

# Introduction to Biometrical Genetics {in the classical twin design}

Conor Dolan et al.  
Vrije Universiteit

Boulder 2018 Workshop

**Slide acknowledgement:** Manuel Ferreira, Pak Sham, Shaun Purcell, Sarah Medland, and Sophie van der Sluis

Slides: 3-7 .... what's it all about? individual differences

Slides: 8-15 ..... how to quantify individual differences

Slides: 16-18 ... genetic terminology, QTL

Slides: 19-27 ... mean & variance as function of QTL

Slides: 28- 30... interpretation of variance components in regression

Slides: 31- 43 ... covariance as function of QTL and IBD

Slides: 44-48 ... intro to practical (there is no practical!)

“Having 5 fingers genetically determined”

*“DNA includes a blueprint to build a hand”*





normal



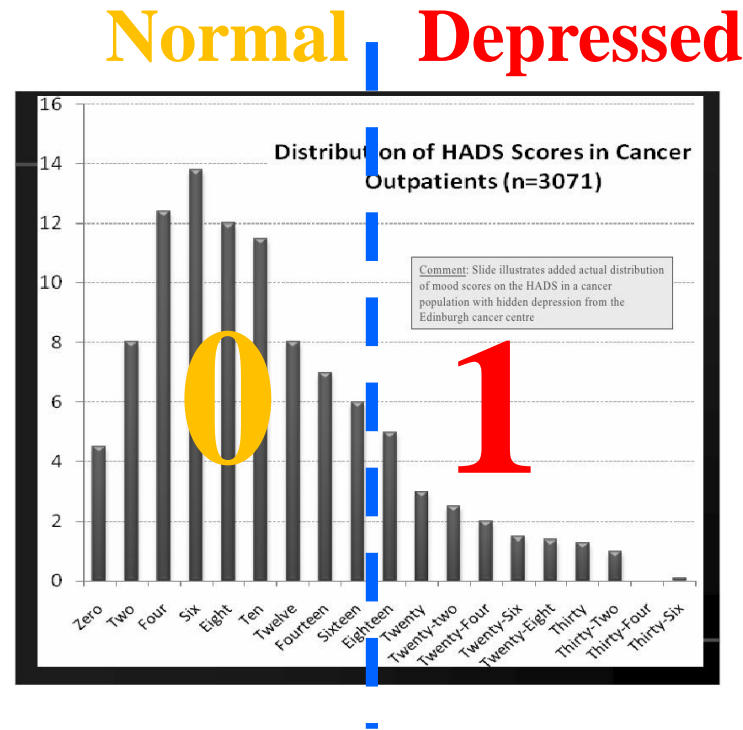
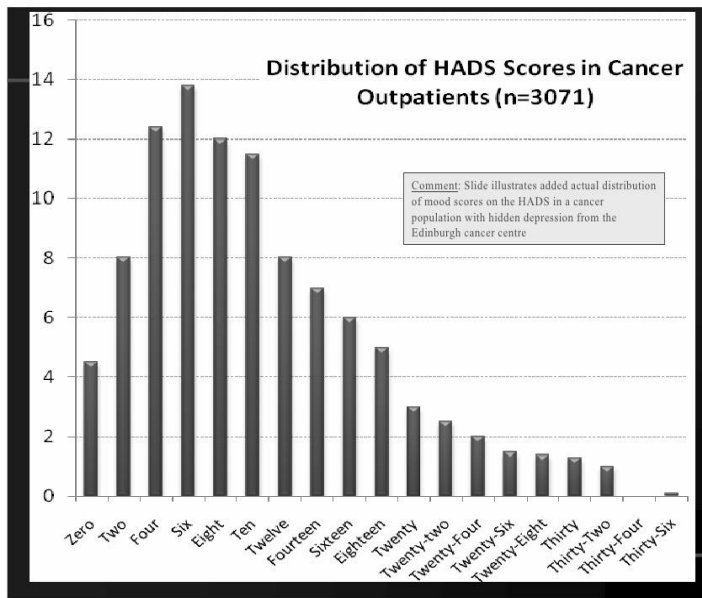
polydactyly



angry dog

Behavior genetic research is concerned with relating **individual differences** in **phenotypes** to individual differences at the **genetic level** and individual differences in **environmental influences**

Phenotype: **continuously varying, genetically complex**,  
e.g. (ideally) normally distributed  
e.g., binary (dichotomous, 0-1 coded) phenotype  
(based on continuous phenotype; liability threshold model).



The phenotype is a **quantitative** trait, a **metric** trait, a **complex** trait

# Genetically complex:

Individual differences in the phenotype are subject to the effects of many genes of small effects, a.k.a. **polygenes, minor genes**. How many? Hundreds (Educational Attainment, Height) ... Thousands....?

Phenotypic individual differences are attributable to genetic individual differences in a large number of polygenes, a.k.a. **QTLs (quantitative trait loci)**.

Polygenicity implies phenotypic continuous distributions (Nick Martin's intro talk)

# People differ phenotypically

## Q. How to quantify individual differences?

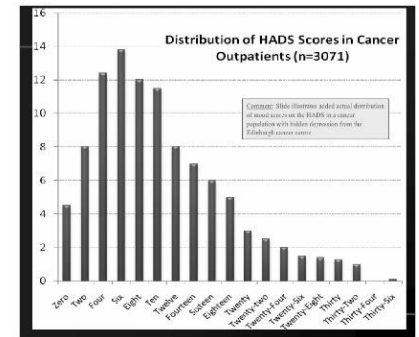
The variance:  $s^2$ ,  $\sigma^2$ ,  $\sigma^2_X$ ,  $\text{var}(X)$ ,  $V_X$

**mean (X)**

$$\mu = \frac{1}{N} \sum_{i=1}^N x_i$$

**variance (X)**

$$\sigma^2 = \frac{1}{N} \sum_{i=1}^N (x_i - \mu)^2$$



$x_i$  is the phenotypic value of person  $i$  ( $i=1, \dots, N$ )

Formula to find the mean for X

$$\mu_x = \frac{\sum_{i=1}^n x_i}{n}$$

$$\mu = \frac{1}{N} \sum_{i=1}^N x_i$$

Formula to find the mean for Y

$$\mu_y = \frac{\sum_{i=1}^n y_i}{n}$$

Formula to find covariance of X & Y

$$\text{cov}(X, Y) = \frac{\sum_{i=1}^n (x_i - \mu_x)(y_i - \mu_y)}{(n - 1)}$$

We need the covariance: express the phenotypic relatedness among family members



# Means, Variances and Covariances

$$\mu = E(X) = \sum_i x_i f(x_i)$$

$$\mu = \frac{1}{N} \sum_{i=1}^N x_i$$

$$\begin{aligned} \text{Var}(X) &= E(X - \mu)^2 \\ &= \sum_i (x_i - \mu)^2 f(x_i) \end{aligned}$$

$$\sigma^2 = \frac{1}{N} \sum_{i=1}^N (x_i - \mu)^2$$

$$\begin{aligned} \text{Cov}(X, Y) &= E(X - \mu_X)(Y - \mu_Y) \\ &= \sum_i (x_i - \mu_X)(y_i - \mu_Y) f(x_i, y_i) \end{aligned}$$

1,1,2,2,3,4,5,5,6,6

$$\text{mean} = (1+1+2+2+3+4+5+5+6+6)/10 \\ = 36/10 = 3.6$$

$$\mu = \frac{\sum_{i=1}^N x_i}{N}$$

f(1) = 2/10 = .2	.2*1 +
f(2) = 2/10 = .2	.2*2 +
f(3) = 1/10 = .1	.1*3 +
f(4) = 1/10 = .1	.1*4 +
f(5) = 2/10 = .2	.2*5 +
f(6) = 2/10 = .2	.2*6
	-----
	3.5

$$\mu = E(X) = \sum_i x_i f(x_i)$$

# in R  
x=c(1,1,2,2,3,4,5,5,6,6)  
mean(x)

1,1,2,2,2,3,4,5,5,5,6,6

mean = 3.5

$f(1) = 2/10 = .2$	$.2*(1-3.5)^2 +$
$f(2) = 2/10 = .2$	$.2*(2-3.5)^2 +$
$f(3) = 1/10 = .1$	$.1*(3-3.5)^2 +$
$f(4) = 1/10 = .1$	$.1*(4-3.5)^2 +$
$f(5) = 2/10 = .2$	$.2*(5-3.5)^2 +$
$f(6) = 2/10 = .2$	$.2*(6-3.5)^2$

-----

variance = 3.45

stdev =  $\sqrt{\text{variance}}$

stdev =  $\sqrt{3.45} = 1.857$

$$\mu = E(X) = \sum_i x_i f(x_i)$$

$$\begin{aligned} \text{Var}(X) &= E(X - \mu)^2 \\ &= \sum_i (x_i - \mu)^2 f(x_i) \end{aligned}$$

# in R

$x=c(1,1,2,2,3,4,5,5,6,6)$

$\text{var}(x)$

## covariance

$$\begin{aligned} \text{Cov}(X, Y) &= E(X - \mu_X)(Y - \mu_Y) \\ &= \sum_i (x_i - \mu_X)(y_i - \mu_Y) f(x_i, y_i) \end{aligned}$$

## correlation

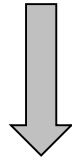
$$\begin{aligned} \text{Cor}(X, Y) &= \text{Cov}(X, Y) / \sqrt{[\text{Var}(X) * \text{var}(Y)]} = \\ &= \text{Cov}(X, Y) / [\text{stdev}(X) * \text{stdev}(Y)] \end{aligned}$$

$\text{Cor}(X, Y)$  is – stand-alone - interpretable

MZ covariance is 291.... uninterpretable

MZ correlation is .80 .... interpretable

To what extent, and how, are  
*individual differences* in genetic makeup, and  
*individual differences* in environmental factors, related to  
*phenotypic (observed) individual differences* ?



To what extent, and how, do  
*individual differences* in genetic makeup, and  
*individual differences* in environmental factors, explain  
*phenotypic (observed) variance* ?

$$\sigma_x^2 = \frac{\sum_{i=1}^N (x_i - \mu)^2}{N - 1}.$$

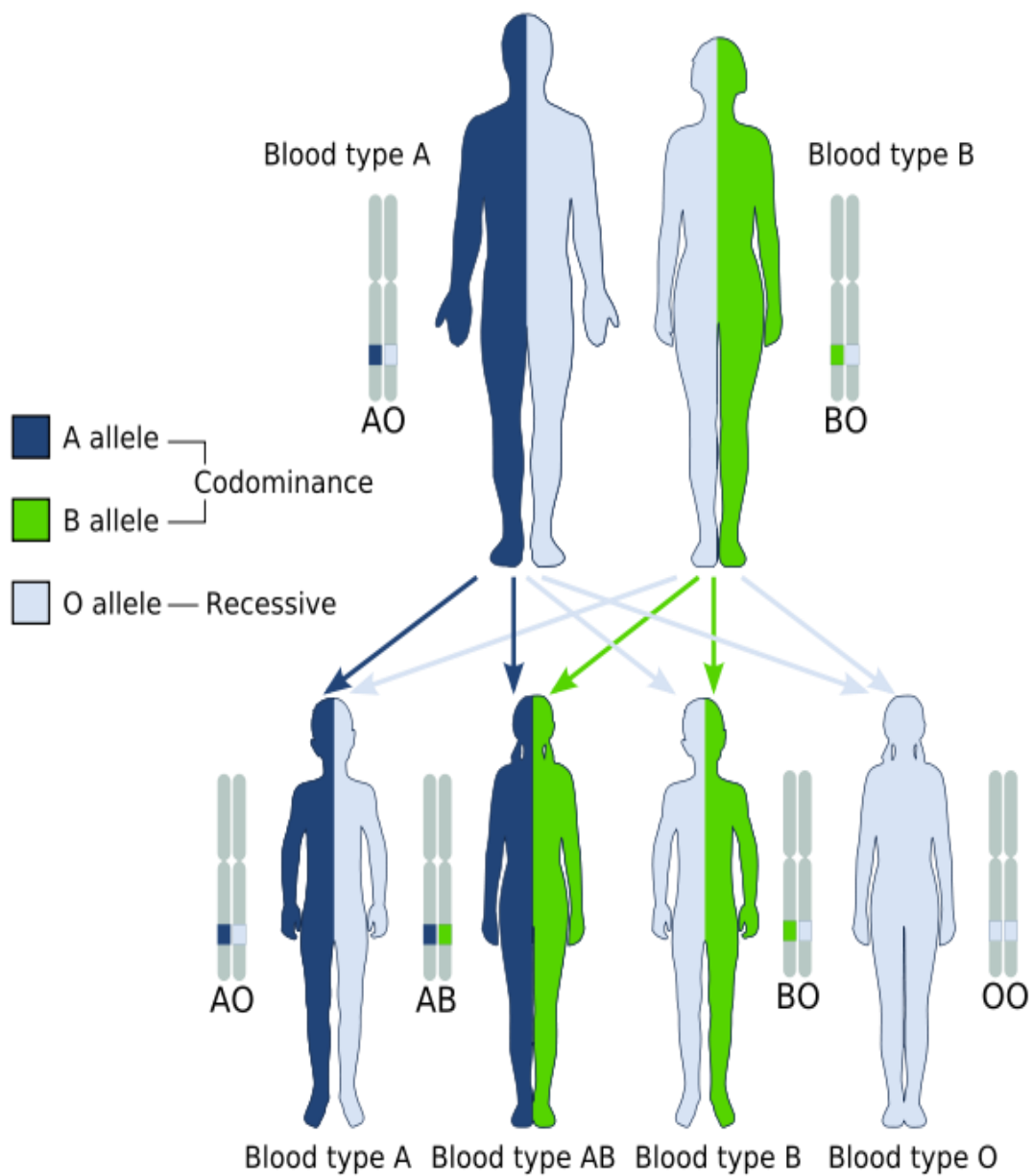
$$\begin{aligned} \text{Var}(X) &= E(X - \mu)^2 \\ &= \sum_i (x_i - \mu)^2 f(x_i) \end{aligned}$$

## terminology

- **QTL Quantative trait locus:** a sequence of DNA base pairs (may be a SNP: single base pair).
- **Locus:** the site of the specific QTL on a chromosome (22 pairs + XY). Humans are diploid (22 pairs autosomal chromosomes + sex chromosomes XY or XX).
- **Allele:** an alternative form of a gene at a locus
- **Genotype:** the combination of alleles at a particular locus
- **Phenotype:** an observed characteristic, which displays individual differences (in part due to genetic differences)

chromosome 9  
location 9q34.2

Mendel's law of  
segregation



## Example of a QTL: FNBP1L gene

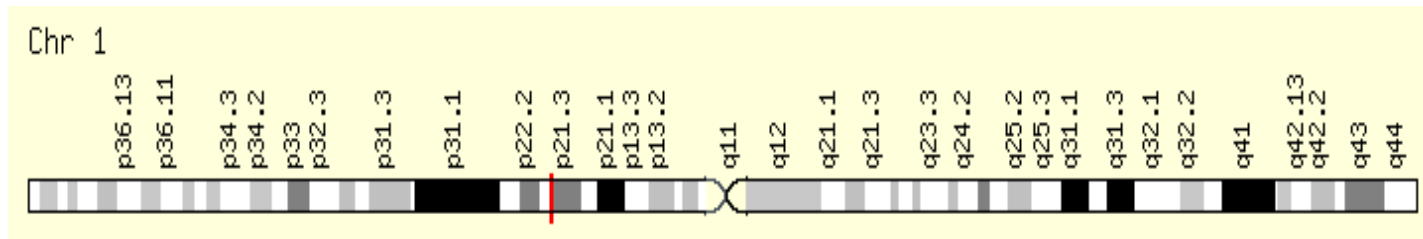
The FNBP1L gene has been associated with intelligence in two studies:

\* "Genome-wide association studies establish that human intelligence is highly heritable and polygenic". 2011, *Mol. Psychiatry* **16** (10): 996–1005. doi:10.1038/mp.2011.85)

\* "Childhood intelligence is heritable, highly polygenic and associated with FNBP1L". *Mol. Psychiatry* **19**(2): 2538. doi:10.1038/mp.2012.184.

Authors include Sarah Medland & Nick Martin.

This gene is on **chromosome 1** (1p22,1), and it comprises **106531 bases** (106.5Kb). Within this gene the SNP **rs236330** specifically is associated with intelligence.





# Population level

---

## 1. Allele frequencies (QTL: diallelic autosomal; e.g., SNP rs236330)

▷ A single locus, with two alleles

- Biallelic a.k.a. diallelic

- in GWAS: Single nucleotide polymorphism, SNP

▷ Alleles **A** and **a**

- Frequency of **A** is **p**

- Frequency of **a** is **q = 1 - p**

} frequencies in the population

▷ Every individual inherits two alleles

- A genotype is the combination of the two alleles

- e.g. **AA**, **aa** (the homozygotes) or **Aa** (the heterozygote)

.....genotype frequencies?

# Biometrical model for single biallelic QTL

▷ Biallelic locus

- Genotypes: **AA, Aa, aa**
- Genotype frequencies:  **$p^2, 2pq, q^2$**

Genotype frequencies  
(Random mating)

		Mother's gametes (egg)	
		<b>A (p)</b>	<b>a (q)</b>
Father's gametes sperm	<b>A (p)</b>	<b>AA (p<sup>2</sup>)</b>	<b>Aa (pq)</b>
	<b>a (q)</b>	<b>aA (qp)</b>	<b>aa (q<sup>2</sup>)</b>

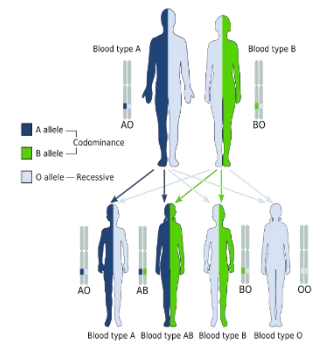
## Hardy-Weinberg Equilibrium frequencies

$$P(AA) = p^2$$

$$P(Aa) = 2pq$$

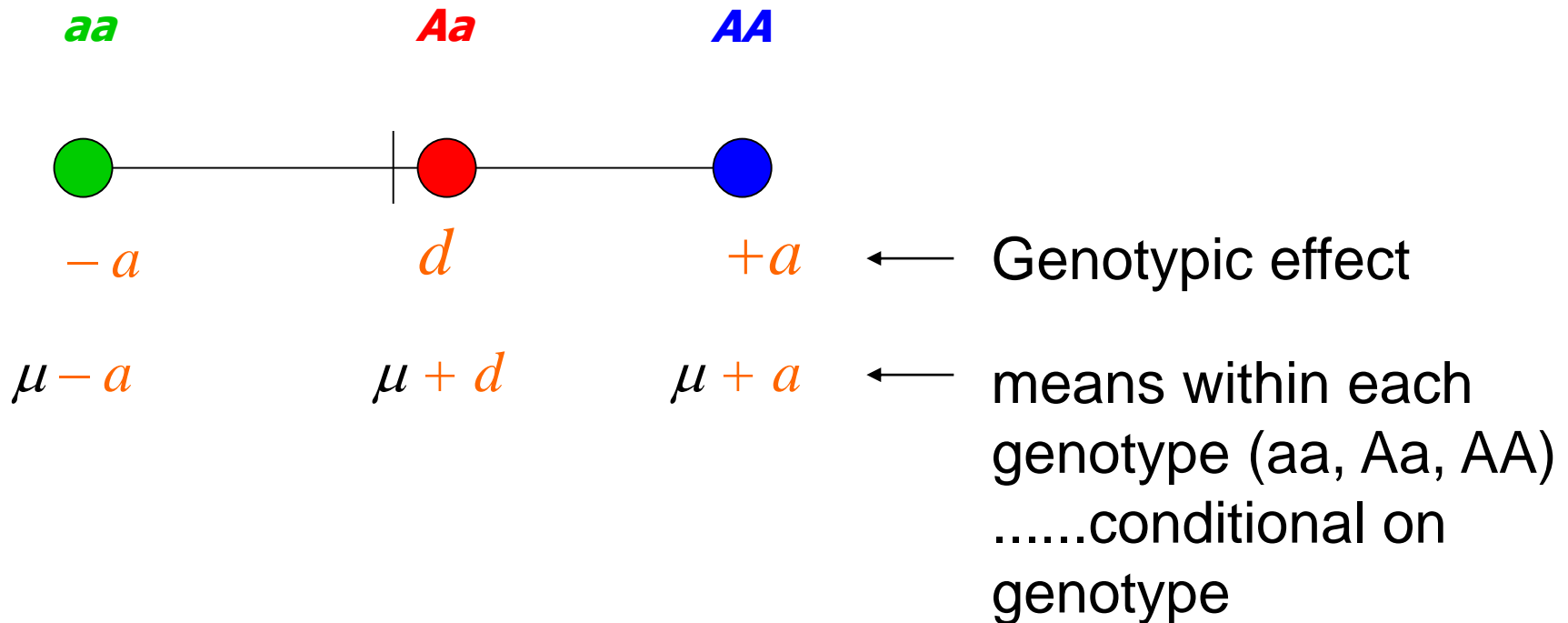
$$P(aa) = q^2$$

$$p^2 + 2pq + q^2 = 1$$



# Phenotype level: contribution to continuous variation

## Biometric Model



take all  $aa$  individuals and calculate their mean phenotypic value:  
 $\mu - a$  (the phenotypic mean **conditional** on genotype  $aa$ )

# Biometrical model for single biallelic QTL

---

## 1. Contribution of the QTL to the Mean

Genotypes	<b>AA</b>	<b>Aa</b>	<b>aa</b>
Effect, $x$	$\mu + \mathbf{a}$	$\mu + \mathbf{d}$	$\mu - \mathbf{a}$
Frequencies, $f(x)$	$\mathbf{p}^2$	$\mathbf{2pq}$	$\mathbf{q}^2$

$$\begin{aligned}(\mu + \mathbf{a})(\mathbf{p}^2) + (\mu + \mathbf{d})(\mathbf{2pq}) + (\mu - \mathbf{a})(\mathbf{q}^2) &= \\ \mu + \mathbf{a}(\mathbf{p}^2) + \mathbf{d}(\mathbf{2pq}) - \mathbf{a}(\mathbf{q}^2) &= \\ \mu + \mathbf{a}(\mathbf{p} - \mathbf{q}) + \mathbf{2pqd}. & \text{ (pop pheno mean)}\end{aligned}$$

$$\mu = E(X) = \sum_i x_i f(x_i)$$

**contribution of the QTL to the population phenotypic mean**

$$m = \mathbf{a}(\mathbf{p} - \mathbf{q}) + \mathbf{2pqd}$$

# Biometrical model for single biallelic QTL

---

## 2. Contribution of the QTL to the Variance (X)

Genotypes	<b>AA</b>	<b>Aa</b>	<b>aa</b>
Effect (x)	$\mu + \mathbf{a}$	$\mu + \mathbf{d}$	$\mu - \mathbf{a}$
Frequencies, $f(x)$	<b><math>p^2</math></b>	<b><math>2pq</math></b>	<b><math>q^2</math></b>

$$s^2_{QTL} = (\mathbf{a}-m)^2 p^2 + (\mathbf{d}-m)^2 2pq + (-\mathbf{a}-m)^2 q^2$$

$$m = \mathbf{a}(p-q) + 2pq\mathbf{d}$$

$$\begin{aligned} \text{Var}(X) &= E(X - \mu)^2 \\ &= \sum_i (x_i - \mu)^2 f(x_i) \end{aligned}$$

# Biometrical model for single biallelic QTL

---

$$\begin{aligned} s^2_{QTL} &= (a-m)^2 p^2 + (d-m)^2 2pq + (-a-m)^2 q^2 \\ &= \underline{2pq[a+(q-p)d]^2} + \underline{(2pqd)^2} \\ &= s^2_{QTL(A)} + s^2_{QTL(D)} \end{aligned}$$

Additive or linear effects give rise to variance component

$$s^2_{QTL(A)} = 2^* pq[a+(q-p)d]^2$$

Dominance or *within local allelic interaction* effects give rise to variance component

$$s^2_{QTL(D)} = (2pqd)^2$$

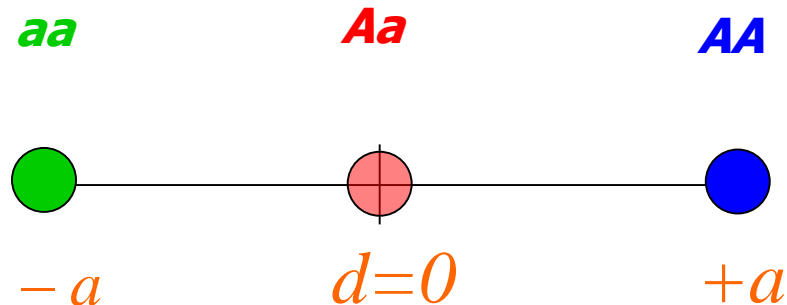
# Biometrical model for single biallelic QTL

---

$$\begin{aligned} s^2_{QTL} &= (a-m)^2 p^2 + (d-m)^2 2pq + (-a-m)^2 q^2 \\ &= \underline{2pq[a+(q-p)d]^2} + \underline{(2pqd)^2} \\ &= s^2_{QTL(A)} + s^2_{QTL(D)} \end{aligned}$$

Additive effects:  $s^2_{QTL(A)} = 2 * pq[a]^2$

Dominance effects:  $s^2_{QTL(D)} = 0$



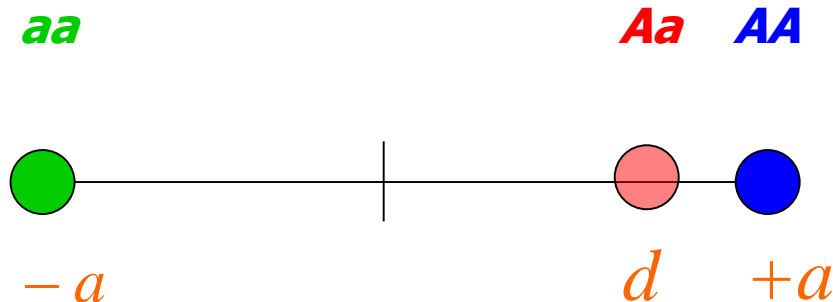
# Biometrical model for single biallelic QTL

---

$$\begin{aligned} s^2_{QTL} &= (a-m)^2 p^2 + (d-m)^2 2pq + (-a-m)^2 q^2 \\ &= \underline{2pq[a+(q-p)d]^2} + \underline{(2pqd)^2} \\ &= s^2_{QTL(A)} + s^2_{QTL(D)} \end{aligned}$$

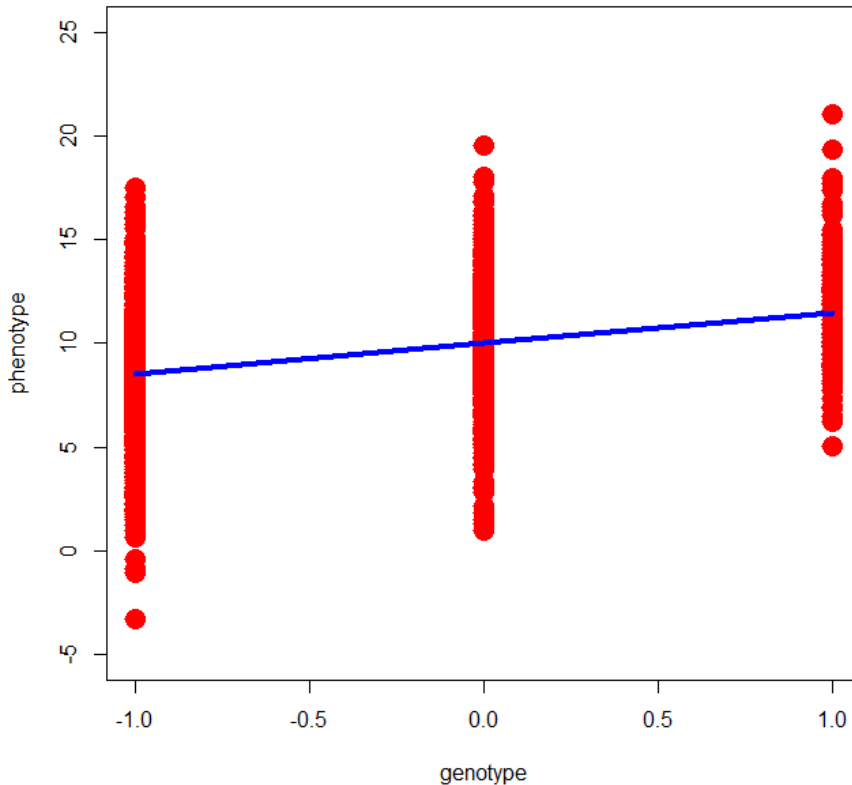
Additive effects:  $s^2_{QTL(A)} = 2^* pq[a+(q-p)d]^2$

Dominance effects:  $s^2_{QTL(D)} = (2pqd)^2$



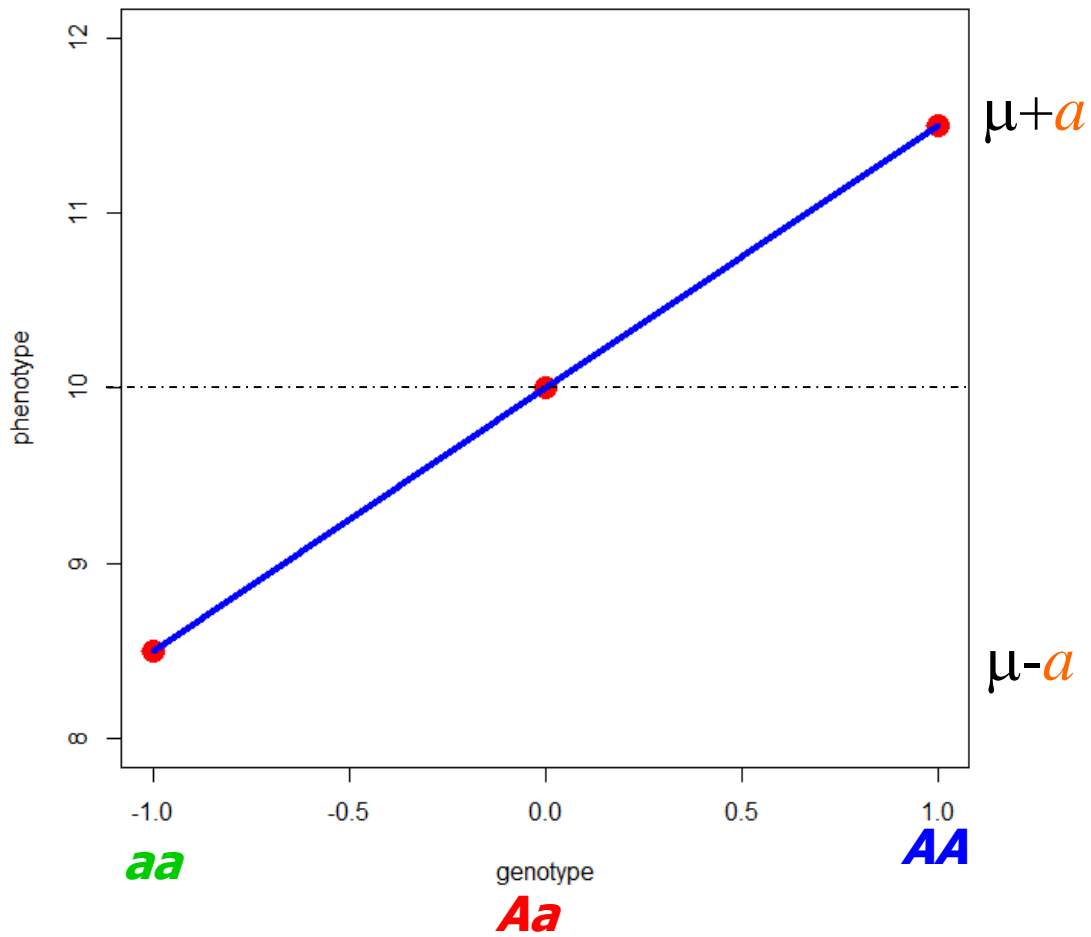


Suppose we measure the QTL and the phenotype and regress  $X$  on QTL. The scatterplot of the data (aa coded -1; Aa coded 0; AA coded 1).



we ask:  
*how much of the phenotypic variance is explained by the predictor (QTL)*

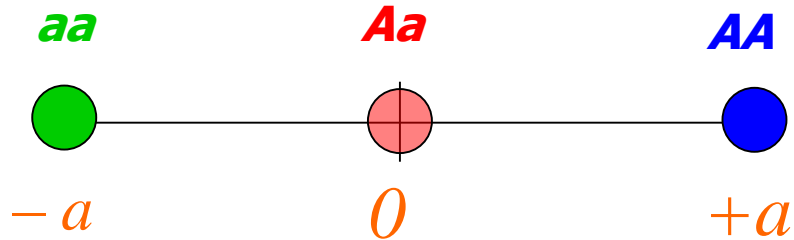
In the following slides we look at the regression lines only (not plotting the **residuals** – just to avoid clutter).

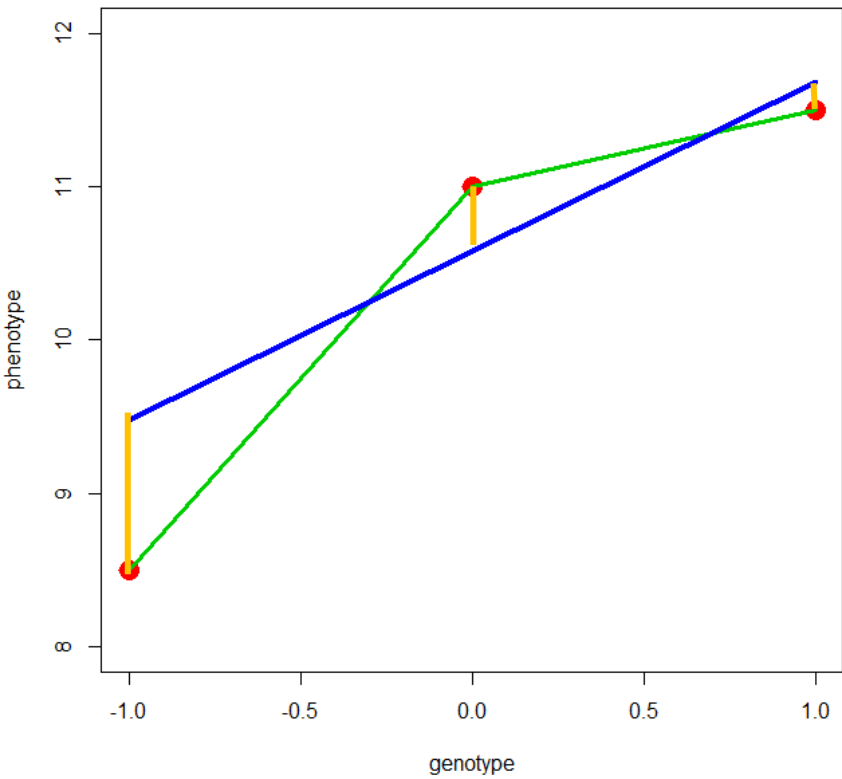


*Explained variance:*

$$s^2_{QTL(A)} = 2 * pq[a]^2$$

$$s^2_{QTL(D)} = 0$$





$\mu + a$

$\mu + d$

*Explained variance (blue line):*

$$S^2_{QTL(A)} = 2 * pq [a + (q-p)d]^2$$

*Not explained*

$$S^2_{QTL(D)} = (2pqd)^2$$

$\mu - a$

**aa**

**AA**

**Aa**

**aa**

**Aa**

**AA**

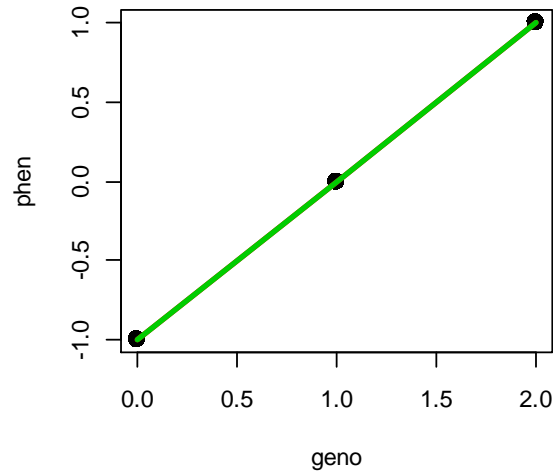


$-a$

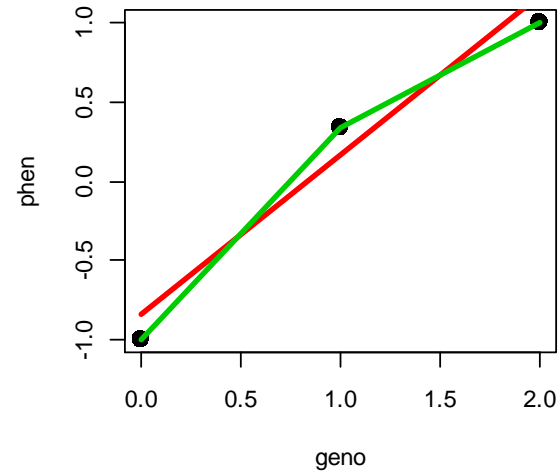
$d$

$+a$

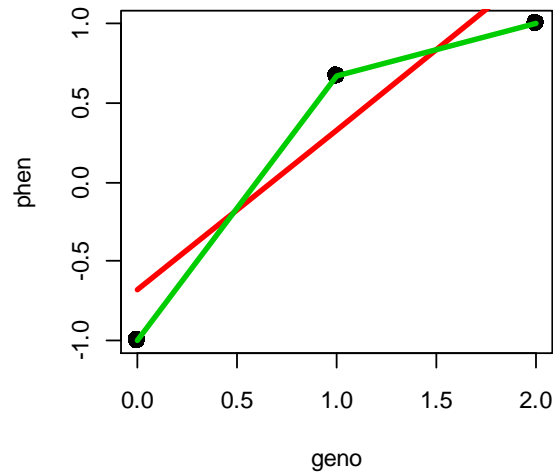
explained by additive model 1



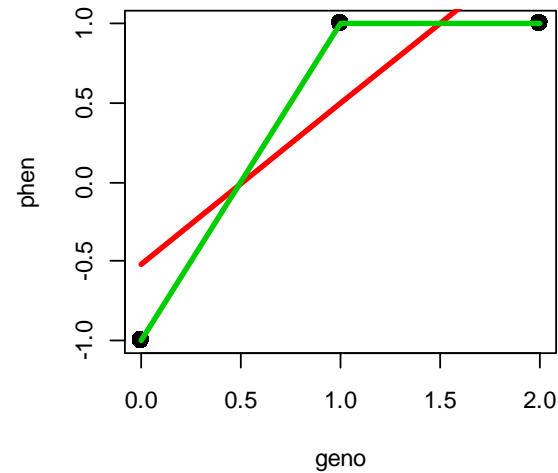
explained by additive model 0.949



explained by additive model 0.823



explained by additive model 0.676



$s^2_{QTL(A)}$  always greater than zero

$s^2_{QTL(D)}$  can be zero (additive model  $d=0$ )

# Biometrical model for single biallelic QTL

---

## 3. Contribution of the QTL to the Cov (X, Y)

$$\text{Cov}(X, Y) = \sum_i (x_i - \mu_X)(y_i - \mu_Y) f(x_i, y_i)$$

	<b>AA</b> ( <b>a-m</b> )	<b>Aa</b> ( <b>d-m</b> )	<b>aa</b> ( <b>-a-m</b> )
<b>AA</b> ( <b>a-m</b> )	$(a-m)^2$	$(a-m)(d-m)$	$(a-m)(-a-m)$
<b>Aa</b> ( <b>d-m</b> )	$(a-m)(d-m)$	$(d-m)^2$	$(d-m)(-a-m)$
<b>aa</b> ( <b>-a-m</b> )	$(a-m)(-a-m)$	$(d-m)(-a-m)$	$(-a-m)^2$

$$m = a(p-q) + 2pqd$$

What about the  $f(x_i, y_i)$ ?

# Biometrical model for single biallelic QTL

---

## 3A. Contribution of the QTL to the Cov (X, Y) – MZ twins

$$\text{Cov}(X, Y) = \sum_i (x_i - \mu_X)(y_i - \mu_Y) f(x_i, y_i)$$

	<b>AA</b> (a-m)	<b>Aa</b> (d-m)	<b>aa</b> (-a-m)
<b>AA</b> (a-m)	$p^2(a-m)^2$	$0(a-m)(d-m)$	$0(a-m)(-a-m)$
<b>Aa</b> (d-m)	$0(a-m)(d-m)$	$2pq(d-m)^2$	$0(d-m)(-a-m)$
<b>aa</b> (-a-m)	$0(a-m)(-a-m)$	$0(d-m)(-a-m)$	$q^2(-a-m)^2$

$$\begin{aligned} \text{Covar}(X_i, X_j) &= (a-m)^2 p^2 + (d-m)^2 2pq + (-a-m)^2 q^2 \\ &= 2pq[a + (q-p)d]^2 + (2pqd)^2 = S^2_{\text{QTL}(A)} + S^2_{\text{QTL}(D)} \end{aligned}$$

# Biometrical model for single biallelic QTL

---

## 3B. Contribution of the QTL to the Cov (X, Y) – Parent-Offspring

$$Cov(X, Y) = \sum_i (x_i - \mu_X)(y_i - \mu_Y) f(x_i, y_i)$$

---

	<b>AA</b> ( <i>a-m</i> )	<b>Aa</b> ( <i>d-m</i> )	<b>aa</b> ( <i>-a-m</i> )
<b>AA</b> ( <i>a-m</i> )	$p^3(a-m)^2$	$p^2q(a-m)(d-m)$	$0(a-m)(-a-m)$
<b>Aa</b> ( <i>d-m</i> )	$p^2q(a-m)(d-m)$	$pq(d-m)^2$	$pq^2(d-m)(-a-m)$
<b>aa</b> ( <i>-a-m</i> )	$0(a-m)(-a-m)$	$pq^2(d-m)(-a-m)$	$q^3(-a-m)^2$

---

given an  $AA$  parent, an  $AA$  offspring can come from either  $AA$  x  $AA$  or  $AA$  x  $Aa$  parental mating types

$AA$  x  $AA$  will occur  $p^2 \times p^2 = p^4$   
and have  $AA$  offspring  $\text{Prob}(AA)=1$

$AA$  x  $Aa$  will occur  $p^2 \times 2pq = 2p^3q$   
and have  $AA$  offspring  $\text{Prob}(AA)=0.5$   
and have  $Aa$  offspring  $\text{Prob}(Aa)=0.5$

$$\begin{aligned}\text{Therefore, P}(AA \text{ parent \& } AA \text{ offspring}) &= p^4 + .5 * 2 * p^3q \\ &= p^3(p+q) \\ &= p^3\end{aligned}$$



So can be complicated, but can also be simple ....

		Parent		
		$AA$ ( $a-m$ )	$Aa$ ( $d-m$ )	$aa$ ( $-a-m$ )
Offspring	$AA$ ( $a-m$ )	$p^3(a-m)^2$	$p^2q(a-m)(d-m)$	$0(a-m)(-a-m)$
	$Aa$ ( $d-m$ )	$p^2q(a-m)(d-m)$	$pq(d-m)^2$	$pq^2(d-m)(-a-m)$
	$aa$ ( $-a-m$ )	$0(a-m)(-a-m)$	$pq^2(d-m)(-a-m)$	$q^3(-a-m)^2$

why zero probability?

# Biometrical model for single biallelic QTL

## 3B. Contribution of the QTL to the Cov (X, Y) – Parent-Offspring

	<b>AA</b> (a-m)	<b>Aa</b> (d-m)	<b>aa</b> (-a-m)
<b>AA</b> (a-m)	$p^3(a-m)^2$	$p^2q(a-m)(d-m)$	$0(a-m)(-a-m)$
<b>Aa</b> (d-m)	$p^2q(a-m)(d-m)$	$pq(d-m)^2$	$pq^2(d-m)(-a-m)$
<b>aa</b> (-a-m)	$0(a-m)(-a-m)$	$pq^2(d-m)(-a-m)$	$q^3(-a-m)^2$

$$\begin{aligned}
 \text{Cov}(X_i, X_j) &= (a-m)^2 p^3 + \dots + (-a-m)^2 q^3 \\
 &= pq[a+(q-p)d]^2 = \frac{1}{2} S^2_{\text{QTL}(A)}
 \end{aligned}$$

# Biometrical model for single biallelic QTL

---

## 3C. Contribution of the QTL to the Cov (X, Y) – Unrelated individuals

	$p^2$ <b>AA</b> (a-m)	$2pq$ <b>Aa</b> (d-m)	$q^2$ <b>aa</b> (-a-m)
$p^2$ <b>AA</b> (a-m)	$p^4(a-m)^2$	$2p^3q(a-m)(d-m)$	$p^2q^2(a-m)(-a-m)$
$2pq$ <b>Aa</b> (d-m)	$2p^3q(a-m)(d-m)$	$4p^2q^2(d-m)^2$	$2pq^3(d-m)(-a-m)$
$q^2$ <b>aa</b> (-a-m)	$p^2q^2(a-m)(-a-m)$	$2pq^3(d-m)(-a-m)$	$q^4(-a-m)^2$

---

$$\begin{aligned} \text{Cov}(X_i, X_j) &= (a-m)^2 p^4 + \dots + (-a-m)^2 q^4 \\ &= 0 \end{aligned}$$

Follow same method for full sibs and DZ twins  
 Derive genotype frequencies ....

s1	s2	eff	eff		frequency (p(A)=p, p(a)=1-p)
AA	AA	a	a	r1	$p^{**4}+p^{**3}*q+p^{**2}*q^{**2}/4$
aa	aa	-a	-a	r2	$p^{**2}*q^{**2}/4+p*q^{**3}+q^{**4}$
Aa	Aa	d	d	r3	$p^{**3}*q+3*p^{**2}*q^{**2}+p*q^{**3}$
AA	Aa	a	d	r4	$p^{**3}*q+p^{**2}*q^{**2}/2$
Aa	AA	d	a	r4	$p^{**3}*q+p^{**2}*q^{**2}/2$
Aa	aa	d	-a	r5	$p^{**2}*q^{**2}/2+p*q^{**3}$
aa	Aa	-a	d	r5	$p^{**2}*q^{**2}/2+p*q^{**3}$
AA	aa	a	-a	r6	$p^{**2}*q^{**2}/4$
aa	AA	-a	a	r6	$p^{**2}*q^{**2}/4$

# Biometrical model for single biallelic QTL

## 3B. Contribution of the QTL to the Cov (X, Y) – DZ twins

	<b>AA</b> (a-m)	<b>Aa</b> (d-m)	<b>aa</b> (-a-m)
<b>AA</b> (a-m)	<b>r1</b> (a-m) <sup>2</sup>	<b>r4</b> (a-m) (d-m)	<b>r6</b> (a-m) (-a-m)
<b>Aa</b> (d-m)	<b>r4</b> (a-m) (d-m)	<b>r2</b> (d-m) <sup>2</sup>	<b>r5</b> (d-m) (-a-m)
<b>aa</b> (-a-m)	<b>r6</b> (a-m) (-a-m)	<b>r5</b> (d-m) (-a-m)	<b>r3</b> (-a-m) <sup>2</sup>

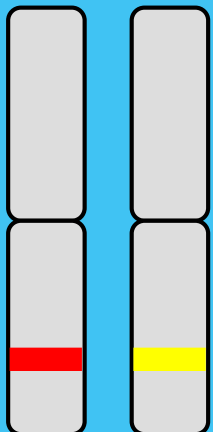
$$\text{Cov}(X_i, X_j) = (a-m)^2 r1 + \dots + (-a-m)^2 r3$$

$$= \frac{1}{2} 2pq[a+(q-p)d]^2 + \frac{1}{4} (2pqd)^2 = \frac{1}{2} S^2_{\text{QTL(A)}} + \frac{1}{4} S^2_{\text{QTL(D)}}$$

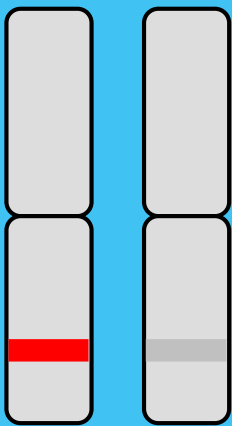
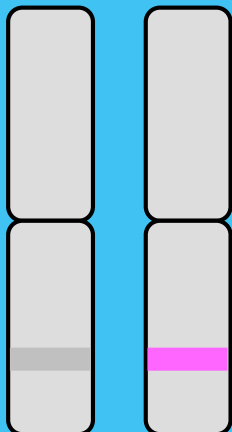


John Cleese ... A famous British person

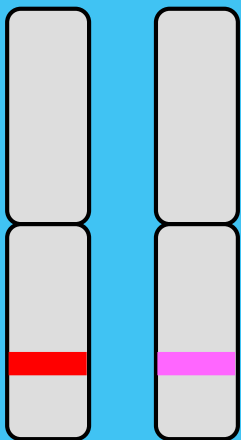
# Random segregation and identity-by-descent (IBD) in sibpairs



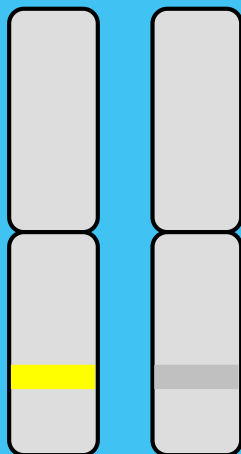
x



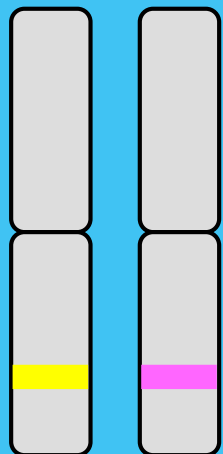
1/4



1/4




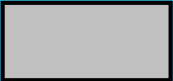
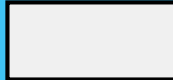
1/4



1/4

# IDENTITY BY DESCENT (IBD) MZs



- 
100% MZ sibs share BOTH parental alleles IBD = 2
- 
0 sibs share ONE parental allele IBD = 1
- 
0 sibs share NO parental alleles IBD = 0



# IDENTITY BY DESCENT (IBD) DZs

Sib 1



Sib 2



2	1	1	0
1	2	0	1
1	0	2	1
0	1	1	2



$4/16 = 1/4$  sibs share BOTH parental alleles IBD = 2

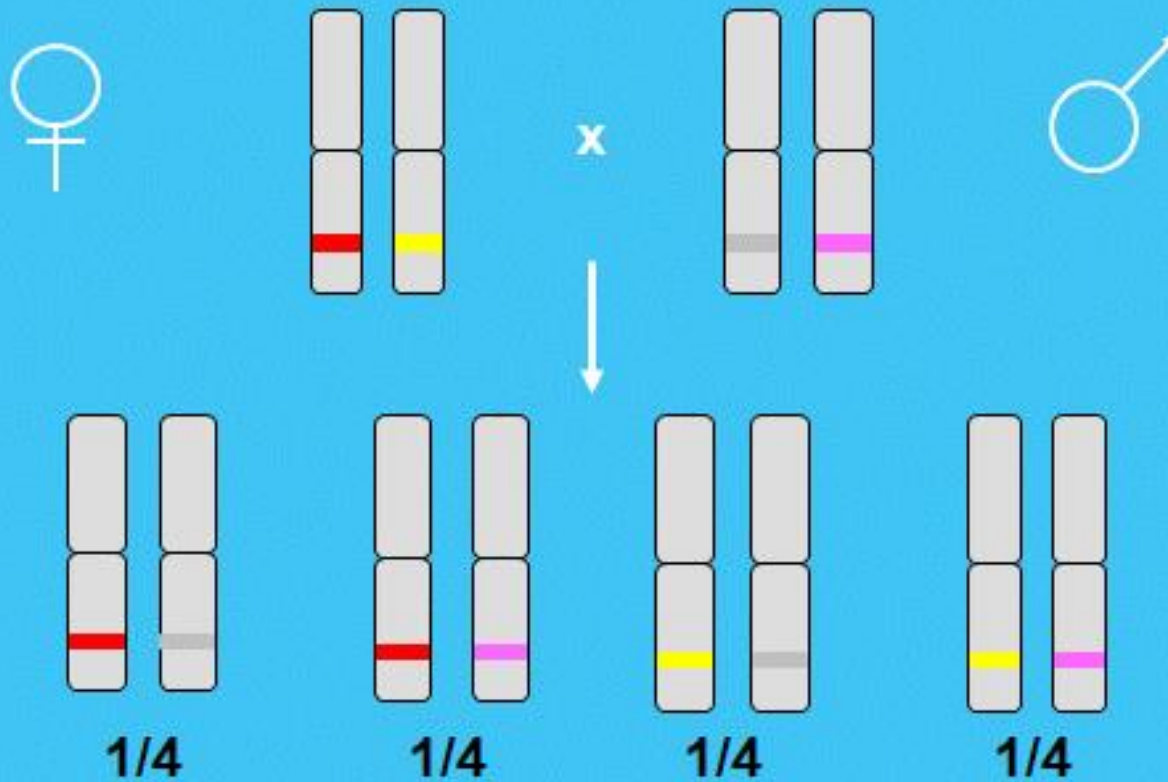


$8/16 = 1/2$  sibs share ONE parental allele IBD = 1



$4/16 = 1/4$  sibs share NO parental alleles IBD = 0

# Random segregation and identity-by-descent (IBD) in sibpairs



What about parent offspring?  
many alleles do they share IBD?

# Biometrical model for single biallelic QTL

## 3D. Contribution of the QTL to the Cov (X, Y) – DZ twins and full sibs

# identical alleles  
inherited from  
parents

**2**

**1**

(father)

**1**

(mother)

**0**

$\frac{1}{4}$  (2 alleles)

+  $\frac{1}{2}$  (1 allele) +

$\frac{1}{4}$  (0 alleles)

*MZ twins*

*P-O*

*Unrelateds*

$$\begin{aligned}
 \text{DZ} &= \frac{1}{4} \text{Cov}(MZ) + \frac{1}{2} \text{Cov}(P-O) + \frac{1}{4} \text{Cov}(Unrel) \\
 \text{Cov}(X_i, X_j) &= \frac{1}{4} (S^2_{QTL(A)} + S^2_{QTL(D)}) + \frac{1}{2} (\frac{1}{2} S^2_{QTL(A)}) + \frac{1}{4} (0) \\
 &= \frac{1}{2} S^2_{QTL(A)} + \frac{1}{4} S^2_{QTL(D)}
 \end{aligned}$$

Biometrical model predicts contribution of a QTL to the mean, variance and covariances of a trait (discarding environmental effects)

### 1 QTL

$$\text{Var}(X) = s^2_{QTL(A)} + s^2_{QTL(D)}$$

$$\text{Cov}(MZ) = s^2_{QTL(A)} + s^2_{QTL(D)}$$

$$\text{Cov}(DZ) = \frac{1}{2} s^2_{QTL(A)} + \frac{1}{4} s^2_{QTL(D)}$$

$$\text{Cov}(P-O) = \frac{1}{2} s^2_{QTL(A)}$$

### Multiple QTL

$$\text{Var}(X) = \sum(s^2_{QTL(A)}) + \sum(s^2_{QTL(D)}) = V_A + V_D$$

$$\text{Cov}(MZ) = \sum(s^2_{QTL(A)}) + \sum(s^2_{QTL(D)}) = V_A + V_D$$

$$\text{Cov}(DZ) = \sum(\frac{1}{2} s^2_{QTL(A)}) + \sum(\frac{1}{4} s^2_{QTL(D)}) = \frac{1}{2} V_A + \frac{1}{4} V_D$$

$$\text{Cov}(P-O) = \sum(\frac{1}{2} s^2_{QTL(A)}) = \frac{1}{2} V_A$$

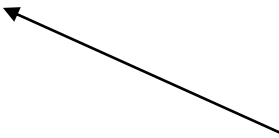
# Contributions of $V_A$ and $V_D$ to covariances between relatives

Relationship	contribution of QTL to covariance	
	$V_A$	$V_D$
Sibling (DZ twin)	$\frac{1}{2}$	$\frac{1}{4}$
MZ twin	1	1
Half-sibling	$\frac{1}{4}$	0
First cousin	$\frac{1}{8}$	0
Parent-offspring	$\frac{1}{2}$	0
Avuncular	$\frac{1}{4}$	0
Grand-parent	$\frac{1}{8}$	0
Unrelated	0	0

average proportion of alleles shared identically by descent

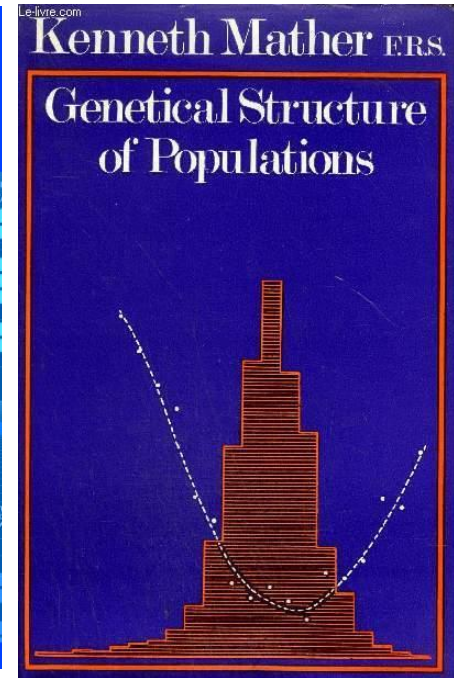
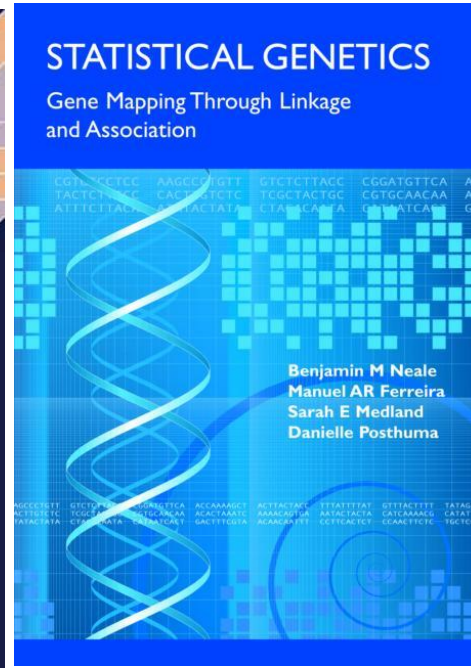
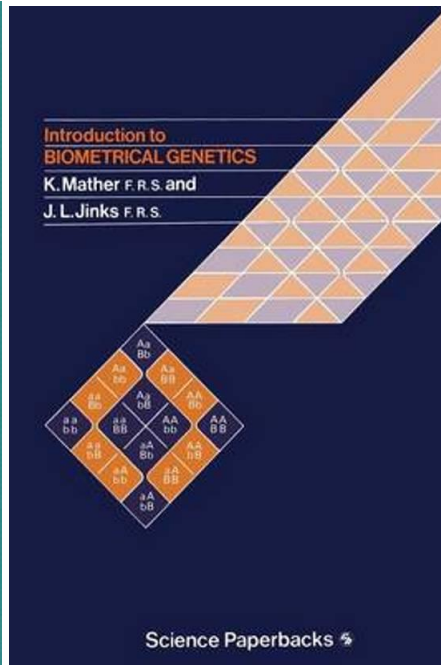
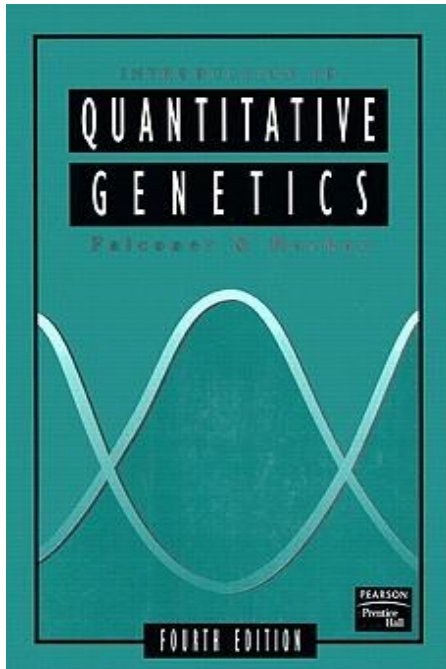


proportion of IBD=2



These proportions tell use how much of  $V_A$  &  $V_D$  contribute to the phenotypic covariance among family members (useful info in extended twin design / extended pedigrees)

**Slide acknowledgement:** Manuel Ferreira, Pak Sham, Shaun Purcell, Sarah Medland, and Sophie van der Sluis



see also Manuel A.R. Ferreira's  
<http://slidegur.com/doc/4322268/biometrical-genetics>

sgene.exe

