b11	mean	VA11	VD11	VE11
	2.7678	20.7346	0.3674	0.2250	0.1690
			1bound	estimate	ubound
oneADEvc.US[1,1]		0.0681	0.3674	0.6530	
oneADEvc.US[1,2]		-0.0459	0.2250	0.5244	
oneADEvc.US[1,3]		0.1503	0.1690	0.1910	

	base	comparison	ep	minus2LL	df	AIC	diffLL	diffdf	p	
1	oneADEvc		<NA>	5	4022.7885	1770	4032.7885	NA	NA	NA
2	oneADEvc	oneAEvc	4	4025.4093	1771	4033.4093	2.6208252	1	1.0546958e-01	
3	oneADEvc	oneEvc	3	4549.6129	1772	4555.6129	526.8243890	2	3.9951902e-115	

	VA	VD	VE	SA	SD	SE
US	0.3674	0.225	0.1690	0.4825	0.2956	0.2220
US	0.5976	0.000	0.1719	0.7766	0.0000	0.2234
US	0.0000	0.000	0.7598	0.0000	0.0000	1.0000

# 2-group Males

Mx:oneSATc os=910 ns=479 ep=11 co=0 df=899 ll=1883.7212 cpu=0.1162 opt=NPSOL ver=2.19.5.1 stc=0

	b11	mMZ1	mMZ2	vMZ1	cMZ21	vMZ2	mDZ1	mDZ2	vDZ1	cDZ21	vDZ2
	6.5889	20.0568	20.0704	0.5456	0.3924	0.5085	20.2591	20.2294	0.6485	0.1692	0.7148

	base	comparison	ep	minus2LL	df	AIC	diffLL	diffdf	p
1	oneSATc	<NA>	11	1883.7212	899	1905.7212	NA	NA	NA
2	oneSATc	oneEM0c	9	1884.0599	901	1902.0599	0.33875227	2	0.844191312
3	oneSATc	oneEMV0c	7	1885.2538	903	1899.2538	1.53260364	4	0.820850873
4	oneSATc	oneEMVZc	5	1899.3977	905	1909.3977	15.67656595	6	0.015599373

	base	comparison	ep	minus2LL	df	AIC	diffLL	diffdf	p
1	oneSATc	<NA>	11	1883.7212	899	1905.7212	NA	NA	NA
2	oneSATc	oneADEvc	5	1899.3977	905	1909.3977	15.676566	6	0.015599373

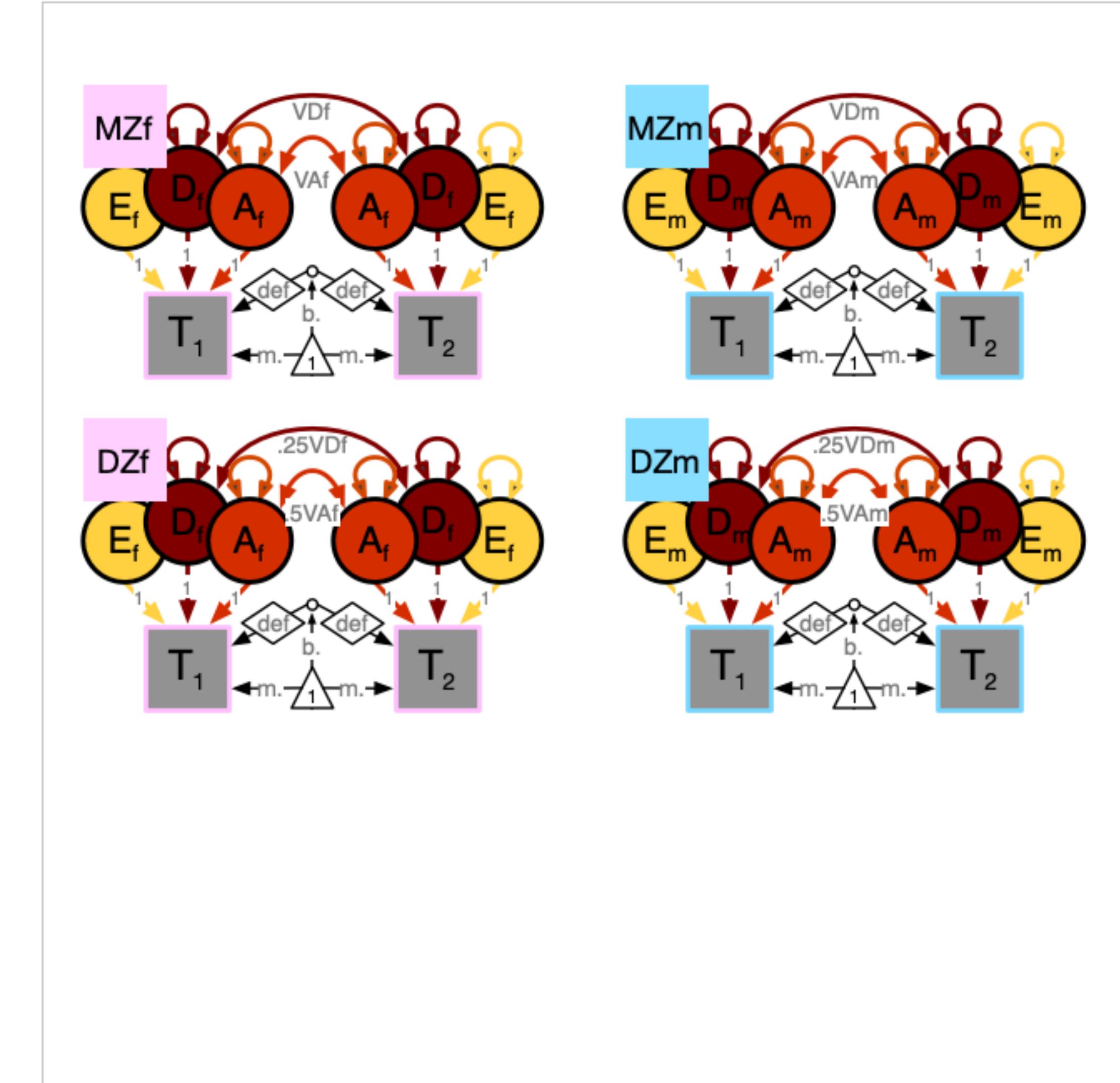
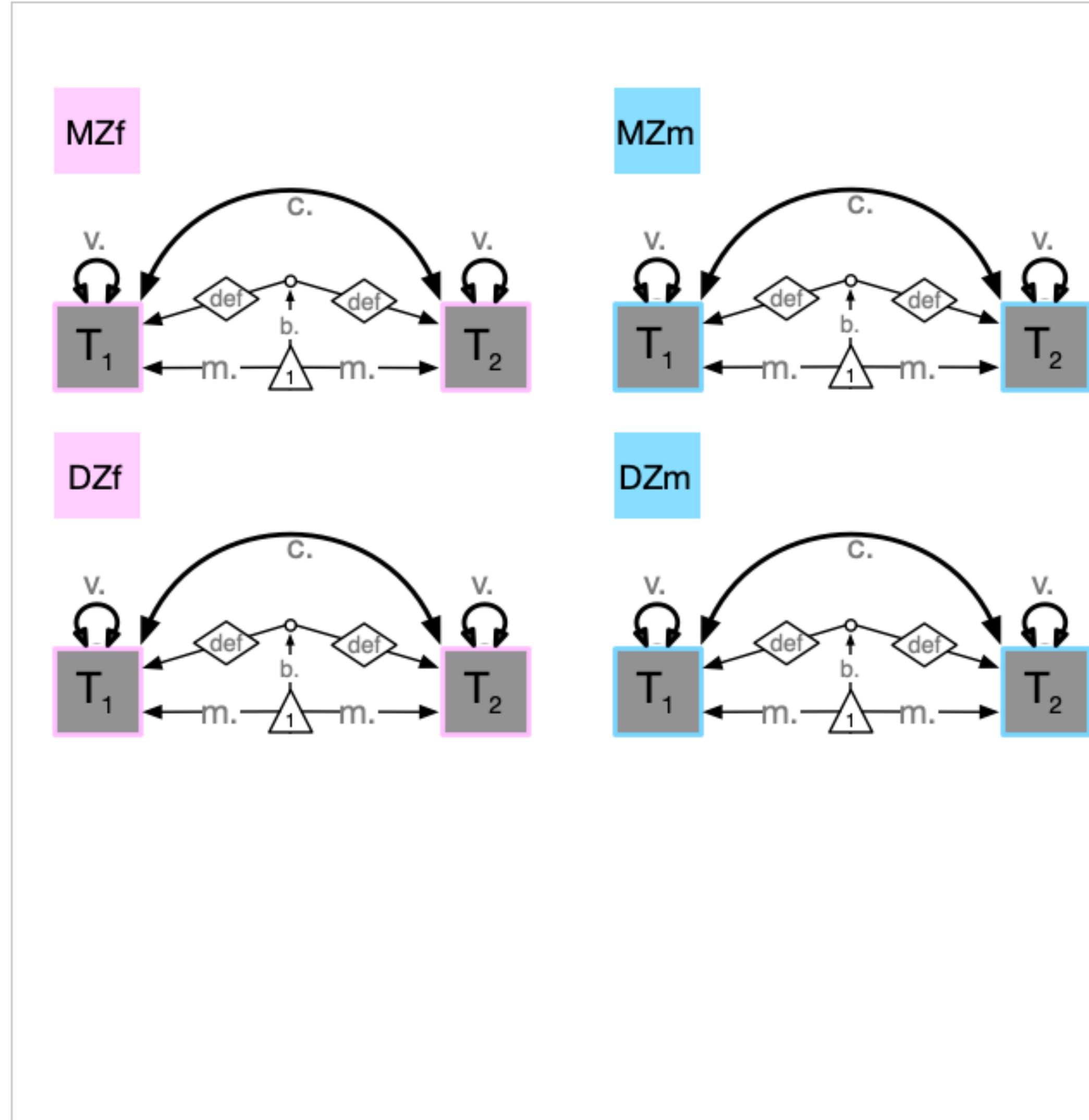
Mx:oneADEvc os=910 ns=479 ep=5 co=0 df=905 ll=1899.3977 cpu=0.2982 opt=NPSOL ver=2.19.5.1 stc=0

	b11	mean	VA11	VD11	VE11
	6.5518	20.1644	0.1032	0.3758	0.1375
			1bound	estimate	ubound
oneADEvc.US[1,1]	-0.2054		0.1032	0.3965	
oneADEvc.US[1,2]	0.0929		0.3758	0.6960	
oneADEvc.US[1,3]	0.1157		0.1375	0.1650	

	base	comparison	ep	minus2LL	df	AIC	diffLL	diffdf	p
1	oneADEvc	<NA>	5	1899.3977	905	1909.3977	NA	NA	NA
2	oneADEvc	oneAEvc	4	1906.4279	906	1914.4279	7.0302024	1	8.0146294e-03
3	oneADEvc	oneEvc	3	2121.8315	907	2127.8315	322.4337643	2	5.0017479e-49

	VA	VD	VE	SA	SD	SE
US	0.1032	0.3758	0.1375	0.1675	0.6096	0.2230
US	0.4854	0.0000	0.1446	0.7705	0.0000	0.2295
US	0.0000	0.0000	0.6028	0.0000	0.0000	1.0000

# 4-group multigroup - quantitative differences



# 4-group

```
Mx:oneSAT4ca os=2685 ns=1398 ep=22 co=0 df=2663 ll=5898.8396 cpu=12.175 opt=NPSOL ver=2.19.5.1 stc=0
bf11 bm11 mMZf1 mMZf2 vMZf1 cMZf21 vMZf2 mDZf1 mDZf2 vDZf1 cDZf21 vDZf2 mMZm1 mMZm2 vMZm1
2.7537 6.5888 20.6887 20.6935 0.7214 0.5841 0.7843 20.7834 20.8081 0.7281 0.2415 0.8030 20.0568 20.0704 0.5456
cMZm21 vMZm2 mDZm1 mDZm2 vDZm1 cDZm21 vDZm2
0.3924 0.5085 20.2591 20.2294 0.6486 0.1692 0.7148
```

```
Mx:oneADEq4vca os=2685 ns=1398 ep=10 co=0 df=2675 ll=5922.1862 cpu=1.6431 opt=NPSOL ver=2.19.5.1 stc=0
bf11 VAf11 VDf11 VEf11 bm11 VAm11 VDm11 VEm11 mZf mZm
2.7677 0.3674 0.2250 0.1690 6.5515 0.1033 0.3758 0.1375 20.7346 20.1645
```

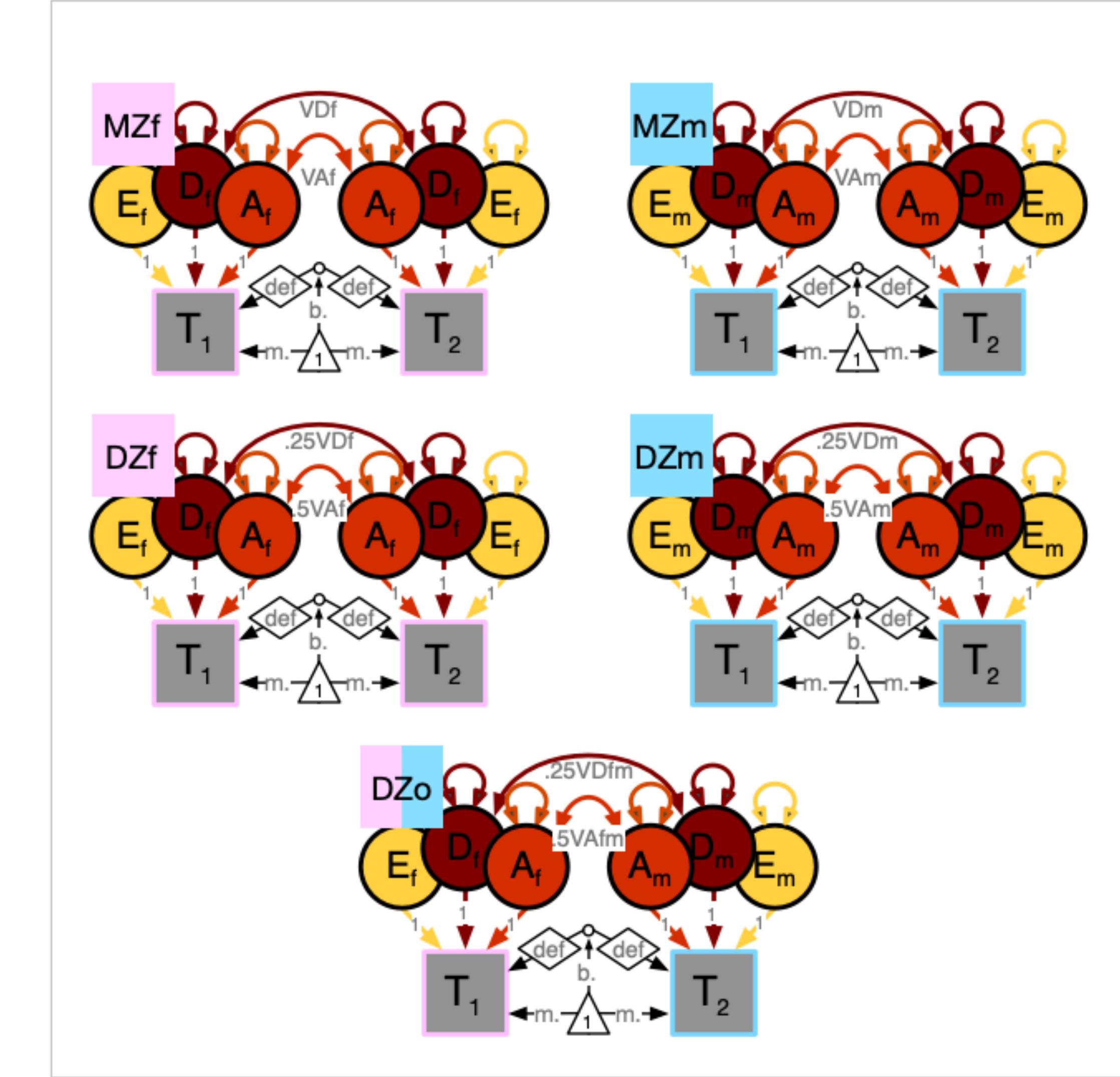
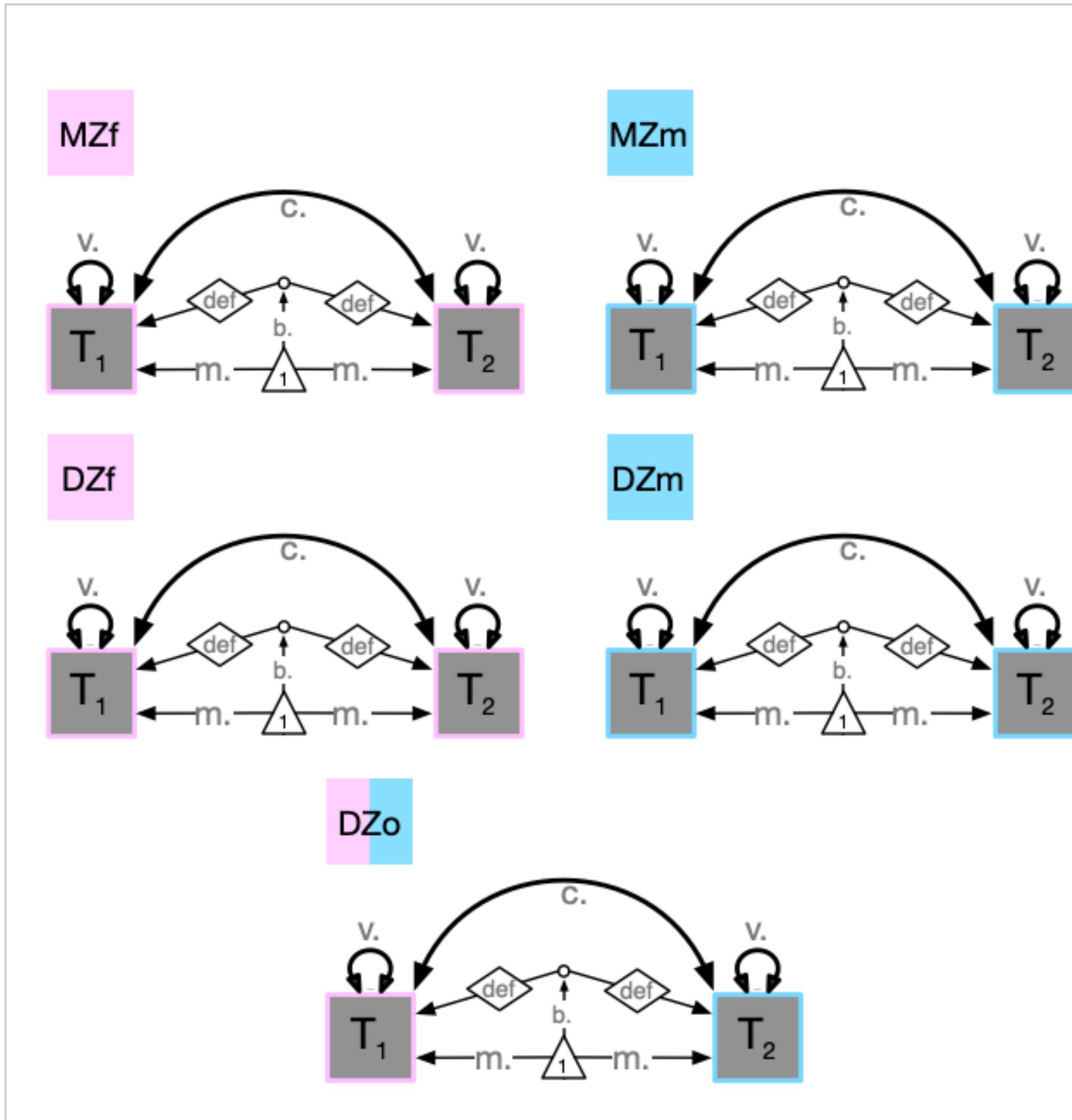
	1bound	estimate	ubound
oneADEq4vca.US[1,1]	0.0681	0.3674	0.6530
oneADEq4vca.US[1,2]	-0.0459	0.2250	0.5244
oneADEq4vca.US[1,3]	0.1503	0.1690	0.1910
oneADEq4vca.US[1,7]	-0.2054	0.1033	0.3965
oneADEq4vca.US[1,8]	0.0929	0.3758	0.6960
oneADEq4vca.US[1,9]	0.1157	0.1375	0.1650

```
Mx:oneADE4vca os=2685 ns=1398 ep=7 co=0 df=2678 ll=5935.0138 cpu=1.3796 opt=NPSOL ver=2.19.5.1 stc=0
bf11 VA11 VD11 VE11 bm11 mZf mZm
2.7736 0.2664 0.2861 0.1588 6.5383 20.7337 20.1656
```

	base	comparison	ep	minus2LL	df	AIC	diffLL	diffdf	p	
1	oneADEq4vca		<NA>	10	5922.1862	2675	5942.1862	NA	NA	NA
2	oneADEq4vca	oneADE4vca	7	5935.0138	2678	5949.0138	12.827571	3	0.0050247266	

	VAf	VDf	VEf	SAf	SDf	SEf	VAm	VDm	VEm	SAm	SDm	SEm
US	0.3674	0.2250	0.1690	0.4825	0.2956	0.2220	0.1033	0.3758	0.1375	0.1675	0.6096	0.2230
US	0.2664	0.2861	0.1588	0.3745	0.4022	0.2233	0.2664	0.2861	0.1588	0.3745	0.4022	0.2233

# 5-group multigroup qualitative & quantitative



# 5-group

Mx:oneADEra5vca os=3654 ns=1907 ep=11 co=0 df=3643 ll=8219.2209 cpu=3.8965 opt=NPSOL ver=2.19.5.1 stc=0

bf11	VAf11	VDf11	VEf11	bm11	VAm11	VDm11	VEm11	VAms11	mZf	mZm
2.7703	0.3389	0.2307	0.1683	5.5588	0.0771	0.3792	0.1373	0.0209	20.7291	20.4263

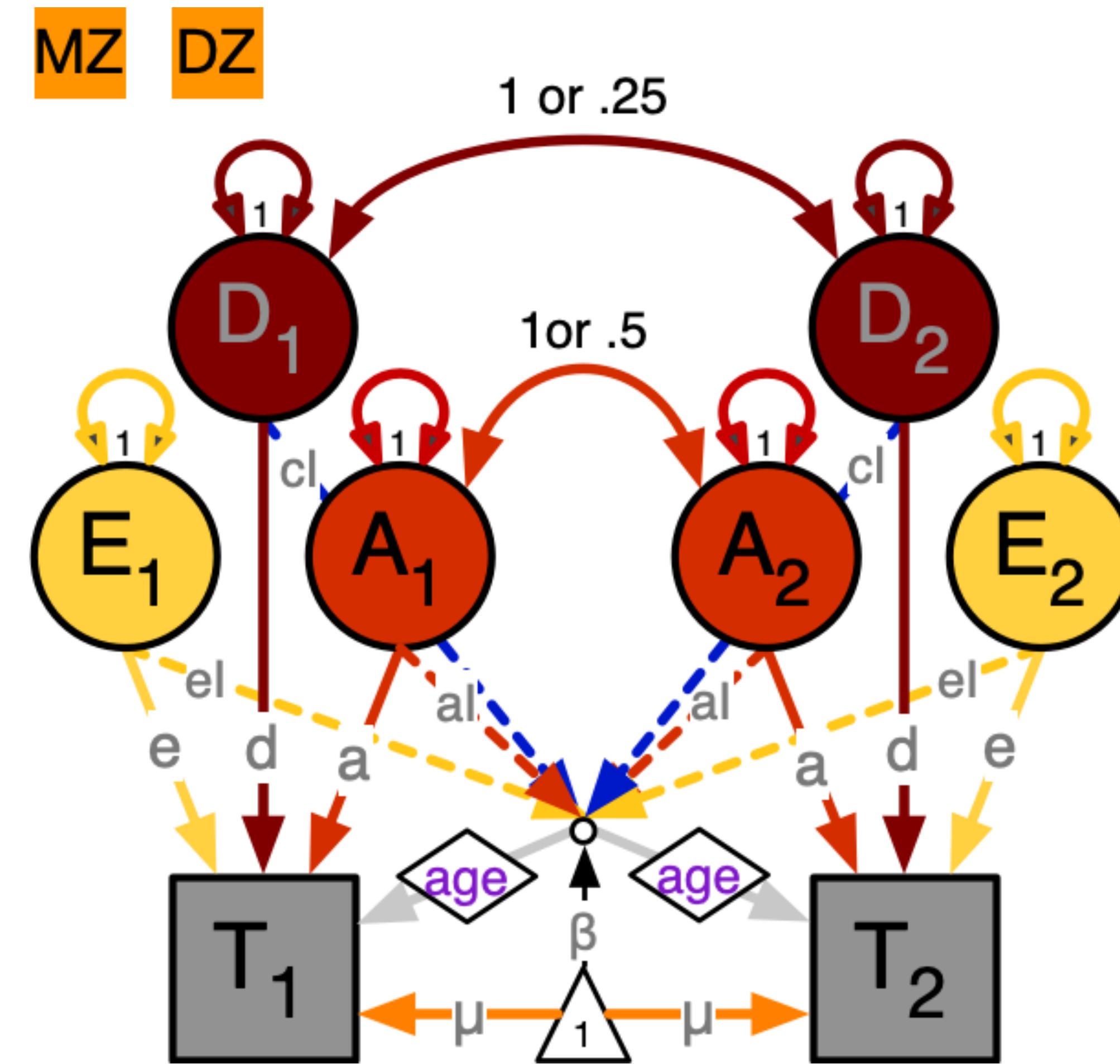
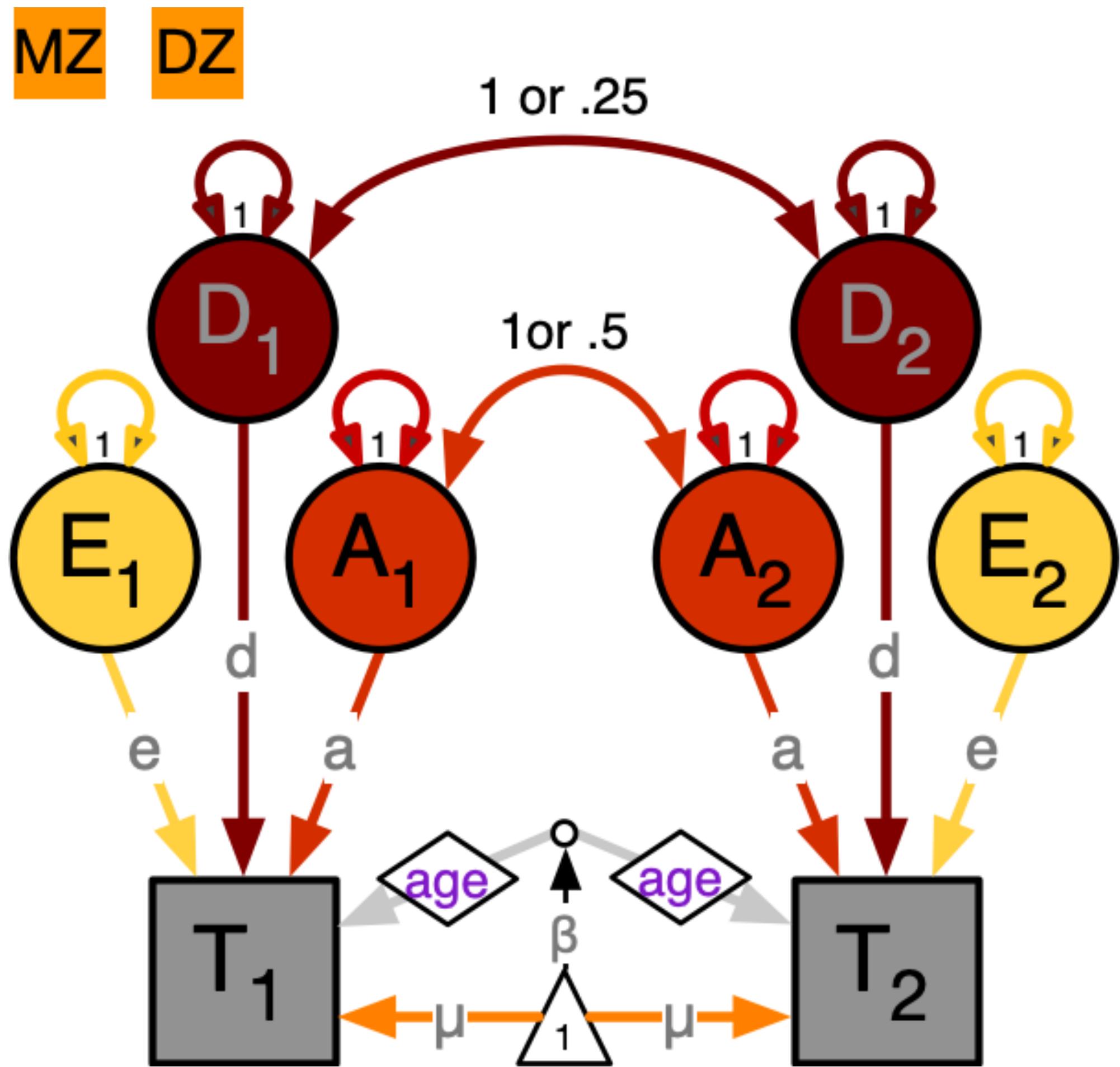
	lbound	estimate	ubound
oneADEra5vca.US[1,1]	NA	0.3389	NA
oneADEra5vca.US[1,2]	0.0000	0.2307	NA
oneADEra5vca.US[1,3]	0.1497	0.1683	0.1900
oneADEra5vca.US[1,7]	NA	0.0980	0.3838
oneADEra5vca.US[1,8]	0.0977	0.3792	NA
oneADEra5vca.US[1,9]	0.1156	0.1373	0.1647
oneADEra5vca.US[1,13]	NA	0.8872	NA
oneADEra5vca.US[1,14]	1.0000	1.0000	1.0000

base	comparison	ep	minus2LL	df	AIC	diffLL	diffdf	p
1 oneADEra5vca		<NA>	11	8219.2209	3643	8241.2209	NA	NA
2 oneADEra5vca	oneADErd5vca	11	8219.2209	3643	8241.2209	-2.1827691e-07	0	NA
3 oneADEra5vca	oneADEq5vca	10	8219.2444	3644	8239.2444	2.3513966e-02	1	0.8781280748
4 oneADEra5vca	oneADE5vca	7	8234.7876	3647	8248.7876	1.5566673e+01	4	0.0036593125

	VAf	VDf	VEf	SAf	SDf	SEf	VAm	VDm	VEm	SAm	SDm	SEm	rg	rd
US oneADEra5vca	0.3389	0.2307	0.1683	0.4592	0.3127	0.2281	0.098	0.3792	0.1373	0.1595	0.6171	0.2234	0.8872	1
US oneADErd5vca	0.3389	0.2307	0.1683	0.4592	0.3127	0.2281	0.098	0.3792	0.1373	0.1594	0.6172	0.2234	1	0.8613
US oneADEq5vca	0.3409	0.2288	0.1683	0.4619	0.31	0.2281	0.0772	0.4	0.1371	0.1256	0.6512	0.2232	1	1
US oneADE5vca	0.1909	0.3405	0.1581	0.2769	0.4938	0.2293	0.1909	0.3405	0.1581	0.2769	0.4938	0.2293	1	1

# 2-group interaction x age

## means & covariances



# 2-group age x ADE interaction

## Females

```
Mx:oneADEcaI os=3853 ns=1982 ep=9 co=0 df=3844 ll=9316.5572 cpu=51.7077 opt=NPSOL ver=2.19.5.1 stc=1
```

	111	q11	meanbmi	a11	d11	e11	aI11	dI11	eI11
	3.8278	-2.0909	20.5932	0.6546	-0.2978	-0.2931	-0.0352	-0.3819	-0.5442

	agevalues	MI	AI	DI	EI	VI
Age15	15	21.120358	0.42166997	0.12607553	0.14044989	0.68819539
Age25	25	21.419506	0.41710835	0.15465499	0.18419682	0.75596015
Age35	35	21.676836	0.41257153	0.18615151	0.23386577	0.83258881
Age45	45	21.892349	0.40805952	0.22056512	0.28945673	0.91808137
Age55	55	22.066046	0.40357232	0.25789580	0.35096970	1.01243782
Age65	65	22.197924	0.39910993	0.29814356	0.41840468	1.11565817
Age75	75	22.287986	0.39467235	0.34130839	0.49176167	1.22774241

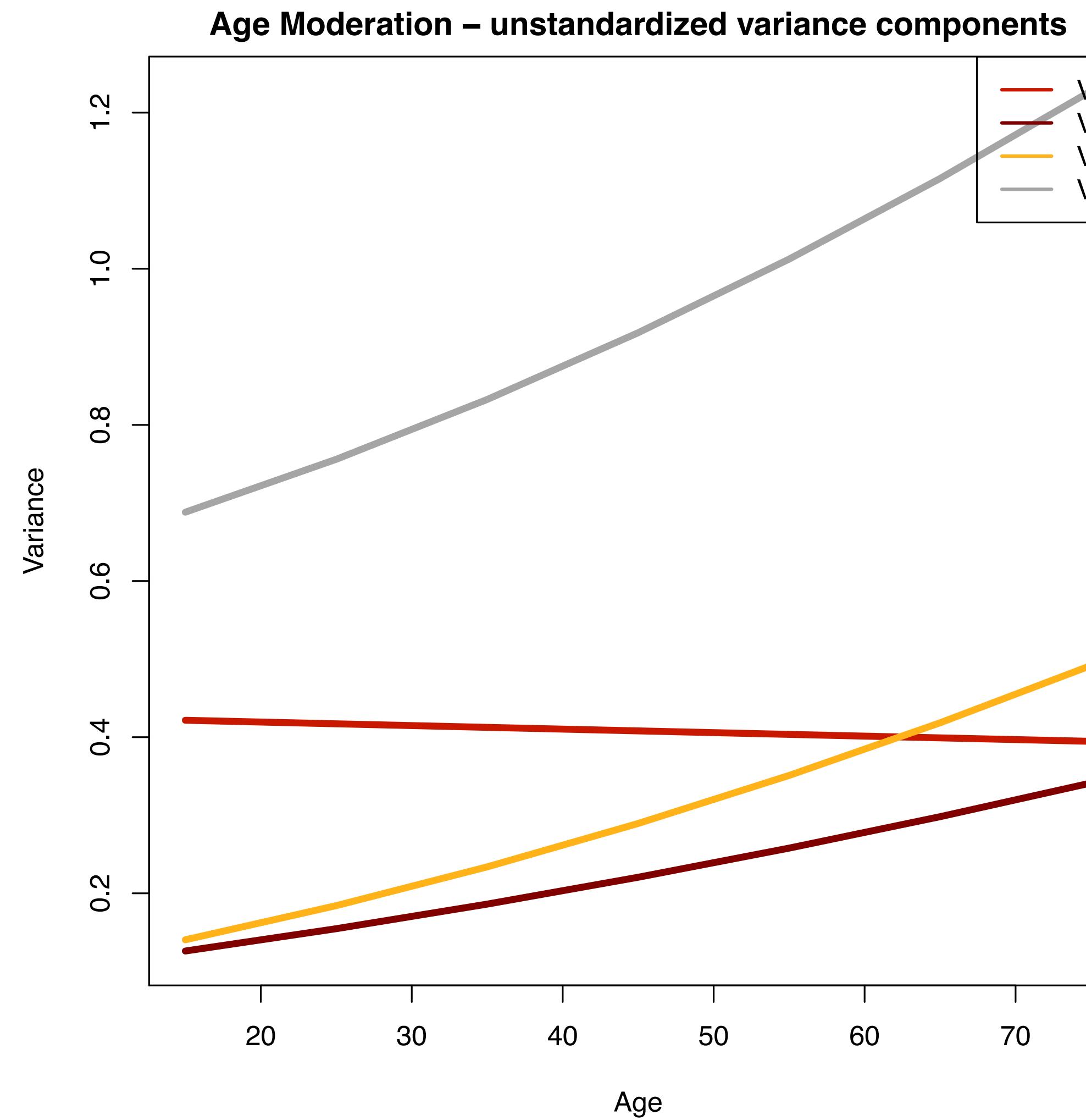
```
> fitADE1qi$SxAge$result
```

	agevalues	MI	AI	DI	EI	VI
Age15	15	21.120358	0.61271839	0.18319729	0.20408432	0.68819539
Age25	25	21.419506	0.55175970	0.20458087	0.24365943	0.75596015
Age35	35	21.676836	0.49552855	0.22358157	0.28088988	0.83258881
Age45	45	21.892349	0.44446989	0.24024572	0.31528439	0.91808137
Age55	55	22.066046	0.39861443	0.25472754	0.34665803	1.01243782
Age65	65	22.197924	0.35773496	0.26723558	0.37502946	1.11565817
Age75	75	22.287986	0.32146185	0.27799674	0.40054141	1.22774241

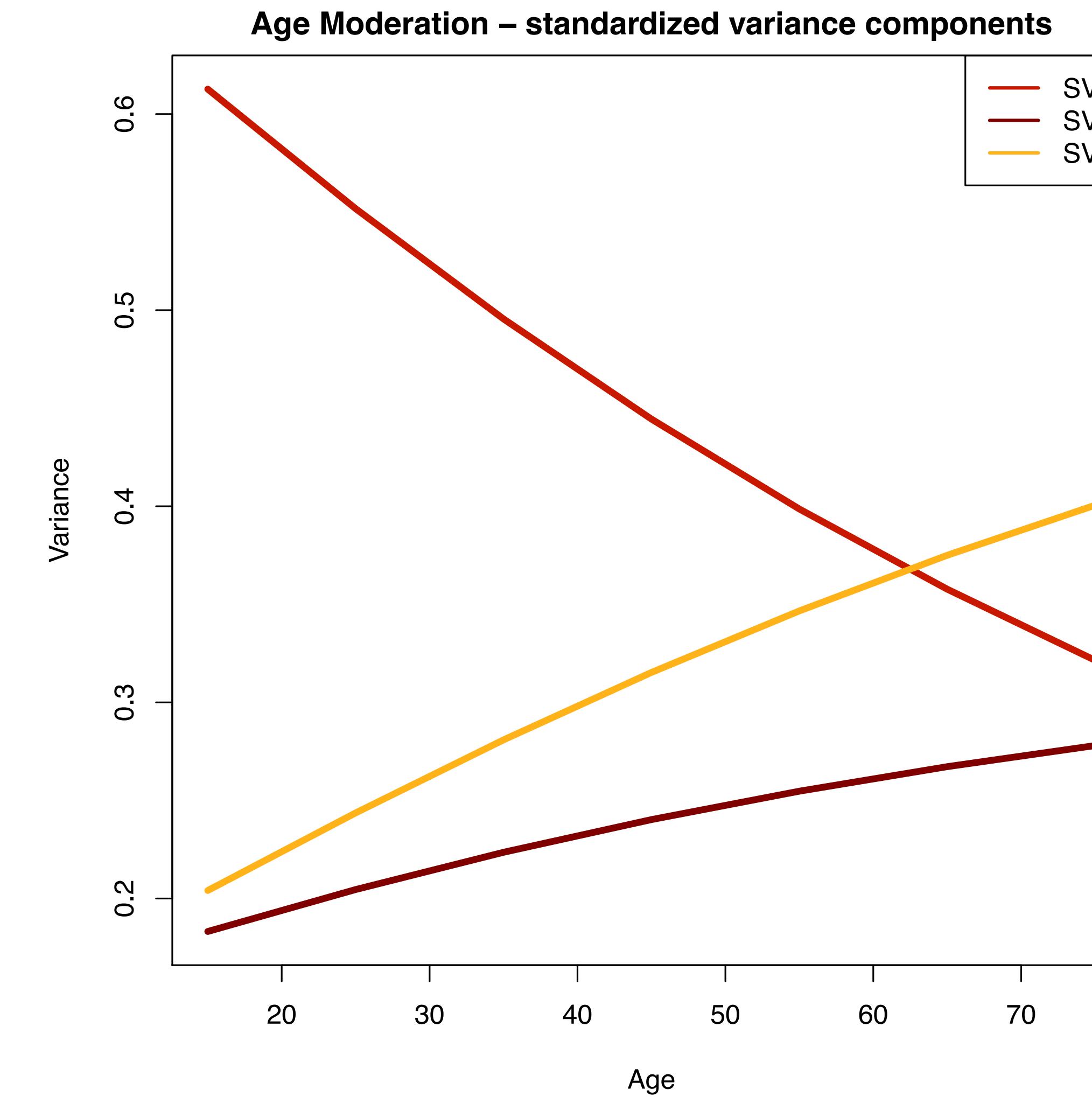
	base	comparison	ep	minus2LL	df	AIC	diffLL	diffdf	p
1	oneADEcaI	<NA>	9	9316.5572	3844	9334.5572	NA	NA	NA
2	oneADEcaI	oneADEcaI2	6	9382.8512	3847	9394.8512	66.2940104	3	2.6517015e-14
3	oneADEcaI	oneADEcaI3	8	9322.6262	3845	9338.6262	6.0690704	1	1.3756934e-02
4	oneADEcaI	oneADEcaI4	7	9551.0273	3846	9565.0273	234.4701786	2	1.2174402e-51

# Age x ADE interaction

## Unstandardized



## Standardized



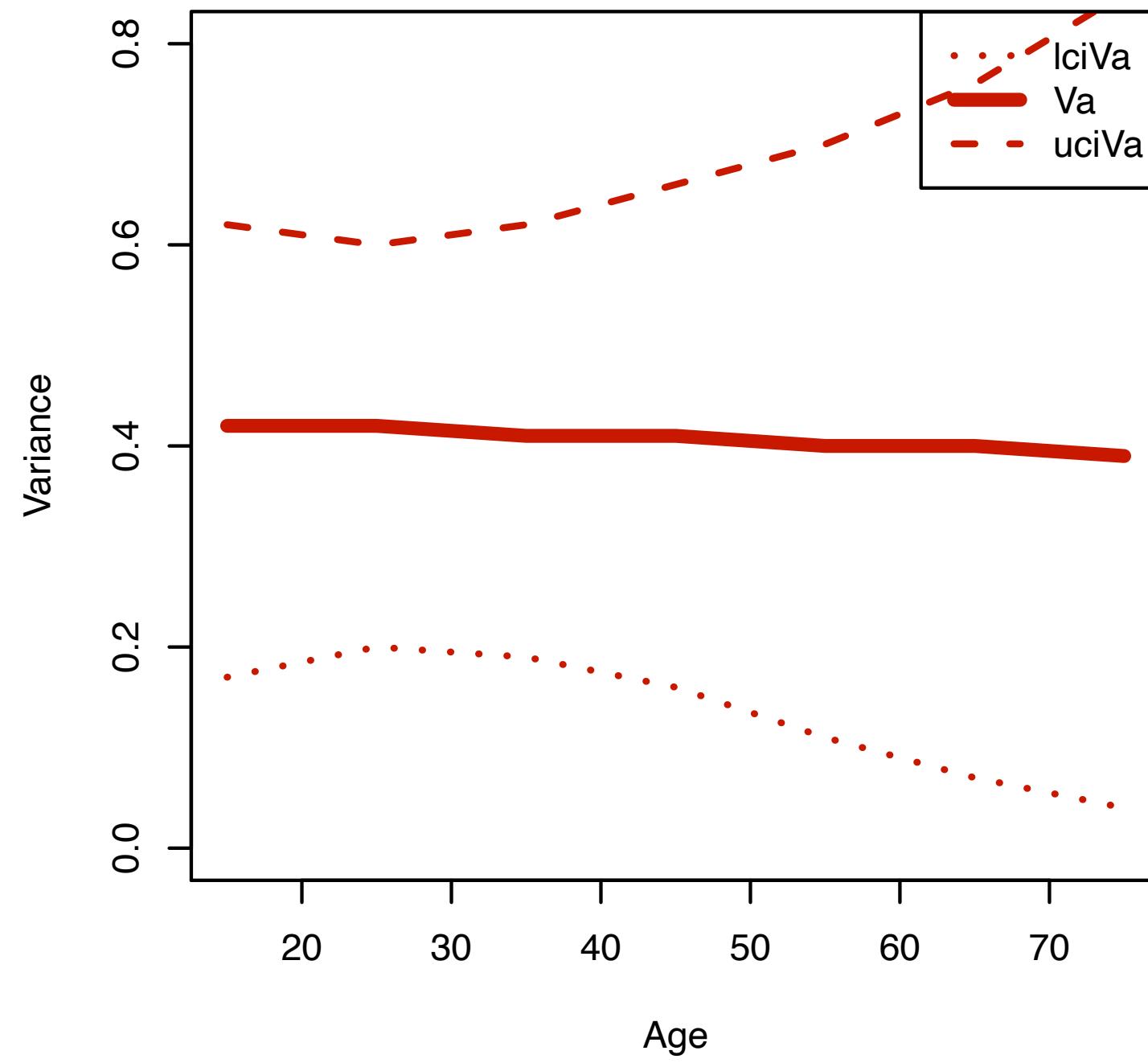
# Age x ADE interaction

VA

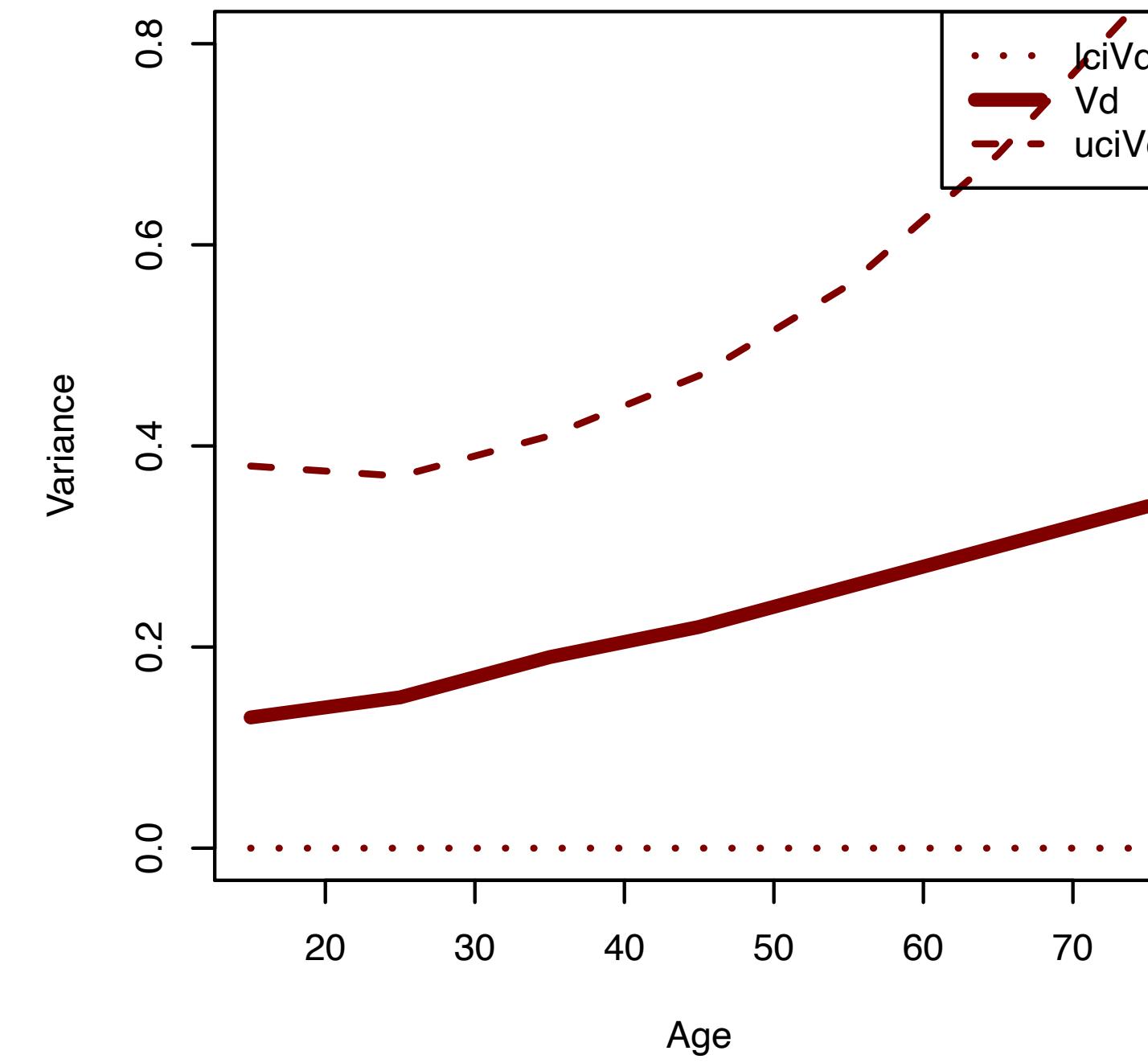
VD

VE

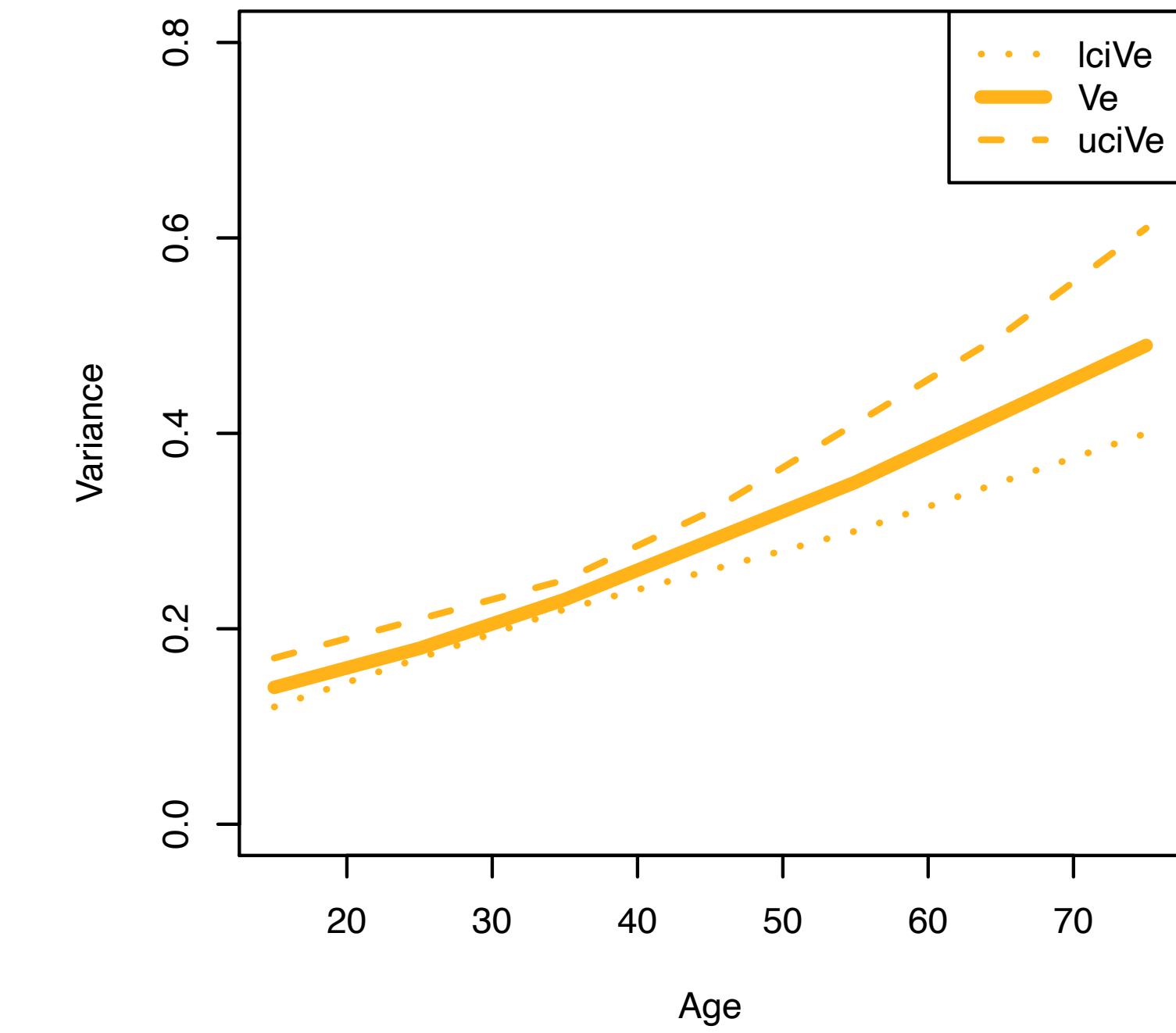
Unstandardized VA variance components +CIs



Unstandardized VD variance components +CIs



Unstandardized VE variance components +CIs



<https://hermine-maes.squarespace.com/pubs/>

The screenshot displays the OpenMx website interface. At the top right are links for "HOME" and "OPENMX". Below this, there are eight boxes representing different modeling approaches:

- HELP**: genetic epidemiology helper functions
- ONE**: classical twin study MZ & DZ twins  
ONE phenotype continuous/binary/ordinal  
SAT | ACE | ADE
- ONEA**: classical twin study MZ & DZ twins  
ONE phenotype continuous/binary/ordinal +covariate age  
SAT | ACE | ADE
- ONEA5**: classical twin study MZ & DZ twins  
ONE phenotype continuous/binary/ordinal +covariate age heterogeneity  
SAT | ACE | ADE
- PUBS**: A thumbnail image of a scientific publication titled "MODEL-FITTING APPROACHES TO THE ANALYSIS OF HUMAN BEHAVIOUR" by L.J. Eaves, KAYTEYN A. LAST, P. A. YOUNG, and N. G. MARTIN. It discusses classical twin studies and beyond.
- ONEA7**: classical twin study twins+ sibling+ genomic relatedness  
ONE phenotype continuous +covariate  
ACE
- TWO**: classical twin study MZ & DZ twins  
TWO phenotypes continuous/binary/ordinal  
SAT | ACE | ADE
- TWO+**: classical twin study MZ & DZ twins  
TWO phenotypes continuous  
biv25

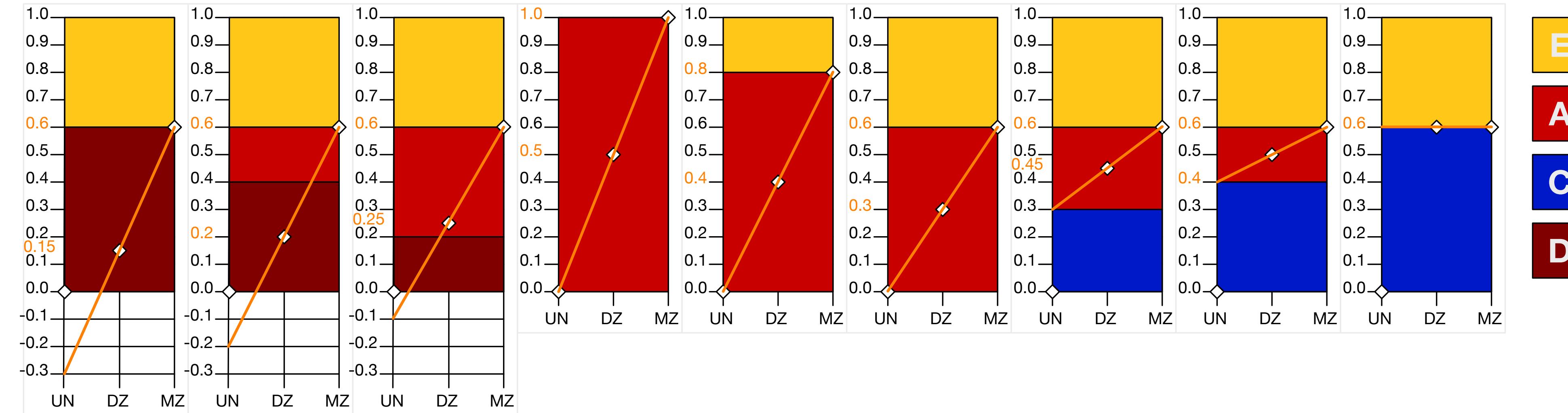
Below the boxes, there is a screenshot of a YouTube channel page for "OpenMx" titled "Beginner's Guide to SEM - Matrix Specification". The channel has 11 videos and 2,413 views. To the right of the channel is a black and white portrait of Lindon J. Eaves, Ph.D., M.A., (Oxon), D.Sc.

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OPENMX

Lindon J. Eaves, Ph.D., M.A., (Oxon), D.Sc.

# From twin correlations to sources of variance



$$D = -2*C$$

$$A' = A + 3C$$

$$C = -1/2*D$$

$$A' = A + 3/2D$$

rMZ: monozygotic twin correlation; rDZ: dizygotic twin correlation;  
 A: additive genetic factors; E: unique environment; C: common environment; D: dominance