

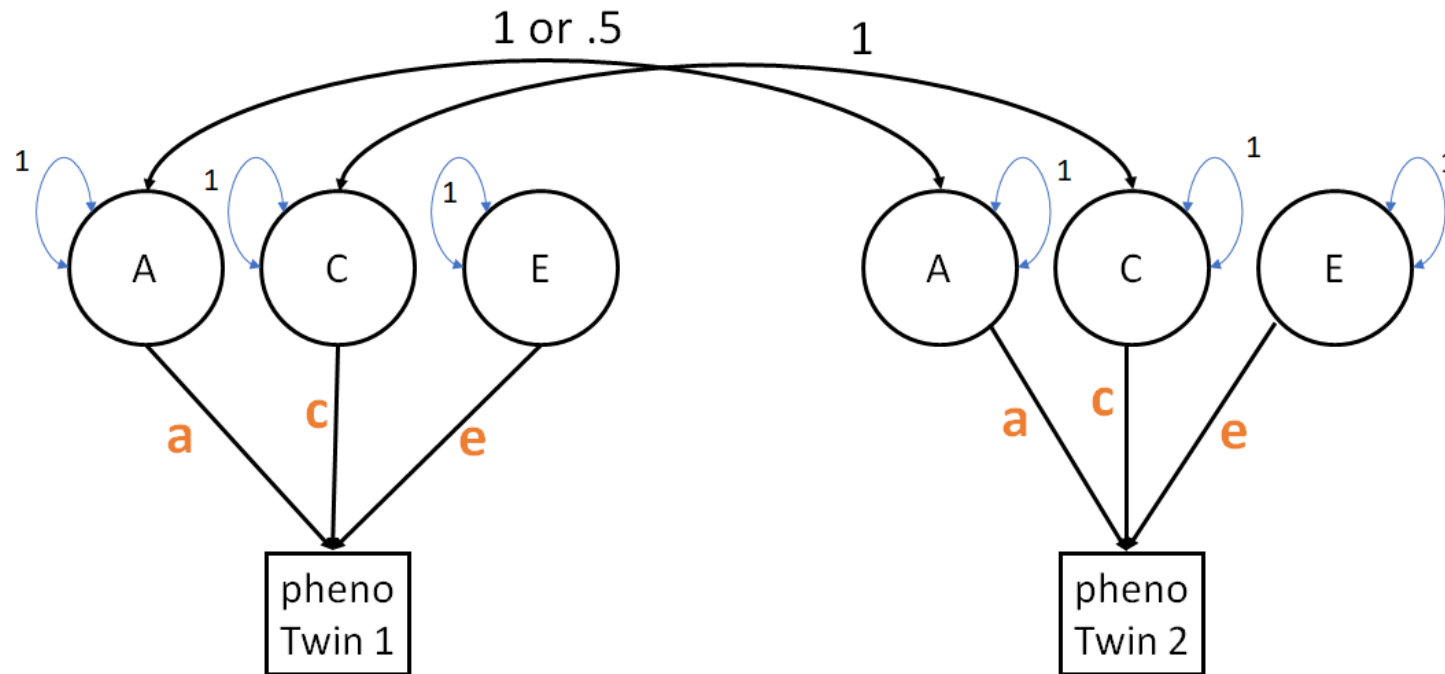
Introducing polygenic risk scores into the twin design

Estimating r_{AC}
and
MR twin model

Conor V. Dolan

Boulder 2020





What is $\text{var}(A)$ again ? (in a simple model assuming linkage equilibrium)

(in a simple model assuming linkage equilibrium)

$$\text{pheno}_i = \mathbf{a}_0 + \mathbf{a}_1 * \text{QTL}_{A1i} + \mathbf{a}_2 * \text{QTL}_{A2i} + \dots + \mathbf{a}_N * \text{QTL}_{ANi} + e_i$$
$$\text{var}(A) = \mathbf{s}^2_{Ph_QTL(A)} = \mathbf{a}_1^2 * \mathbf{s}^2_{QTLA1} + \mathbf{a}_2^2 * \mathbf{s}^2_{QTLA2} + \dots + \mathbf{a}_N^2 * \mathbf{s}^2_{QTLAN}$$

Suppose you have measured QTL₁ to QTL₁₀ and
you have reliable GWAS estimates of \mathbf{a}_1 to \mathbf{a}_{10} : $\hat{\mathbf{a}}_1$ to $\hat{\mathbf{a}}_{10}$

$$\text{PRS}_i = \hat{\mathbf{a}}_1 * \text{QTL}_{A1i} + \hat{\mathbf{a}}_2 * \text{QTL}_{A2i} + \dots + \hat{\mathbf{a}}_{10} * \text{QTL}_{ANi}$$

$$\text{var}(A^*) = \mathbf{a}_{11}^2 * \mathbf{s}^2_{QTLA11} + \mathbf{a}_{12}^2 * \mathbf{s}^2_{QTLA12} + \dots + \mathbf{a}_N^2 * \mathbf{s}^2_{QTLAN}$$

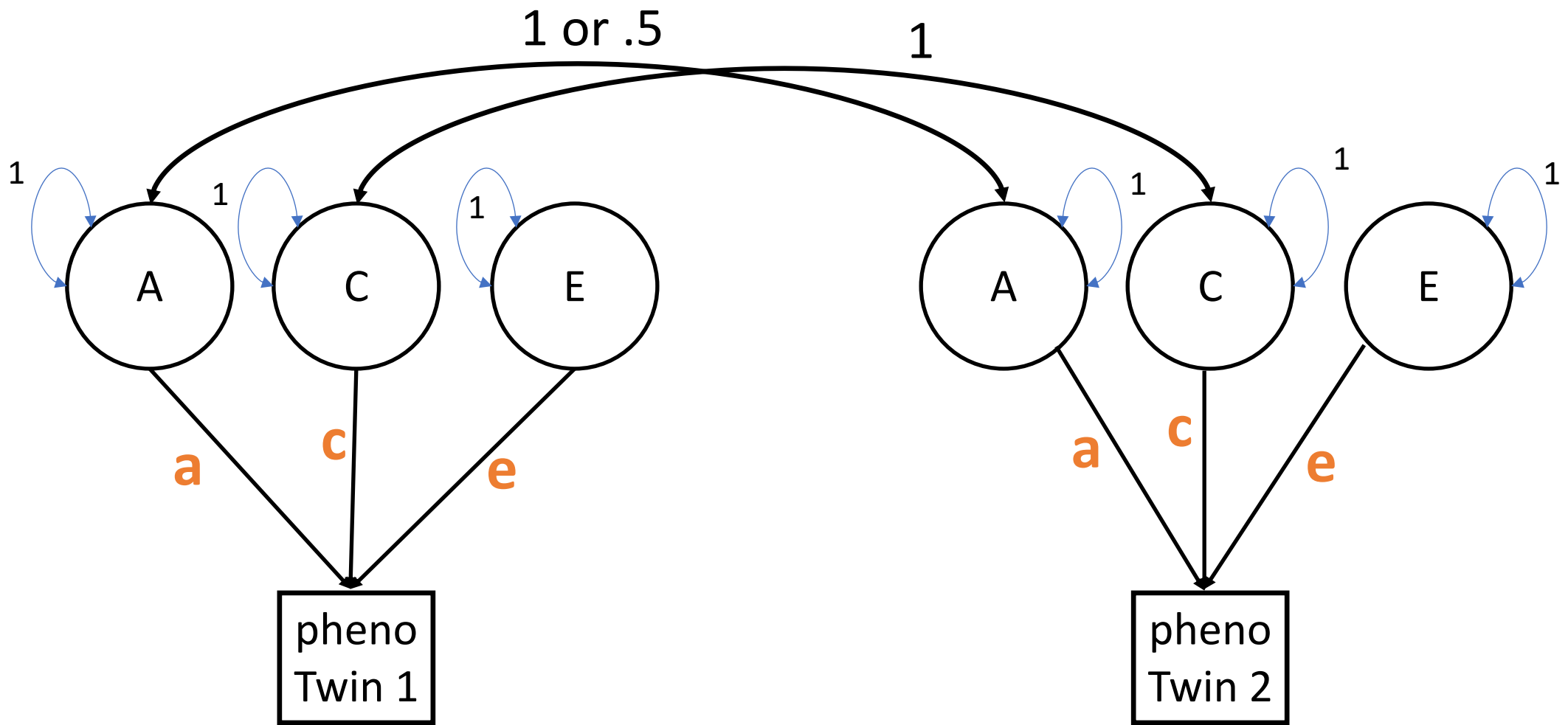
$$\text{var}(A) = \text{var}(P) + \text{var}(A^*)$$

1 to 10

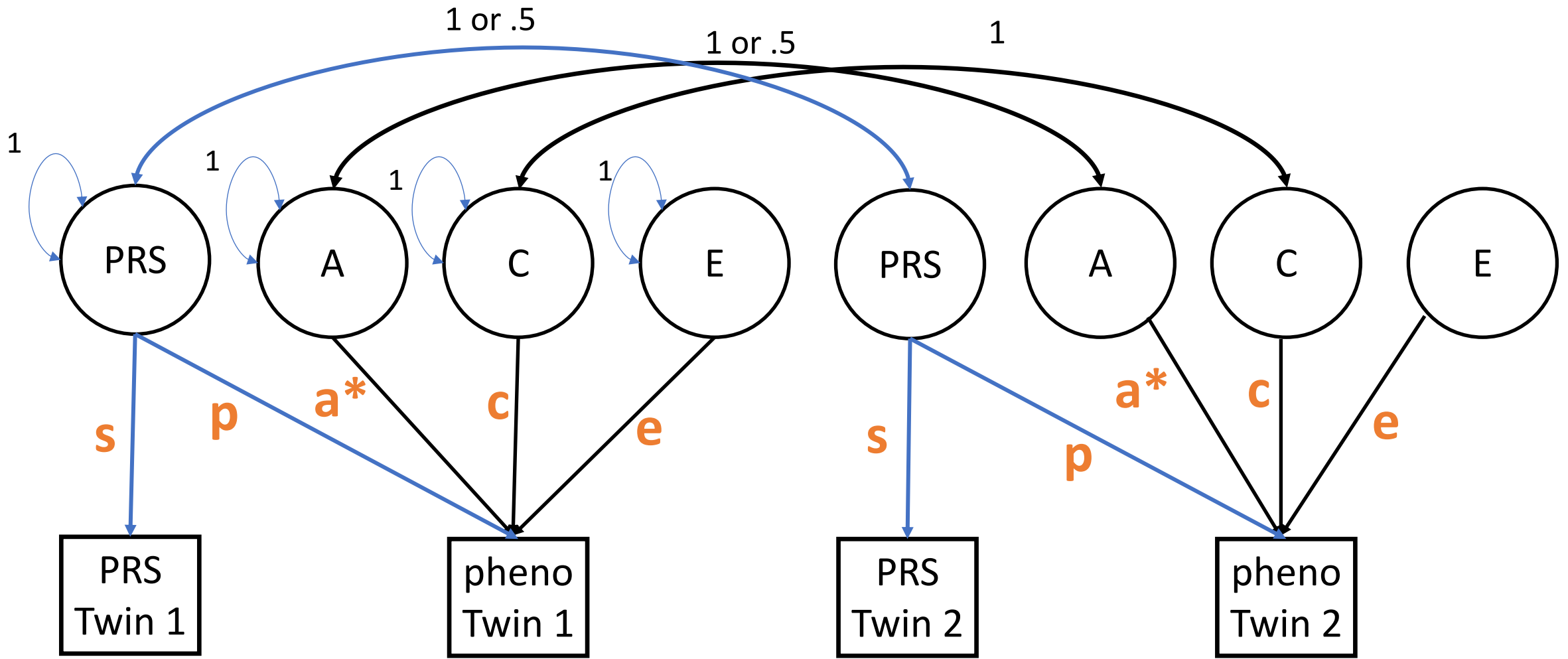
11 to N

The risk score PGS renders part of the **latent A observable**.

Include polygenic risk scores in the ACE twin model to estimate r_{AC}

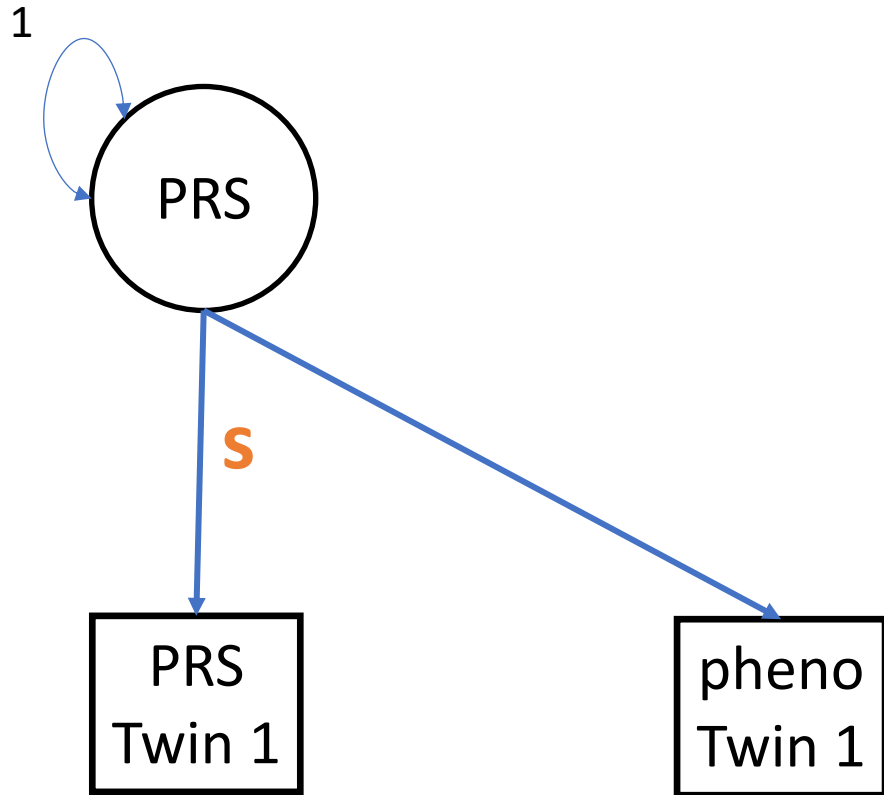


$$\text{var}(\text{pheno}) = a^2 + c^2 + e^2$$

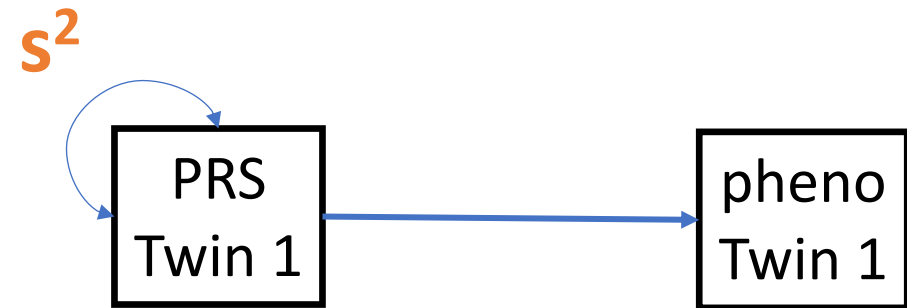


$$\text{var}(\text{pheno}) = a^{*2} + p^2 + c^2 + e^2$$

Identical models



the standard deviation of PRS is s



the variance of PRS is s^2

Nmz = Ndz = 1000 exact data simulation

Standard model ACE

	LB	est	UB
ACE.A[1,1]	0.490	0.600	0.714
ACE.C[1,1]	0.198	0.300	0.399
ACE.E[1,1]	0.471	0.500	0.532

Standard model ACE with P

	LB	est	UB
A*+P	0.492	0.600	0.711
C	0.202	0.300	0.396
E	0.471	0.500	0.532
P	0.047	0.060	0.075
A*	0.433	0.540	0.650

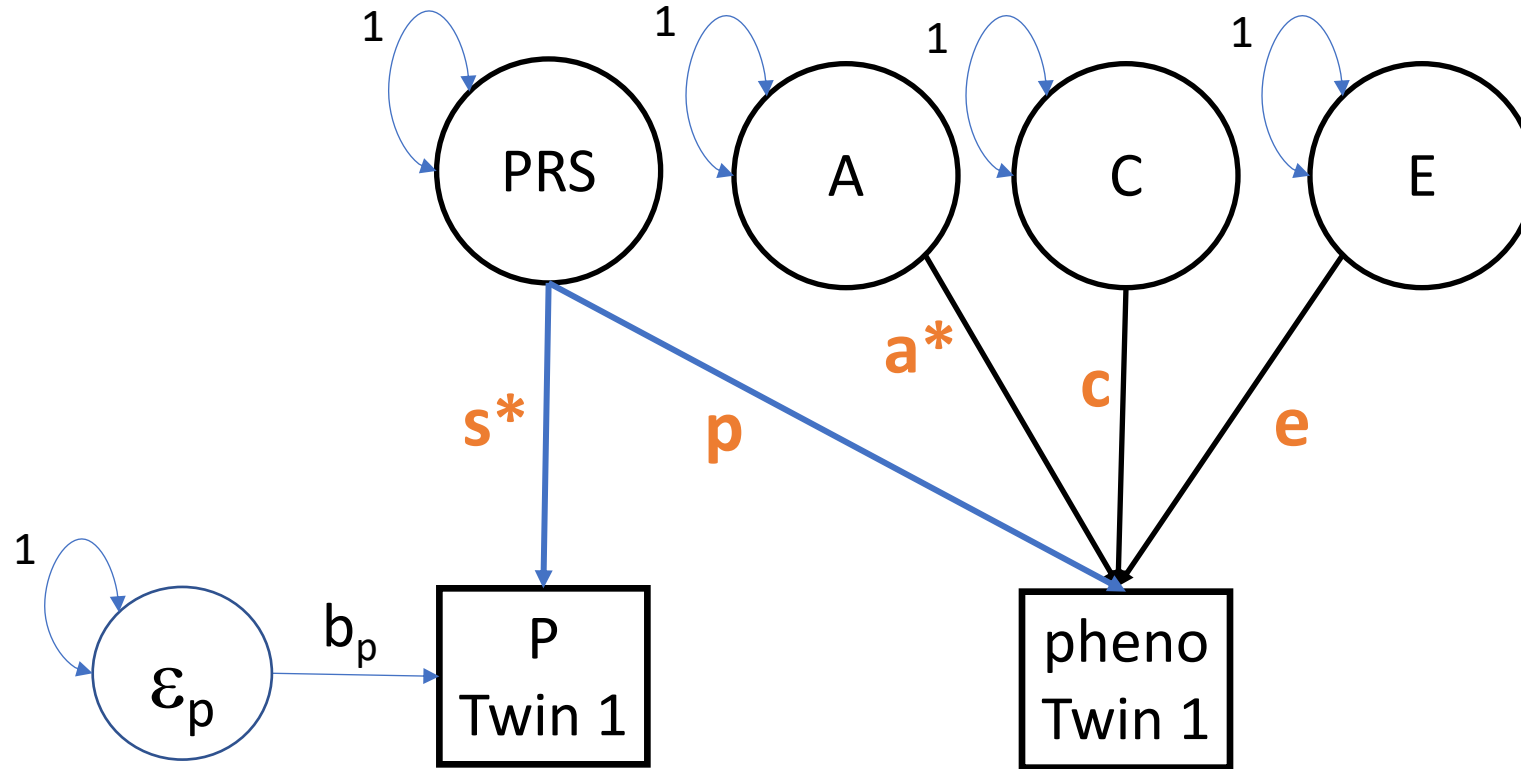
$$\text{var(pheno)} = .60 + .30 + .50$$

$$\text{var(pheno)} = .54 + .06 + .30 + .50$$

.60 in standard model is decomposed into .060 (P) and .540 (A)

$$R^2 = .06/60 \text{ genetic level } 10\%$$

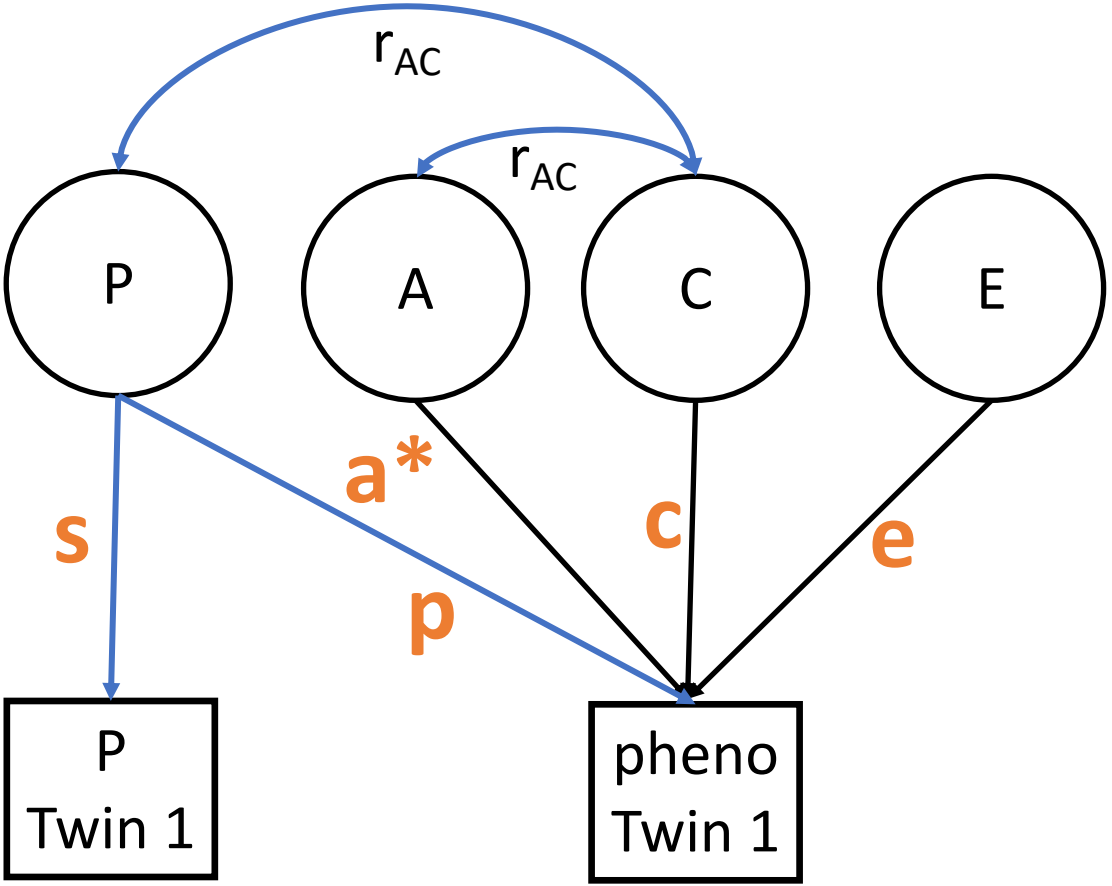
$$R^2 = .06/1.4 \text{ } R^2 \text{ phenotypic level } 4.28\%$$



Phenotype usually includes measurement error which is absorbed by E. Here we assume that the PRS is error free. If it is not and we know the reliability (r_{pp}), we can include that info. The parameter b is fixed to equal:

$$b_p = \sqrt{\text{var}(P) * (1-r_{pp})}$$

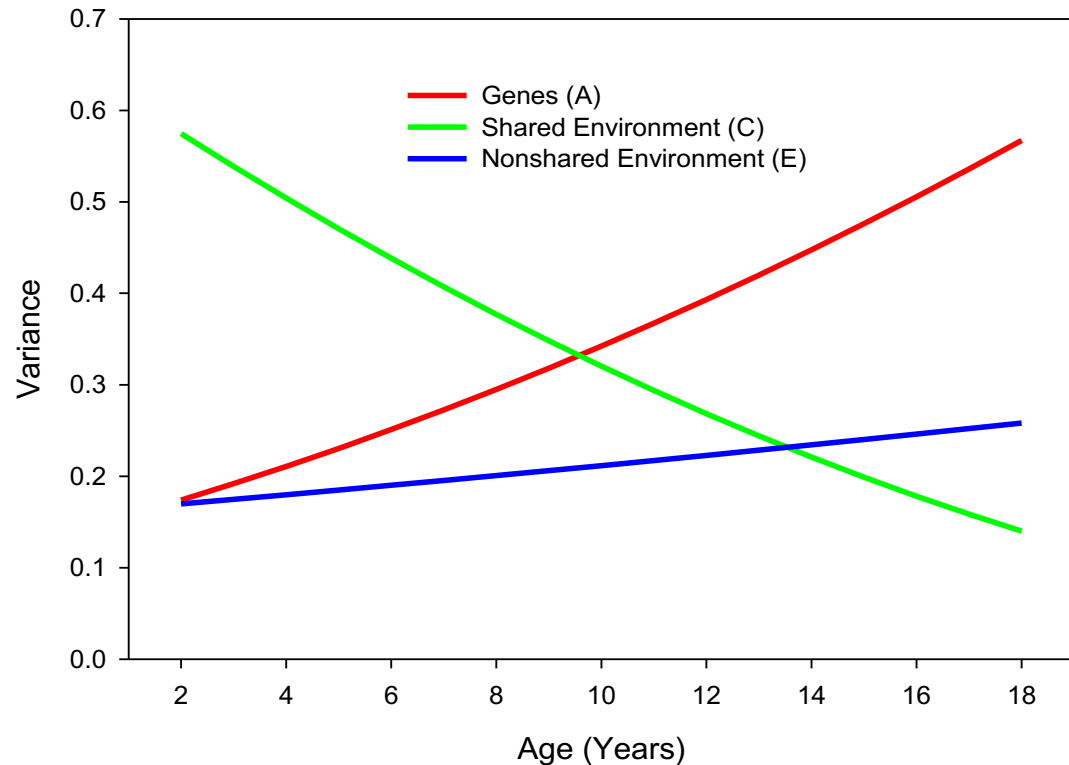
Can we estimate r_{AC} , which not identified in the standard ACE model?



What motivation?

Motivation: Fading C... Is it fading parental influence?

Is the decrease in C due to decrease in C proper + **decrease in r_{AC}**



Tucker-Drob & Bates (2015)

$$c^2 + 2*a*c*r_{AC}$$

What is decreasing?

$$c^2 \text{ or } r_{AC} \text{ in } 2*a*c*r_{AC}$$

probably both

If $r_{AC} > 0$ the C variance in the ACE model is overestimated

If A is correlated with (rather than interacting with) an environmental variable, say C , with correlation r_{AC} then the expected trait variance is $Var(T) = a^2 + c^2 + 2ac \times r_{AC} + e^2$ and the expected twin covariances are

$$Cov(T_1, T_2) = a^2 Cov(A_1, A_2) + c^2 Cov(C_1, C_2) + e^2 Cov(E_1, E_2) + ac Cov(A_1, C_2) + ac Cov(A_2, C_1)$$

$$= a^2 + c^2 + \underline{2ac \times r_{AC}} \text{ for MZ twins}$$

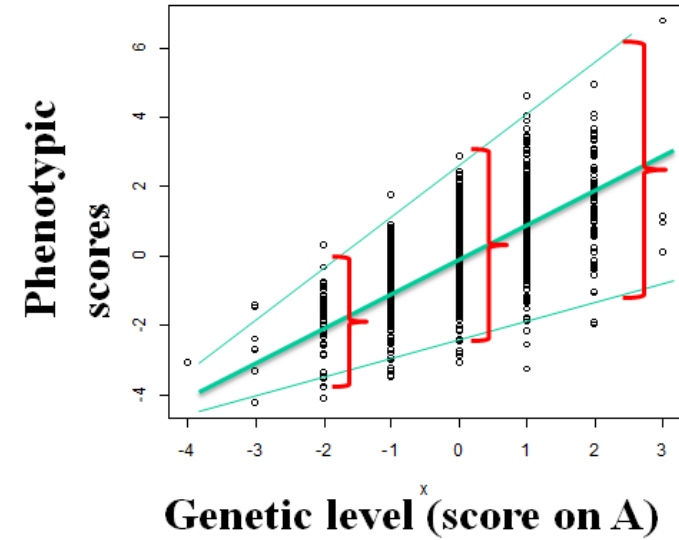
$$= a^2/2 + c^2 + \underline{2ac \times r_{AC}} \text{ for DZ twins}$$

as $Cov(A_1, C_2) = Cov(A_2, C_1) = r_{AC}$.

Homework:
Check

“INTERPLAY”	unmodeled biases
r(AC)	C
r(AE)	A
AxC	A
AxE	E

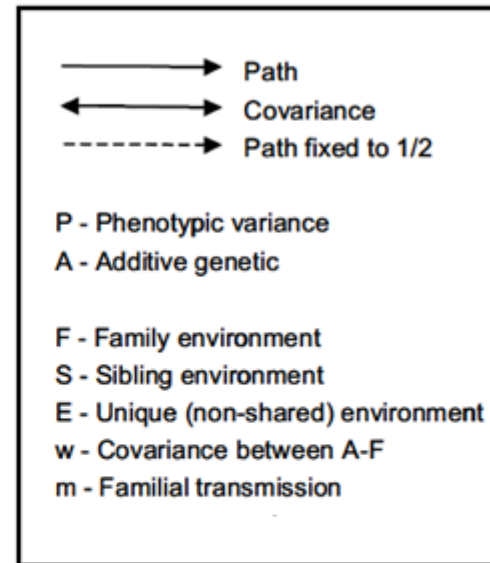
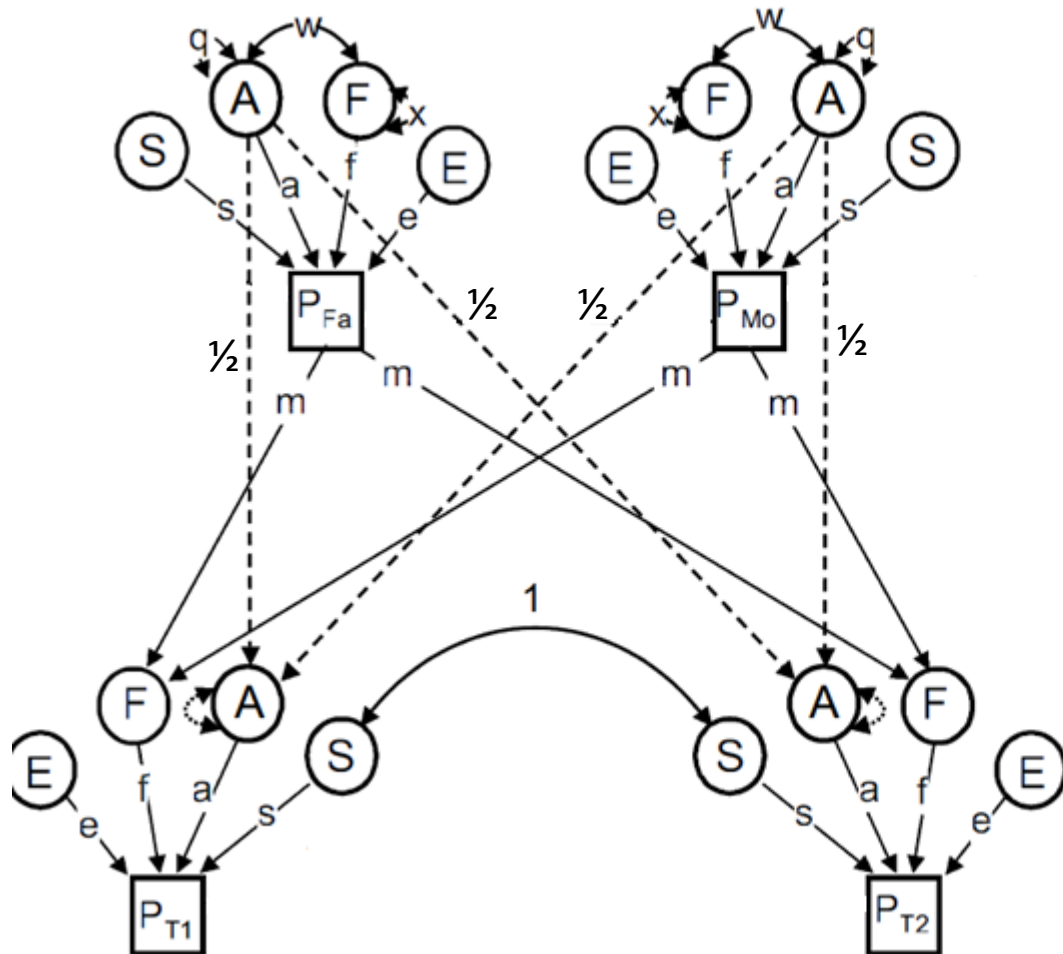
AxE??? what’s that? (Slide 14 mod)



“don’t reify variance components”

you can have a long discussion about the magnitude of variance of A in IQ (but a very short discussion about the proposition $\text{var}(A) = 0$)

Motivation: Is $r_{AC} > 0$ of interest? Parent influence the environmental of children (recent interest via *'transmitted' – 'non transmitted alleles'* design), gives rise to r_{AC} in the offspring (w).

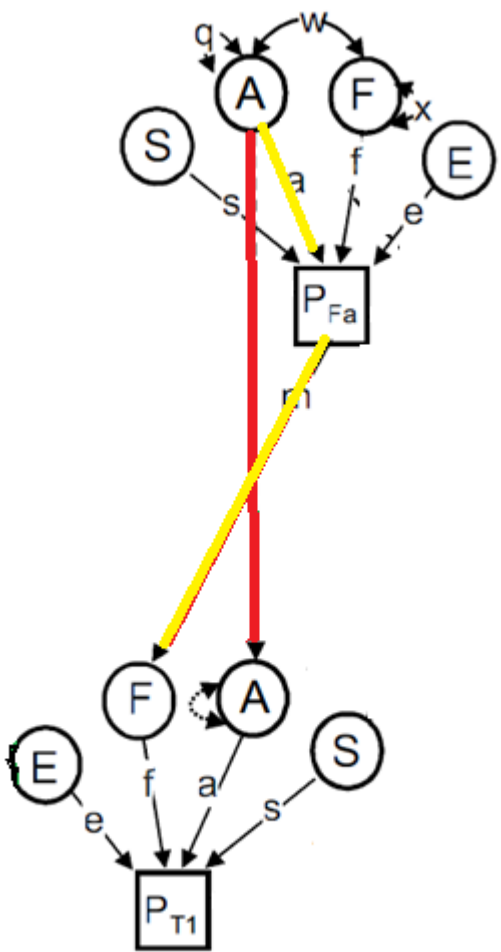
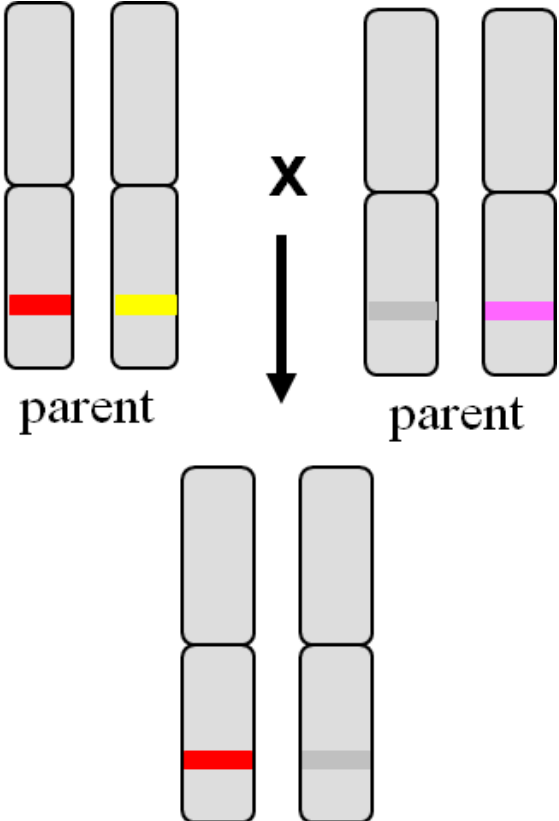


Random mating version

Keller et al (2009) **Twin Research and Human Genetics** Volume 12 Number 1 pp. 8–18.

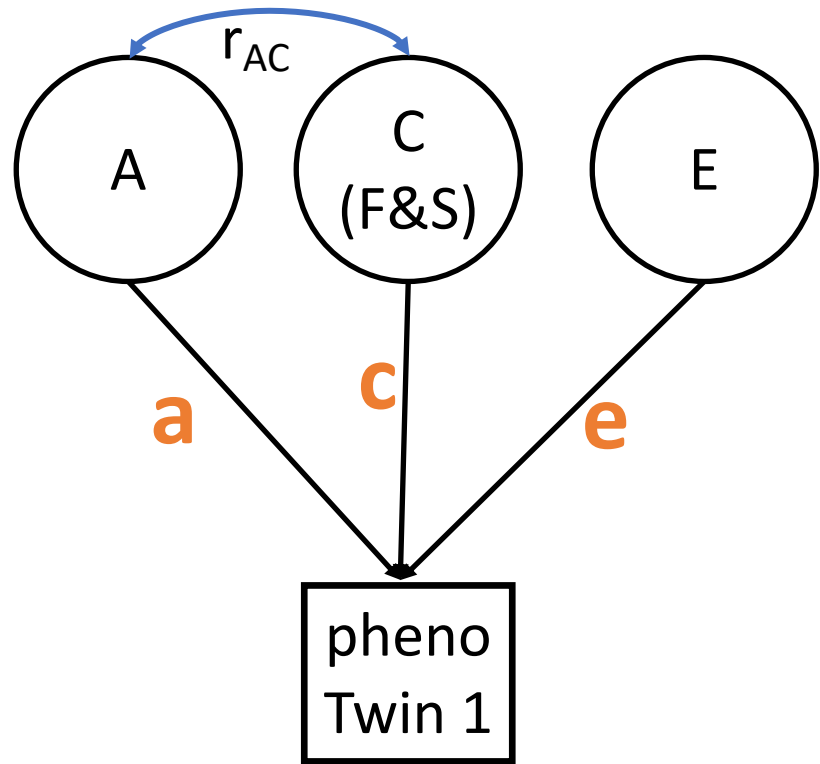
whence w?

recent interest via *'transmitted' – 'non transmitted alleles'* design

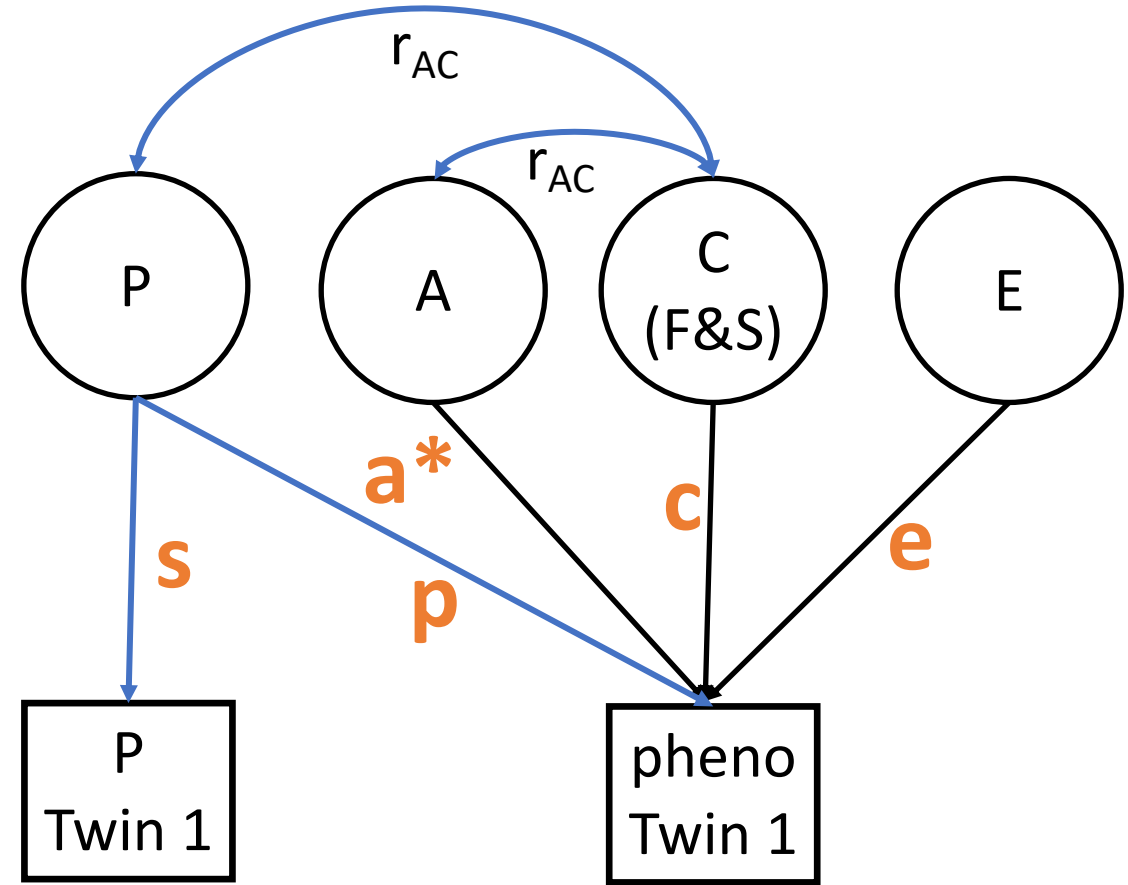


What are the transmitted alleles, what are the untransmitted alleles?

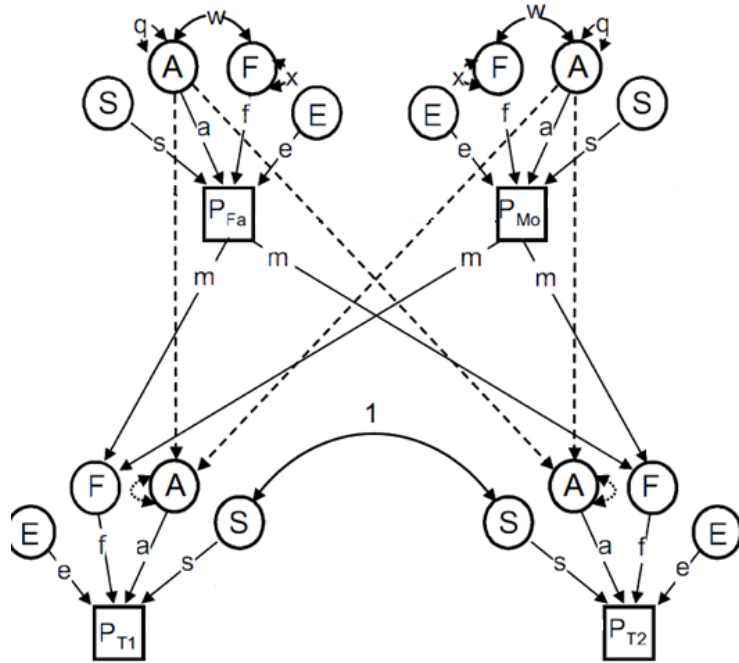
Upshot in the offspring (C = F&S in slide 15)



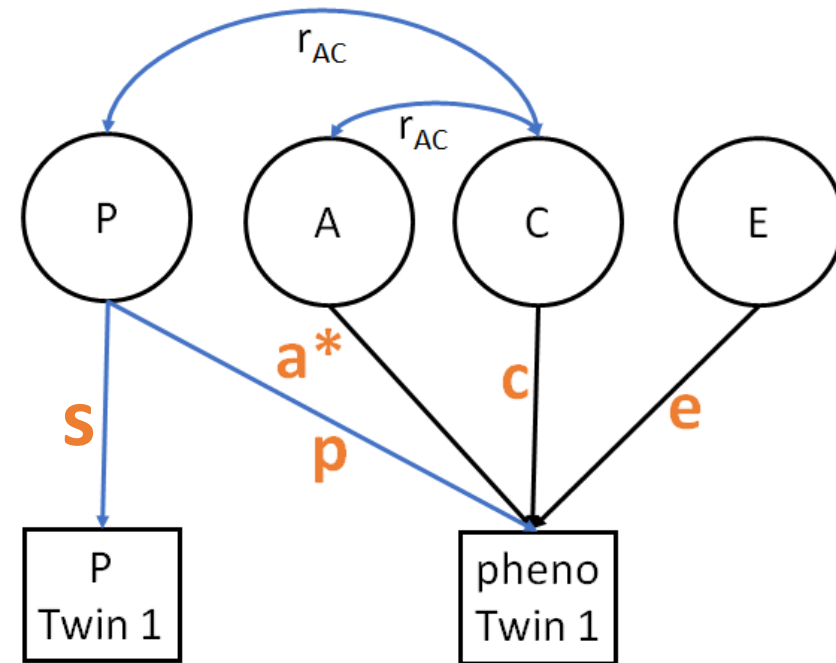
r_{AC} not identified



r_{AC} identified ?



Simulation model



Fitted model

here $r_{AC} = .105$ (model right) and R^2 of the PGS is $.03$
 (that is $.03$ effect in the full model)

DZ covariance matrix (r=.542)

	T1	T2
T1	1.639	0.888
T2	0.888	1.639

Lots of C!

$$2 * .542 - .694 = .39$$

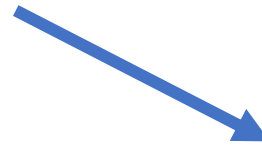
MZ covariance matrix (r=.694)

	T1	T2
T1	1.639	1.139
T2	1.139	1.639

“39% is due to C”

	LB	est	UB
ACE.A[1,1]	0.349	0.500	0.658
ACE.C[1,1]	0.489	0.638	0.784
ACE.E[1,1]	0.459	0.500	0.546

Nmz = Ndz = 1000



	LB	est	UB
rac	-.003	0.105	0.242
E	0.459	0.500	0.546
P	0.023	0.050	0.088
A*	0.302	0.450	0.604
C	0.319	0.505	0.678
A	0.349	0.500	0.658

	LB	est	UB
rac	-	fixed 0	-
E	0.459	0.500	0.546
P	0.057	0.079	0.106
A*	0.306	0.453	0.607
C	0.452	0.596	0.737



Fix $r_{AC} = 0$. Likelihood ratio $T(1) = 3.61057$, $p = 0.0574$

Power to detect r_{AC} here is **.4762 given parameters and $Nmz=Ndz=1000$**

Bad model? Maybe....requires a lot more analyses! (increase r_{AC} , increase %variance due to PGS)

DZ cov matrix

	T1	p1	T2	p2
T1	1.638	0.298	0.888	0.186
p1	0.298	1.000	0.186	0.500
T2	0.888	0.186	1.638	0.298
p2	0.186	0.500	0.298	1.000

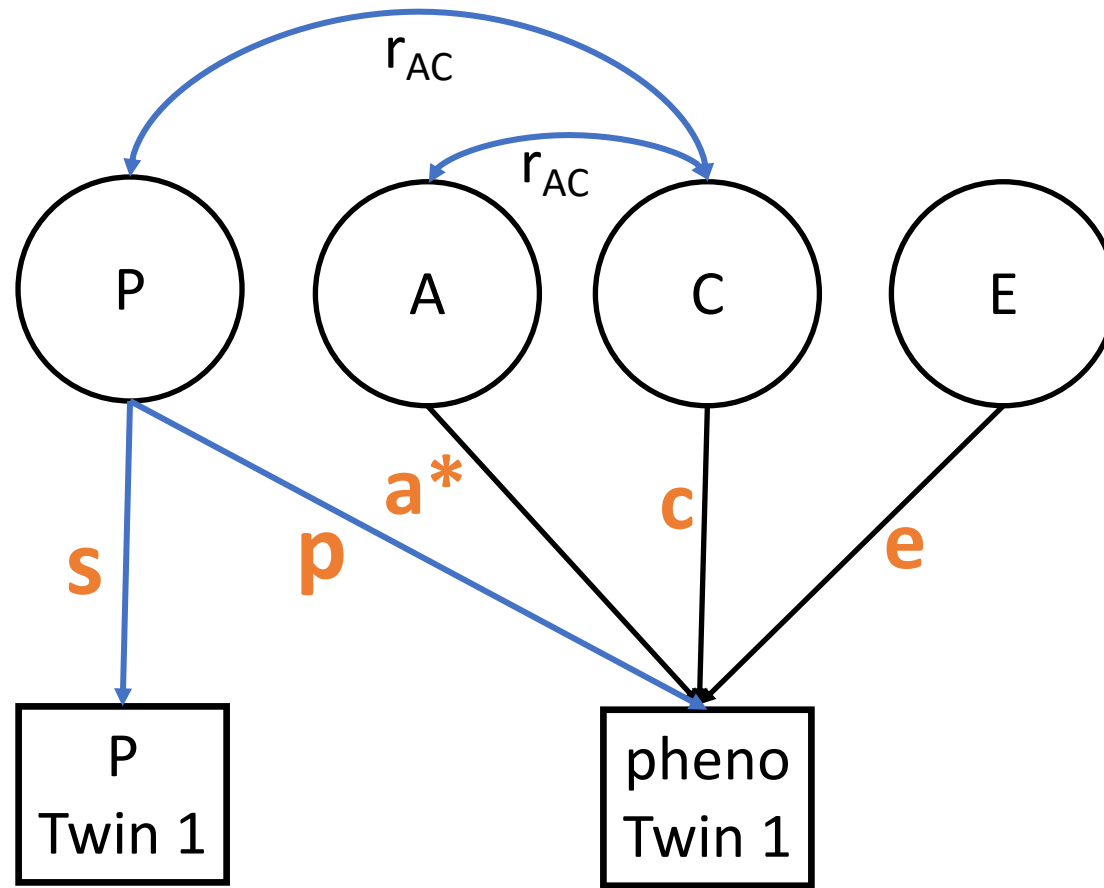
The R^2 in regression of phenotype (T) on p (PGS) is $.298^2 / 1.638 = .054$ (5.4% explained)

	LB	est	UB
rac	-.003	0.105	0.242
E	0.459	0.500	0.546
P	0.023	0.050	0.088
A*	0.302	0.450	0.604
C	0.319	0.505	0.678
A	0.349	0.500	0.658

The R^2 in the true model is $.05 / 1.638 = .030$ (3% of the variance)

3% vs. 5.4%

A question.....



Whence the discrepancy (5.4% vs. 3.0%)?

Extending Causality Tests with Genetic Instruments: An Integration of Mendelian Randomization with the Classical Twin Design

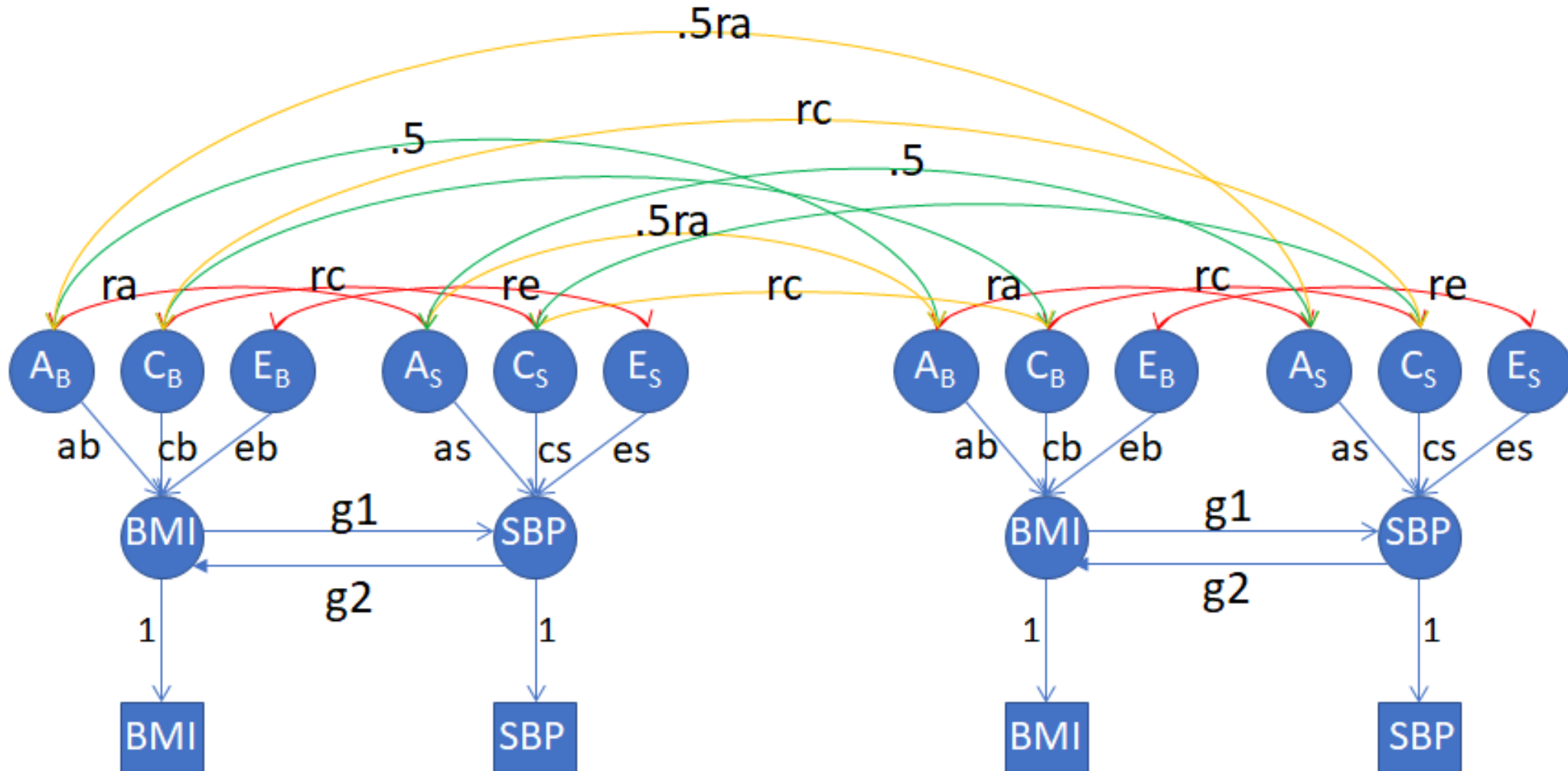
Camelia C Minca & Conor Dolan,

+

Eco de Geus, Dorret Boomsma, Michael C. Neale

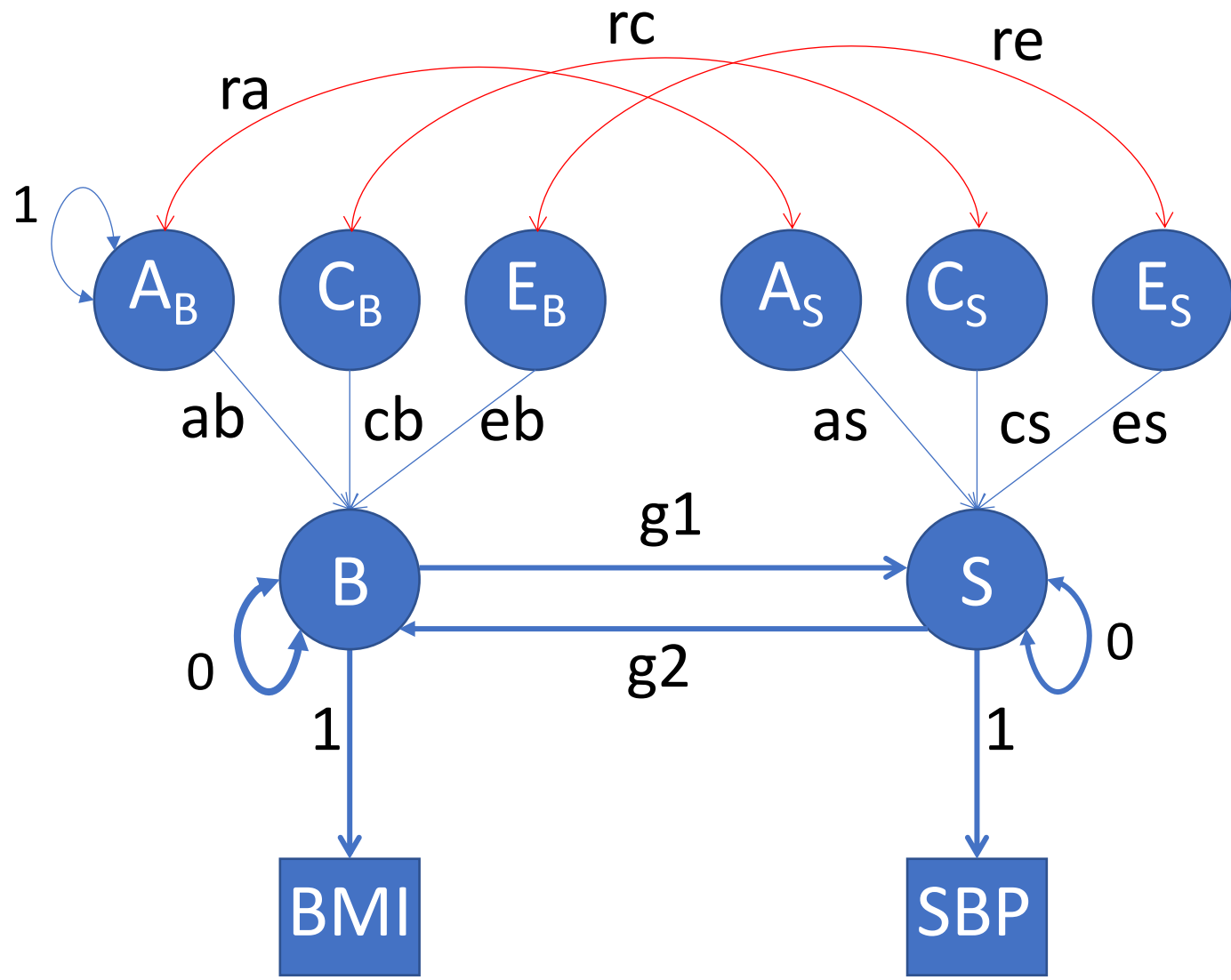
Thanks: David Evans

Modeling causality: Towards the Direction of Causation (DoC) model



not identified

Neale & Cardon, 1992 ; Heath et al., 1993; Duffy & Martin, 1994; Verhulst & Estabrook, 2012.



note “scaling”

Special cases (identified models)

Model 1: $g_1=g_2=0$; $r_e \neq 0$; $r_c \neq 0$; $r_a \neq 0$ (general bivariate model)

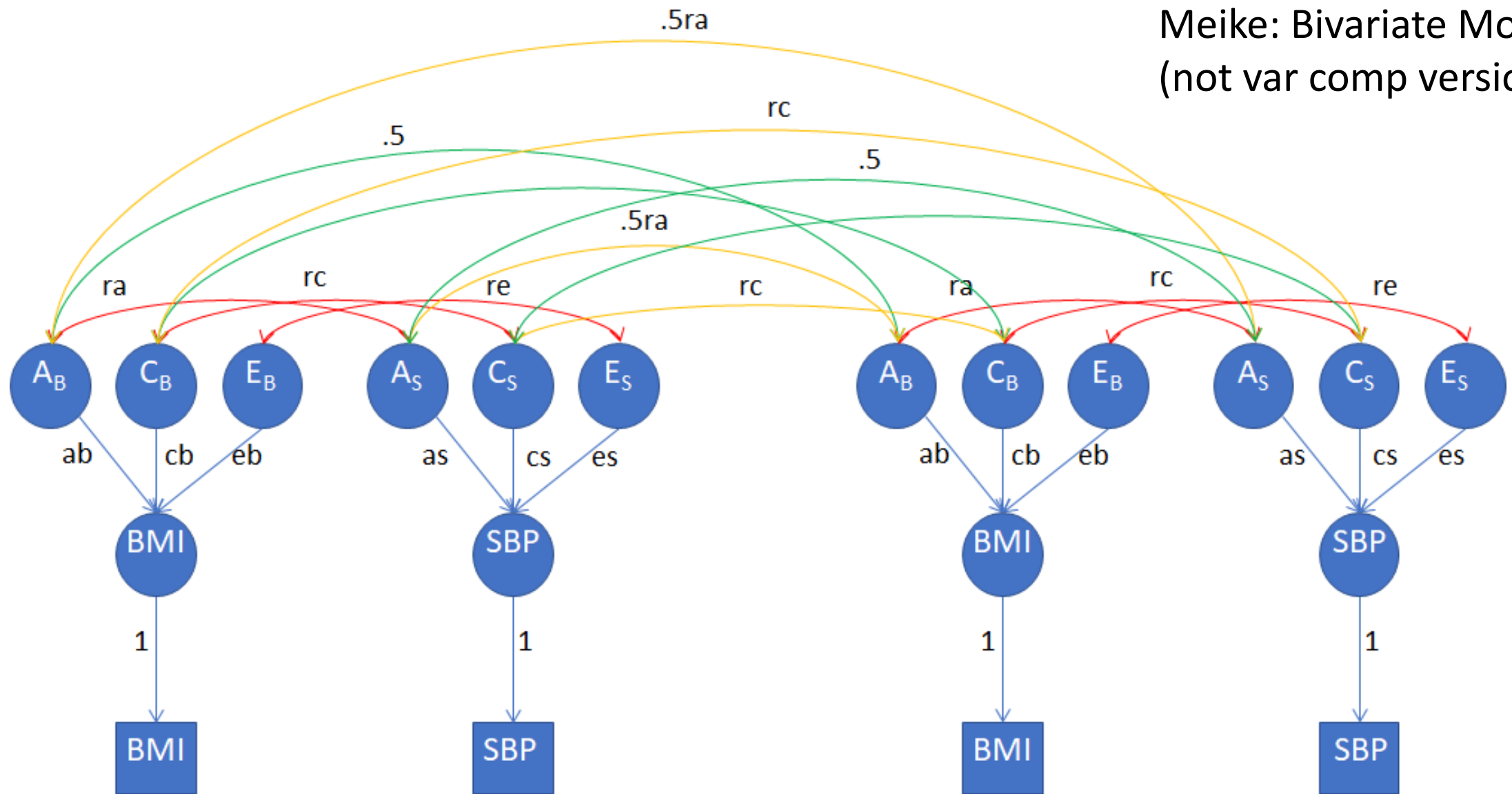
Model 2: $BMI \rightarrow SBP$, $g_1 \neq 0$ & $r_e=r_c=r_a=0$ (unidirectional causation)

Model 3: $SBP \rightarrow BMI$, $g_2 \neq 0$ & $r_e=r_c=r_a=0$ (unidirectional causation)

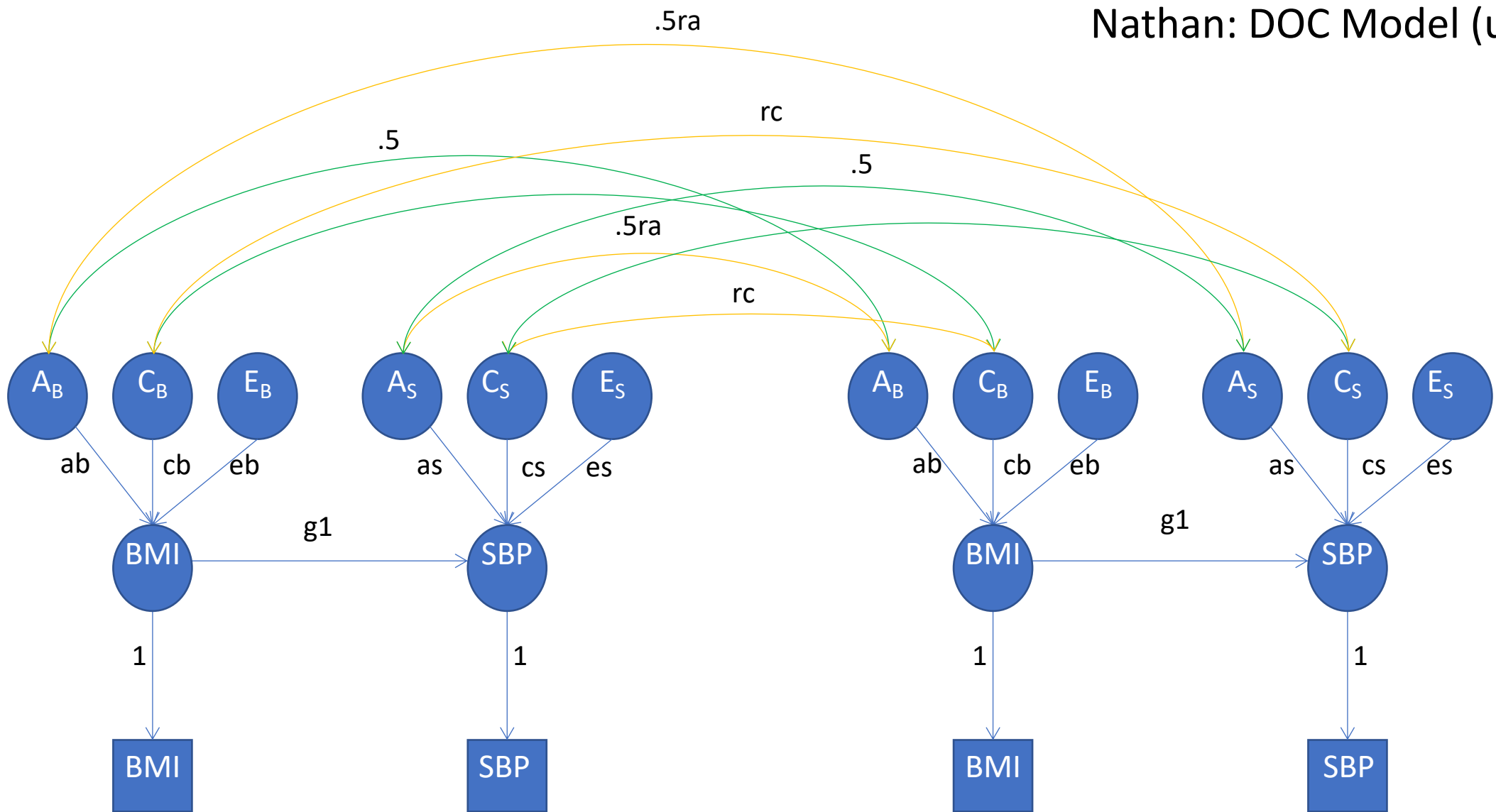
Model 4: $BMI \rightarrow SBP$ & $SBP \rightarrow BMI$; $r_e=r_c=r_a=0$; $g_1 \neq 0$; $g_2 \neq 0$ (reciprocal causation)

Model 5: no association between BMI and SBP

Meike: Bivariate Model
(not var comp version)



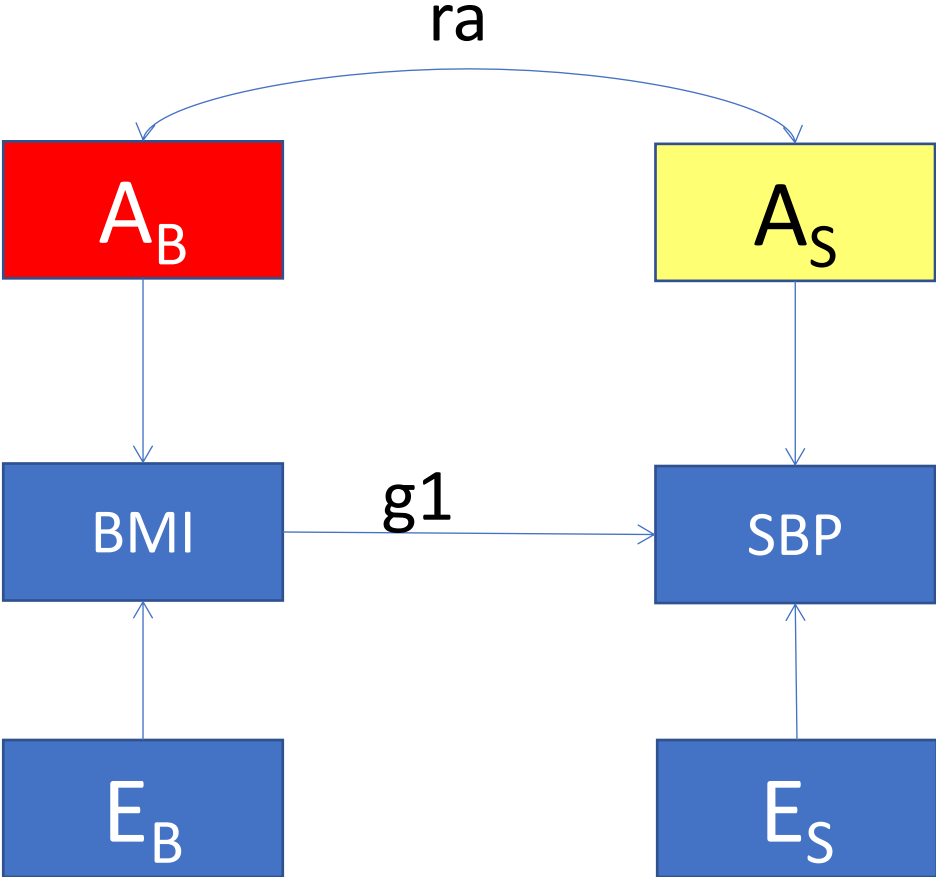
Model 1: $g_1=g_2=0$; $re \neq 0$; $rc \neq 0$; $ra \neq 0$ (general bivariate model)



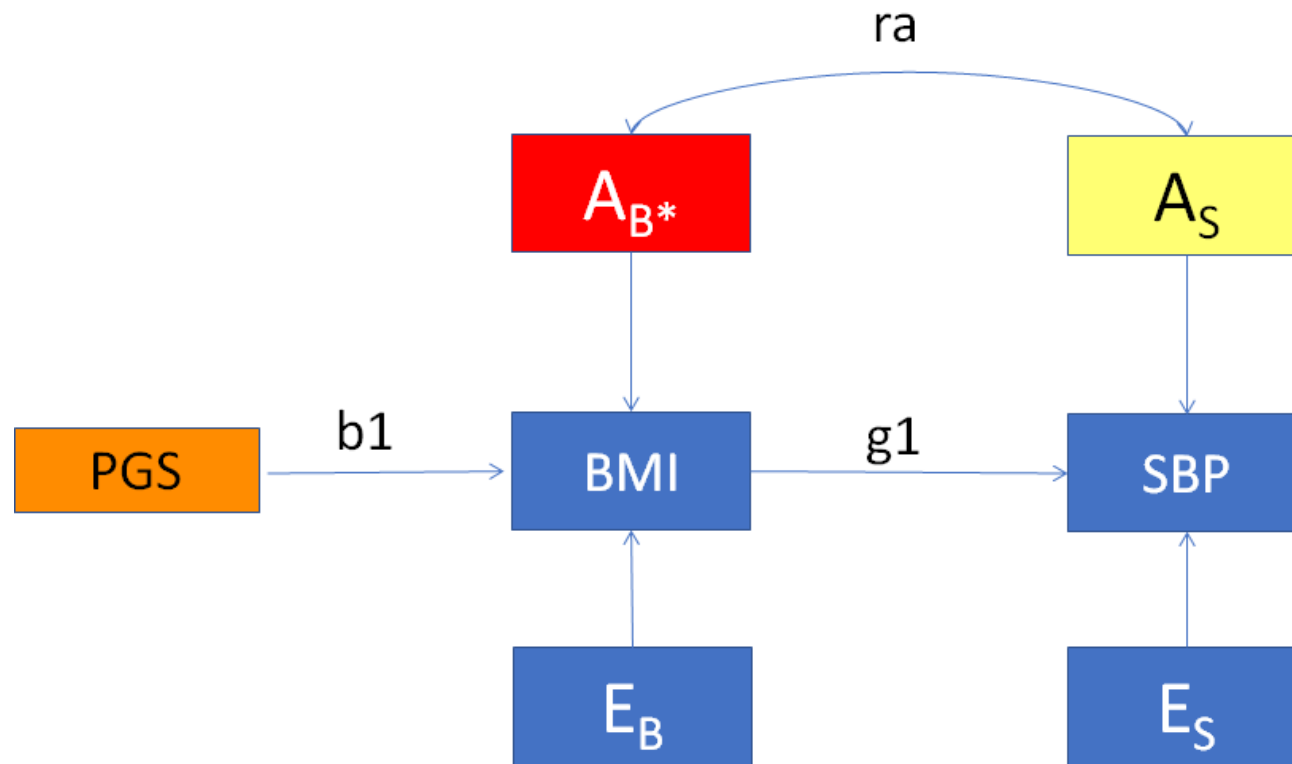
Model 2: BMI \rightarrow SBP, $g1 \neq 0$ & $re=rc=ra=0$ (overidentified unidirectional causation)

MR

Dave: MR

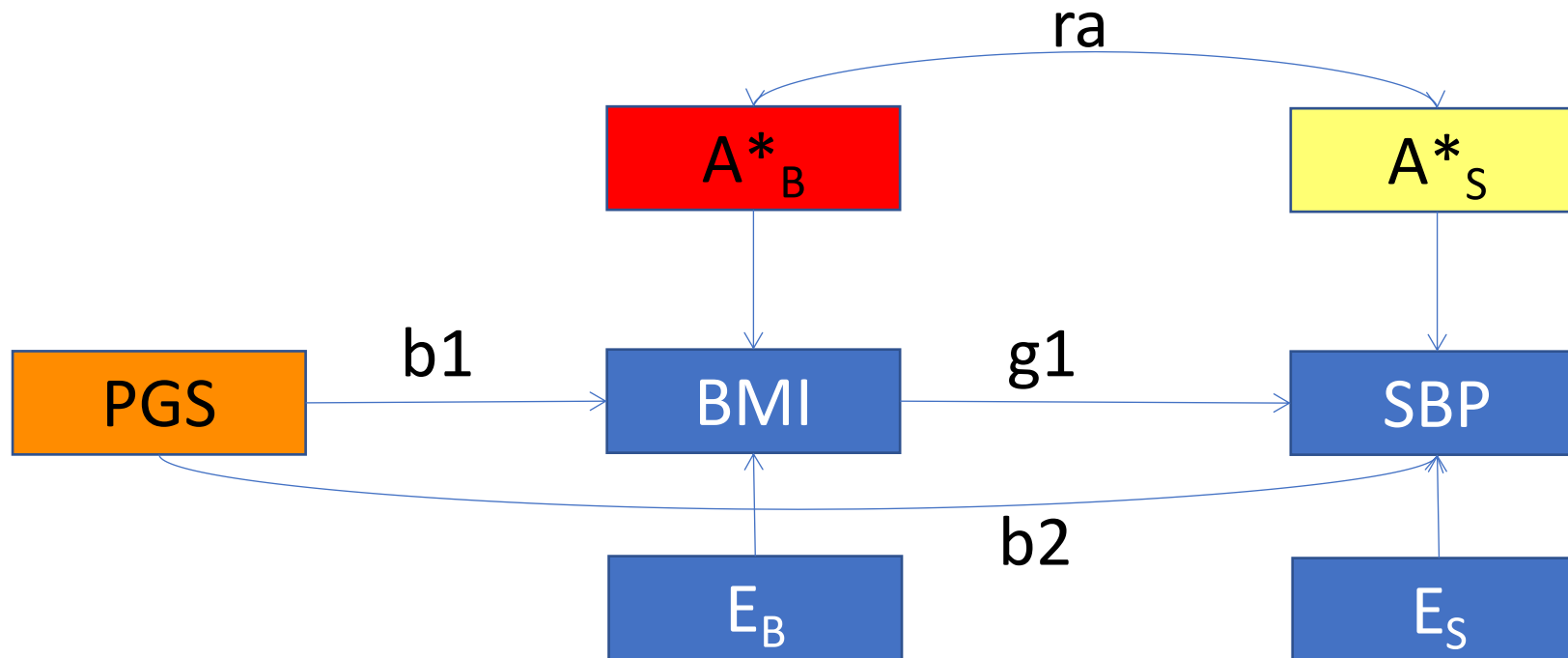


Modeling causality: Mendelian Randomization - model 1 with PGS & **no pleiotropy**

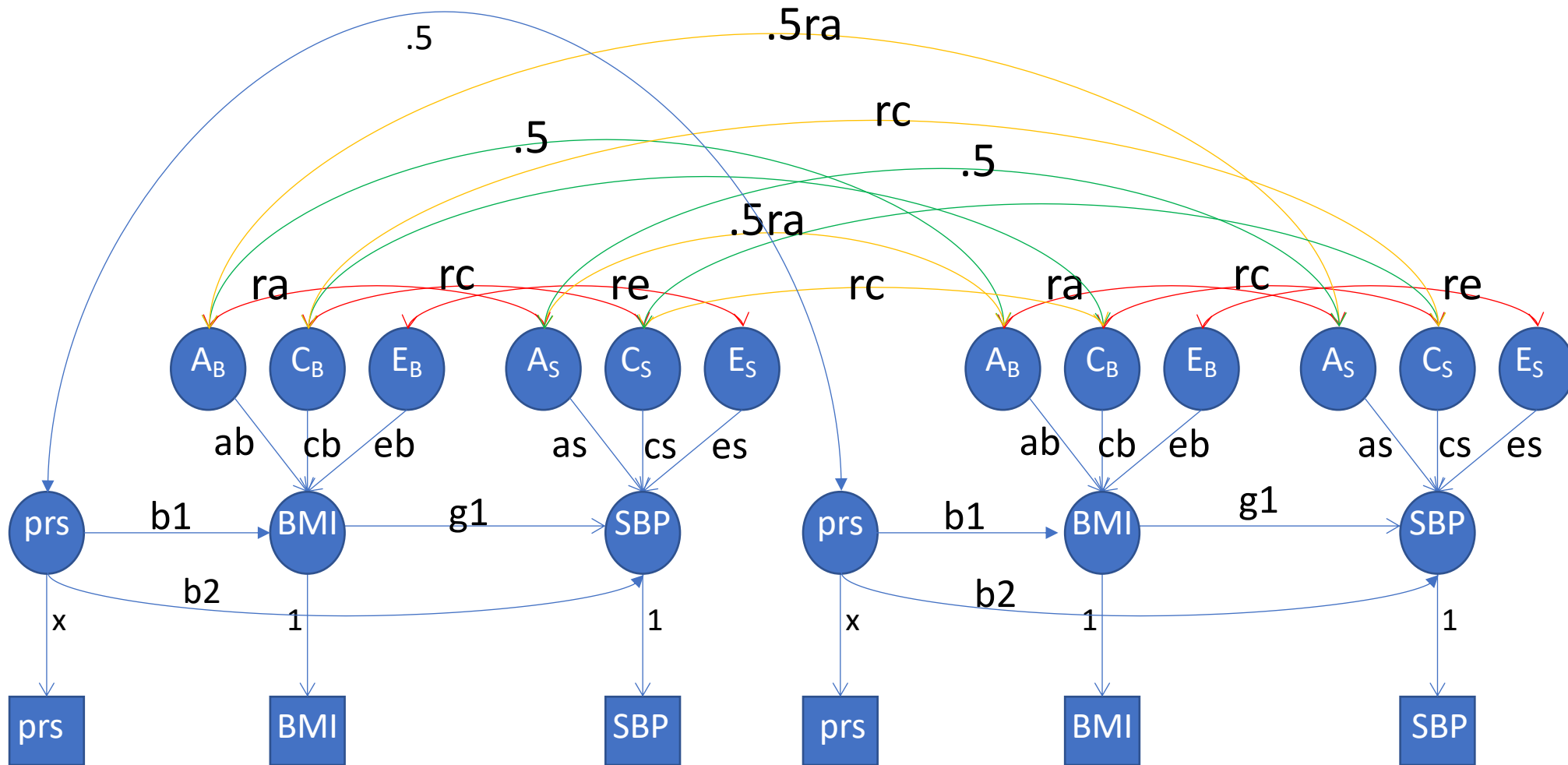


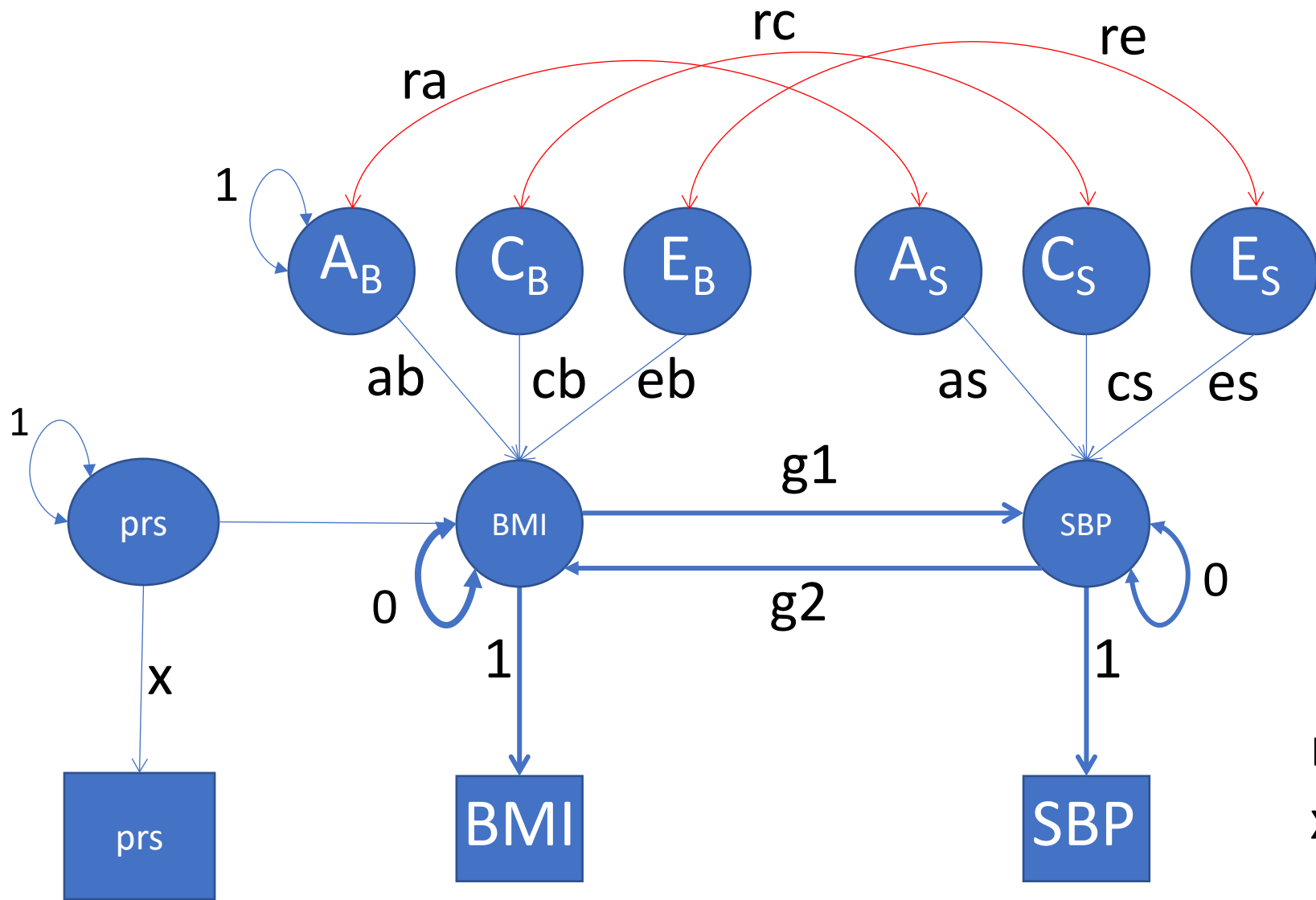
Presence of the “instrumental variable PGS” renders g_1 identified – the larger the b_1 ($R^2 + p$ -value) the more power to reject $g_1=0$

Modeling causality: Mendelian Randomization – model 2 with PGS & **pleiotropy** – violates IV assumption



Relaxing MR's assumptions: Mendelian Randomization meets the Classical Twin Design (MR-Twin model)... DZ model





note “scaling”
 x is the stdev of FTO

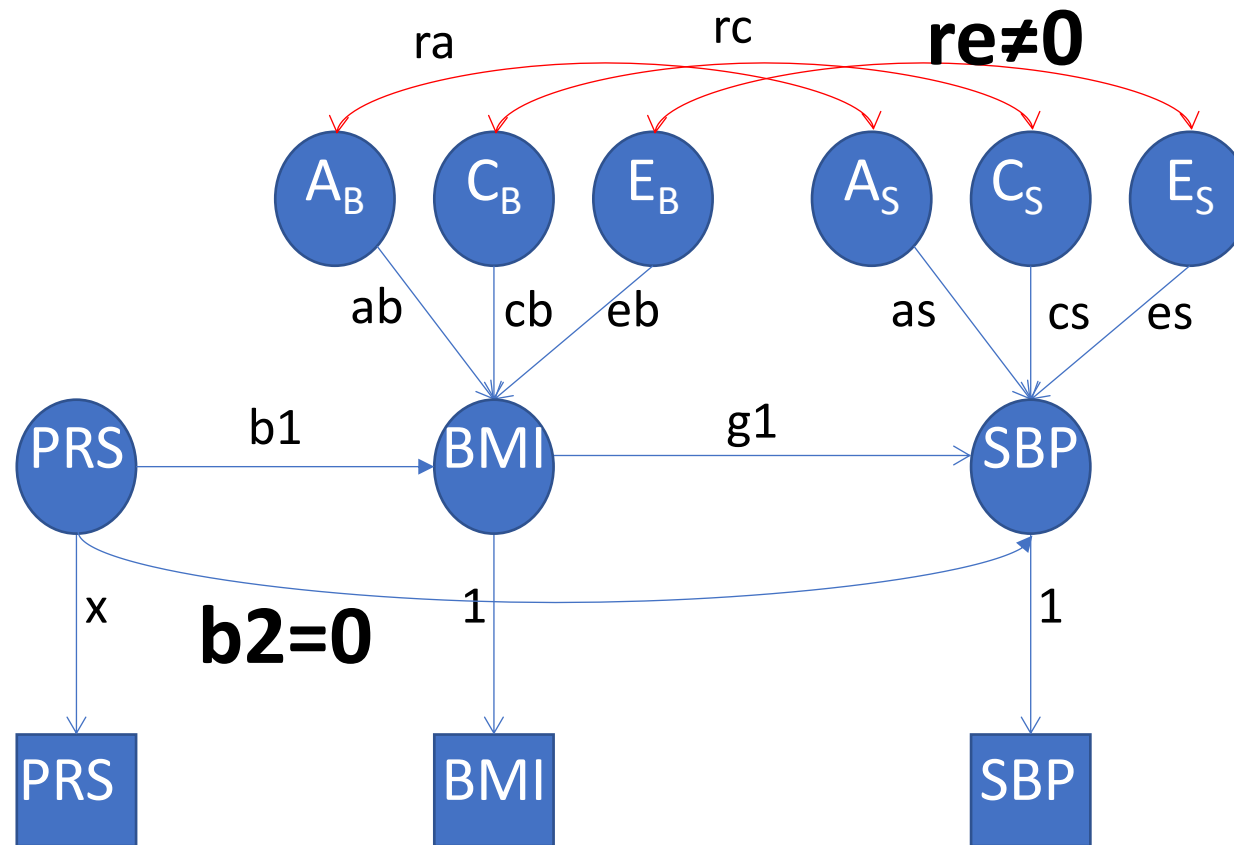
ab cb eb as cs es all freely estimated

	ra	rc	re	b_1	b_2	g_1	ID?
1	fr	fr	fr	fr	fr	fr	No
2	fr	fr	0	fr	fr	fr	Yes
3	fr	fr	fr	fr	0	fr	Yes

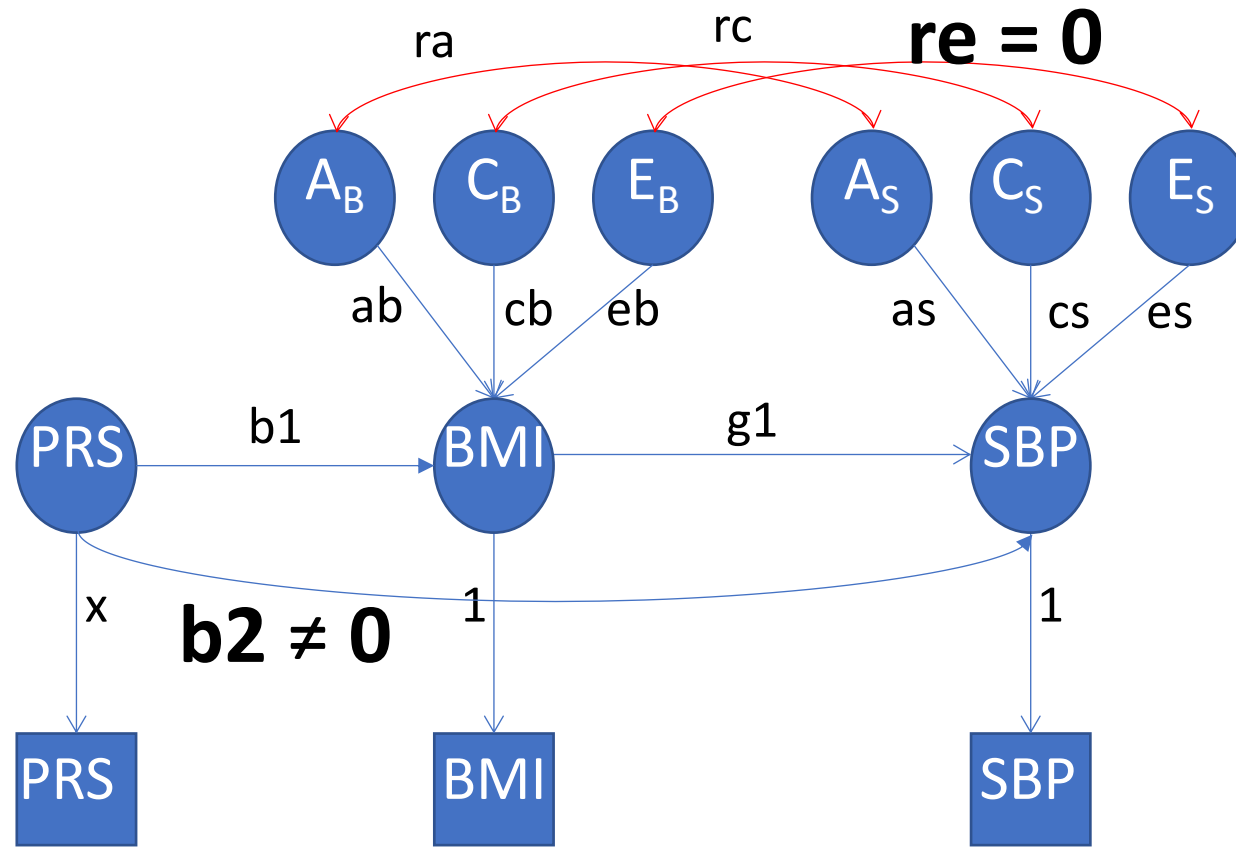
Numerical check: mxCheckIdentification {OpenMx}

Symbolic check: MAPLE, Mathematica, Maxima (free)

Model 1: PGS & no pleiotropy



Model 2: PGS & pleiotropy



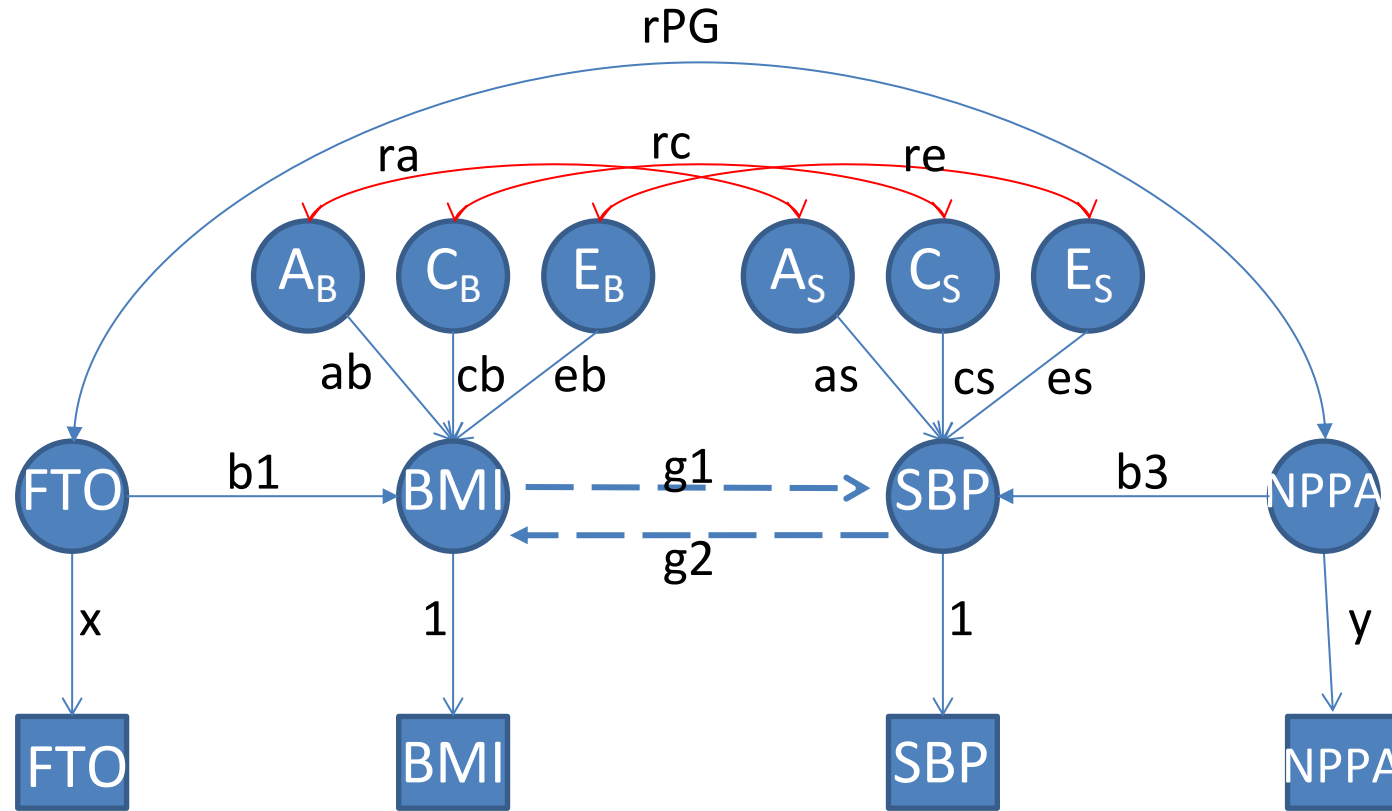


Extending Causality Tests with Genetic Instruments: An Integration of Mendelian Randomization with the Classical Twin Design

Camelia C. Minică¹ · Conor V. Dolan¹ · Dorret I. Boomsma¹ · Eco de Geus¹ · Michael C. Neale^{1,2}

Adding the second IV for SBP (polygenic score NPPA)

Dave: MR



As depicted the model is identified **with $re \neq 0$**
To do: study resolution & power & pleiotropy

[good paper:](#)

[Biodemography Soc Biol.](#) 2011;57(1):88-141.

Social science methods for twins data: integrating causality, endowments, and heritability.

[Kohler HP](#)¹, [Behrman JR](#), [Schnittker J](#).

Camelia C. Minica



previously, a charming
young lady



presently, a charming Nexus
6 Replicant advanced science
model