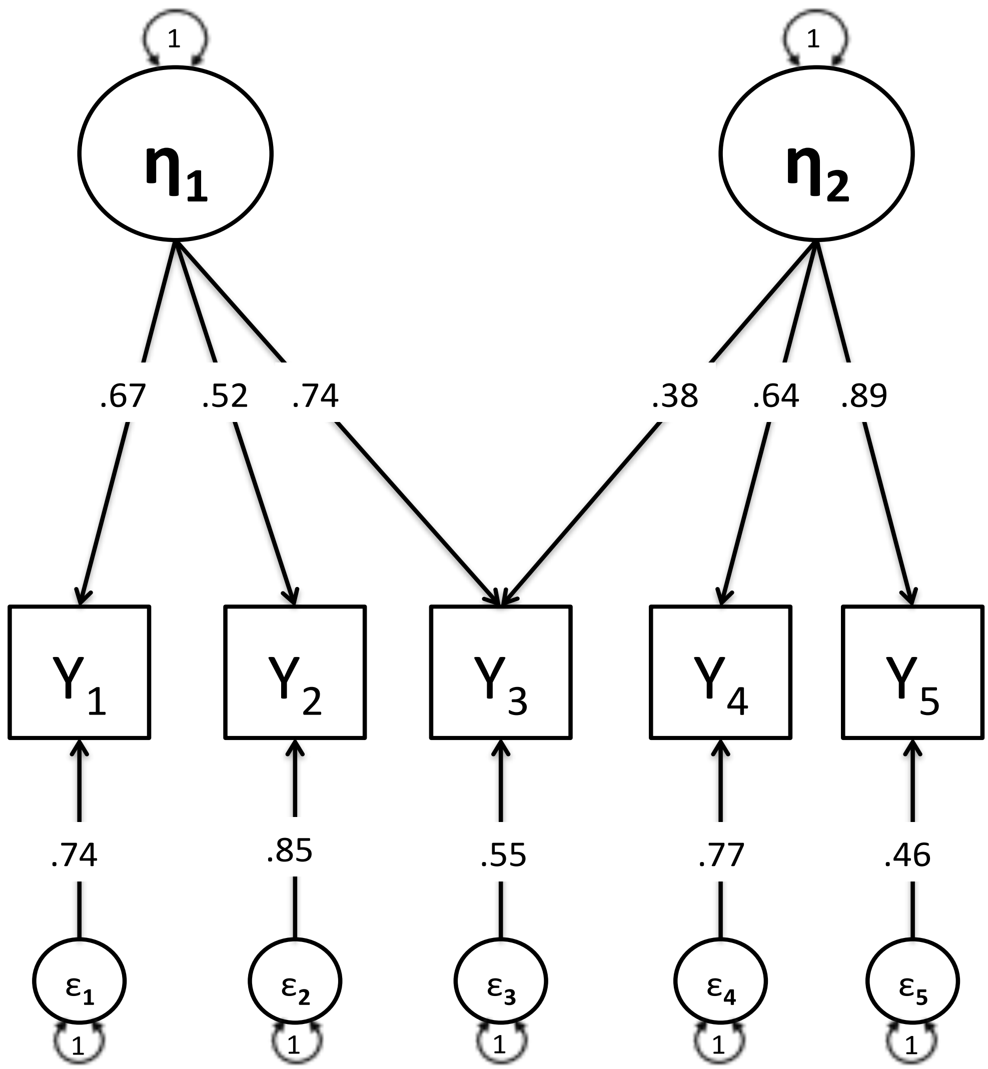
Path Analysis Practice Problems. See page 2 for solutions.

Using the path diagram below, derive the following variances and covariances:

1) Covariance of η_1, and η_2
2) Covariance of Y_3 and Y_5
3) Covariance of Y_1 and Y_3
4) Variance of Y_2
5) Variance of Y_4
6) Variance of Y_3

Using the same path diagram, construct the following:

7) Lambda (λ) matrix of factor loadings from η_1, and η_2 (columns) to Y1…Y5 (rows)
8) Epsilon (ε) diagonal matrix of residual factor loadings
9) Phi (ϕ) matrix of covariances between factors η_1, and η_2
10) If you like, $\lambda\ \phi\ \lambda' + \varepsilon\ \varepsilon'$



3/21 Practice Problem Answers

1) $Cov(\eta_1, \eta_2) = .74 * .38$
$= .28$

2) $Cov(Y_3, Y_5) = .38 * 1 * .89$
$= .34$

3) $Cov(Y_1, Y_3) = .67 * 1 * .74$
$= .50$

4) $Var(Y_2) = (.52 * 1 * .52) + (.85 * 1 * .85)$
$= .52^2 + .85^2$
$=.99$

5) $Var(Y_4) = (.64 * 1 * .64) + (.77 * 1 * .77)$
$= .64^2 + .77^2$
$= 1.00$

6) $Var(Y_3) = (.74 * 1 * .74) + (.38 * 1 * .38) + (.55 * 1 * .55)$
$= .74^2 + .38^2 + .55^2$
$= .99$

7) $$\lambda = \begin{pmatrix} .67 & 0 \\ .52 & 0 \\ .74 & .38 \\ 0 & .64 \\ 0 & .89 \end{pmatrix}$$

8) $$diag(\varepsilon) = \begin{pmatrix} .74 \\ .85 \\ .55 \\ .77 \\ .46 \end{pmatrix}$$

9) Using R to save tedious math:

```
Lambda <- matrix(c(.67,0,.52,0,.74,.38,0,.64,0,.89),5,2,byrow=T)
Phi <- diag(2)
Epsilon <- diag(c(.74,.85,.55,.77,.46))
Lambda%*%Phi%*%t(Lambda) + Epsilon%*%Epsilon
```

Yields (see next page):

	[,1]	[,2]	[,3]	[,4]	[,5]
[1,]	0.9965	0.3484	0.4958	0.0000	0.0000
[2,]	0.3484	0.9929	0.3848	0.0000	0.0000
[3,]	0.4958	0.3848	0.9945	0.2432	0.3382
[4,]	0.0000	0.0000	0.2432	1.0025	0.5696
[5,]	0.0000	0.0000	0.3382	0.5696	1.0037