Simplex Model *Practical*

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&

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Data

562 twin pairs: 261 MZ and 301 DZ twin pairs

Time points: 5.5y, 6.8y, 9.7y and 12.2y

```
> head(data)
  zyg IQ11 IQ21 IQ31 IQ41 IQ12 IQ22 IQ32 IQ42
1   1   NA   NA   NA  111   NA   NA   NA  114
2   1   NA   NA   78   NA   NA   NA   78   NA
3   1   NA   NA   78   NA   NA   NA   89   NA
4   1   NA   NA   78   84   NA   NA   88   94
5   1   NA   NA   79   80   NA   NA   95   87
6   1   NA   NA   79   85   NA   NA   82   78
```

The proportions of observed FSIQ data:
0.812, 0.295, 0.490, 0.828 (MZ twin 1)
0.812, 0.295, 0.490, 0.828 (MZ twin 2)
0.774, 0.379, 0.598, 0.797 (DZ twin 1)
0.774, 0.379, 0.598, 0.797 (DZ twin 2)

Models

1) Saturated Model

2) ACE Cholesky Model:

Smz	=	SA+SC+SE	SA+SC
		SA+SC	SA+SC+SE
Sdz	=	SA+SC+SE	.5*SA+SC
		.5*SA+SC	SA+SC+SE

3) Simplex Model (with simple SE model):

Smz	=	SA+SC+SE	SA+SC
		SA+SC	SA+SC+SE
Sdz	=	SA+SC+SE	.5*SA+SC
		.5*SA+SC	SA+SC+SE

SA= $(I-B_A)\ \Psi_A\ (I-B_A)^t + \Theta_A$

SC= $(I-B_C)\ \Psi_C\ (I-B_C)^t + \Theta_C$

SE= Θ_E

Saturated Model

	IQ11	**IQ21**	**IQ31**	**IQ41**	**IQ12**	**IQ22**	**IQ32**	**IQ42**
IQ11	var1							MZ ≠ DZ
IQ21	cov21	var2						
IQ31	cov31	cov32	var3					
IQ41	cov41	cov42	cov43	var4				
IQ12	cov51	cov52	cov53	cov54	var5			
IQ22	cov61	cov62	cov63	cov64	cov65	var6		
IQ32	cov71	cov72	cov73	cov74	cov75	cov76	var7	
IQ42	cov81	cov82	cov83	cov84	cov85	cov86	cov87	var8

	IQ11	**IQ21**	**IQ31**	**IQ41**	**IQ12**	**IQ22**	**IQ32**	**IQ42**	
mean	m1	m2	m3	m4	m1	m2	m3	m4	MZ = DZ

N pars: 8 vars (MZ ≠ DZ) + (8*(8-1))/2 covs (MZ ≠ DZ) + 4 means (MZ = DZ) = **76**

Saturated Model

MZ: within-twin cross-time point (phenotypic correlations)

	IQ11	IQ21	IQ31	IQ41	IQ12	IQ22	IQ32	IQ42
IQ11	1							
IQ21	0.650	1						
IQ31	0.523	0.748	1					
IQ41	0.409	0.608	0.775	1				
IQ12	0.769	0.655	0.609	0.549	1			
IQ22	0.510	0.696	0.745	0.723	0.572	1		
IQ32	0.482	0.665	0.84	0.747	0.550	0.782	1	
IQ42	0.455	0.582	0.757	0.799	0.613	0.658	0.760	1

5.5y	6.8y	9.7y	12.2y
108.9	104.0	105.4	99.6

Saturated Model

DZ: within-twin cross-time point (phenotypic correlations)

	IQ11	IQ21	IQ31	IQ41	IQ12	IQ22	IQ32	IQ42
IQ11	1							
IQ21	0.603	1						
IQ31	0.475	0.661	1					
IQ41	0.471	0.673	0.737	1				
IQ12	0.641	0.298	0.283	0.258	1			
IQ22	0.397	0.481	0.374	0.346	0.481	1		
IQ32	0.201	0.317	0.483	0.368	0.361	0.627	1	
IQ42	0.248	0.396	0.469	0.501	0.345	0.635	0.707	1

5.5y	6.8y	9.7y	12.2y
108.9	104.0	105.4	99.6

Saturated Model

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IQ21	0.650	1						
IQ31	0.523	0.748	1					
IQ41	0.409	0.608	0.775	1				
IQ12	0.769	0.655	0.609	0.549	1			
IQ22	0.510	0.696	0.745	0.723	0.572	1		
IQ32	0.482	0.665	0.840	0.747	0.550	0.782	1	
IQ42	0.455	0.582	0.757	0.799	0.613	0.658	0.760	1

Saturated Model

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Saturated Model

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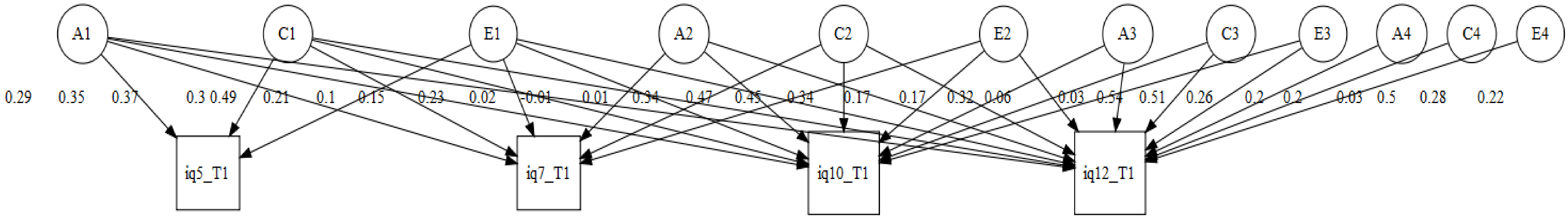
IQ42	0.248	0.396	0.469	0.501	0.345	0.635	0.707	1

ACE Cholesky Model



ACE Cholesky Model
Path Loading Approach

SA_est

5.5y	6.8y	9.7y	12.2y
62.920	69.975	81.814	64.865
69.975	88.266	106.255	91.796
81.814	106.255	128.700	113.079
64.865	91.796	113.079	103.869

SC_est

5.5y	6.8y	9.7y	12.2y
104.240	49.655	23.528	30.271
49.655	63.658	37.981	39.704
23.528	37.981	61.189	45.872
30.271	39.704	45.872	58.479

SE_est

5.5y	6.8y	9.7y	12.2y
48.634	6.059	-2.670	0.948
6.059	66.150	12.827	8.283
-2.670	12.827	45.880	5.780
0.948	8.283	5.780	46.140

RA_est

5.5y	6.8y	9.7y	12.2y
1.000	0.939	0.909	0.802
0.939	1.000	0.997	0.959
0.909	0.997	1.000	0.978
0.802	0.959	0.978	1.000

RC_est

5.5y	6.8y	9.7y	12.2y
1.000	0.610	0.295	0.388
0.610	1.000	0.609	0.651
0.295	0.609	1.000	0.767
0.388	0.651	0.767	1.000

RE_est

5.5y	6.8y	9.7y	12.2y
1.000	0.107	-0.057	0.020
0.107	1.000	0.233	0.150
-0.057	0.233	1.000	0.126
0.020	0.150	0.126	1.000

-2LL = 21378.13

ACE Cholesky Model
Direct Variance Estimation Approach

> SA_est

	iq5_T1	iq7_T1	iq10_T1	iq12_T1
iq5_T1	62.207	75.761	82.891	64.226
iq7_T1	75.761	73.828	107.149	95.969
iq10_T1	82.891	107.149	128.134	114.209
iq12_T1	64.226	95.969	114.209	103.738

> SC_est

	iq5_T1	iq7_T1	iq10_T1	iq12_T1
iq5_T1	104.775	45.507	22.743	30.767
iq7_T1	45.507	73.423	37.561	36.376
iq10_T1	22.743	37.561	61.629	44.951
iq12_T1	30.767	36.376	44.951	58.563

> SE_est

	iq5_T1	iq7_T1	iq10_T1	iq12_T1
iq5_T1	48.815	4.399	-2.917	1.064
iq7_T1	4.399	70.607	12.429	7.145
iq10_T1	-2.917	12.429	46.106	5.583
iq12_T1	1.064	7.145	5.583	46.185

> RA_est

	iq5_T1	iq7_T1	iq10_T1	iq12_T1
iq5_T1	1.000	1.118	0.928	0.800
iq7_T1	1.118	1.000	1.102	1.097
iq10_T1	0.928	1.102	1.000	0.991
iq12_T1	0.800	1.097	0.991	1.000

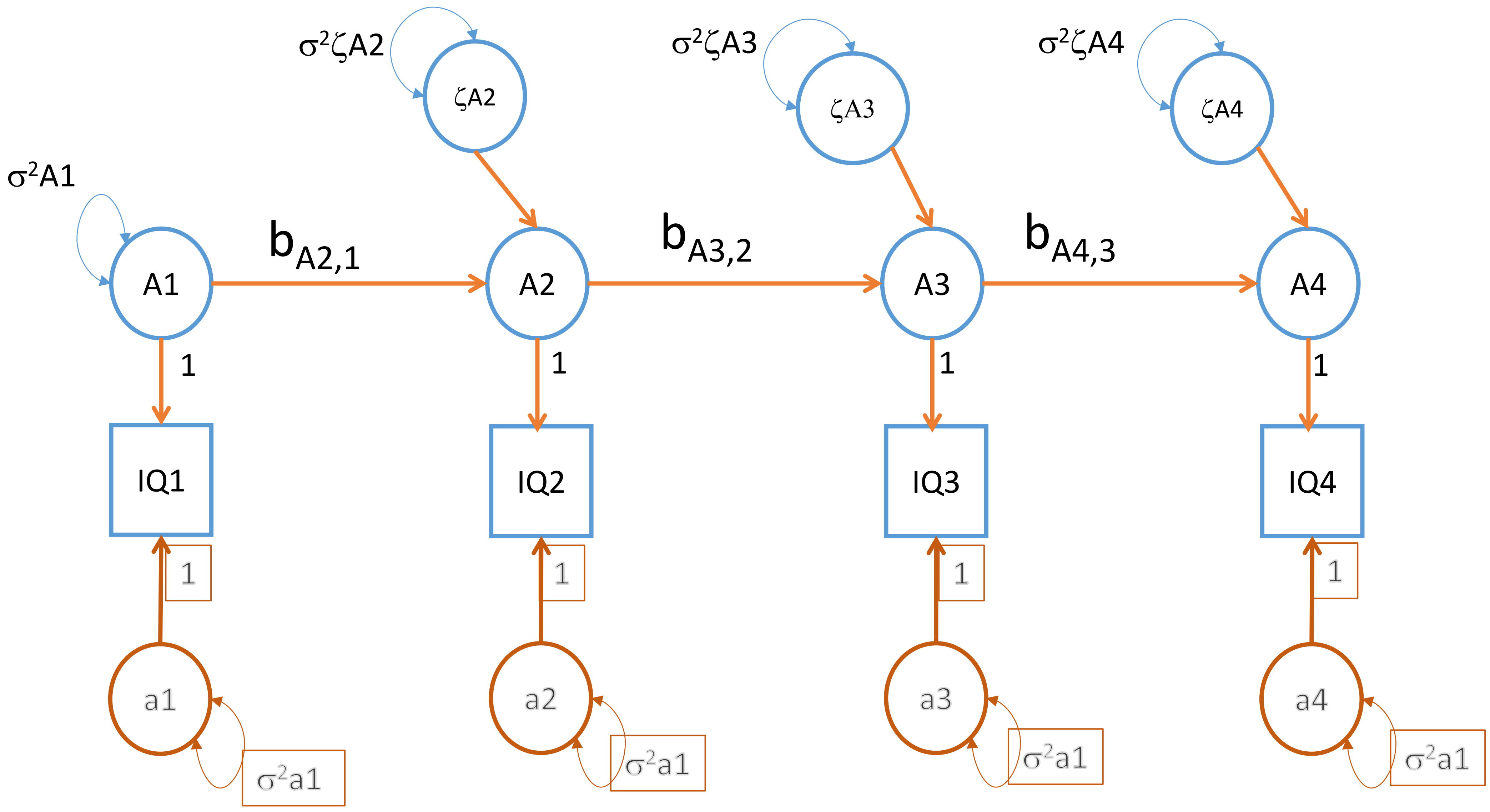
> RC_est

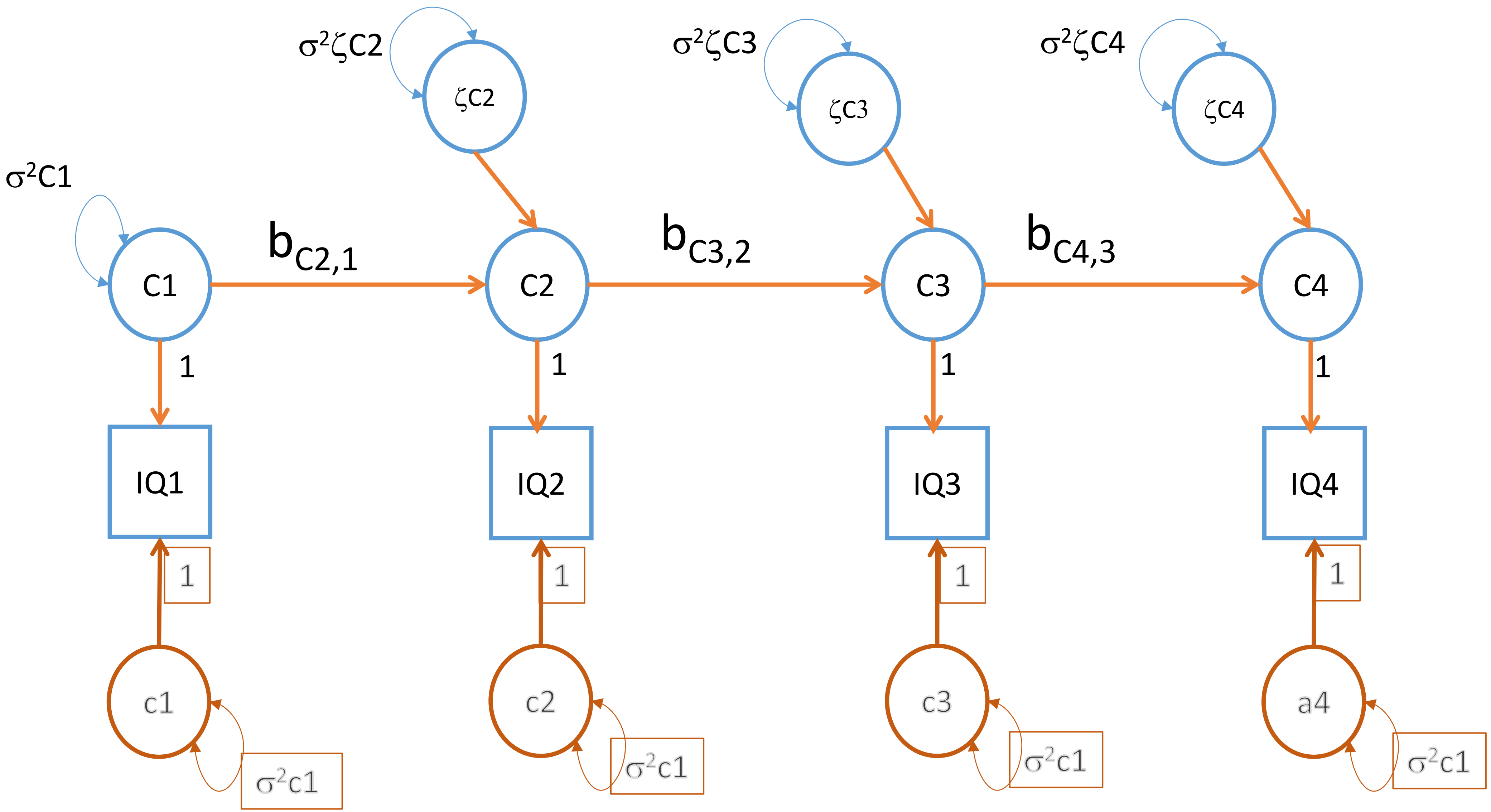
	iq5_T1	iq7_T1	iq10_T1	iq12_T1
iq5_T1	1.000	0.519	0.283	0.393
iq7_T1	0.519	1.000	0.558	0.555
iq10_T1	0.283	0.558	1.000	0.748
iq12_T1	0.393	0.555	0.748	1.000

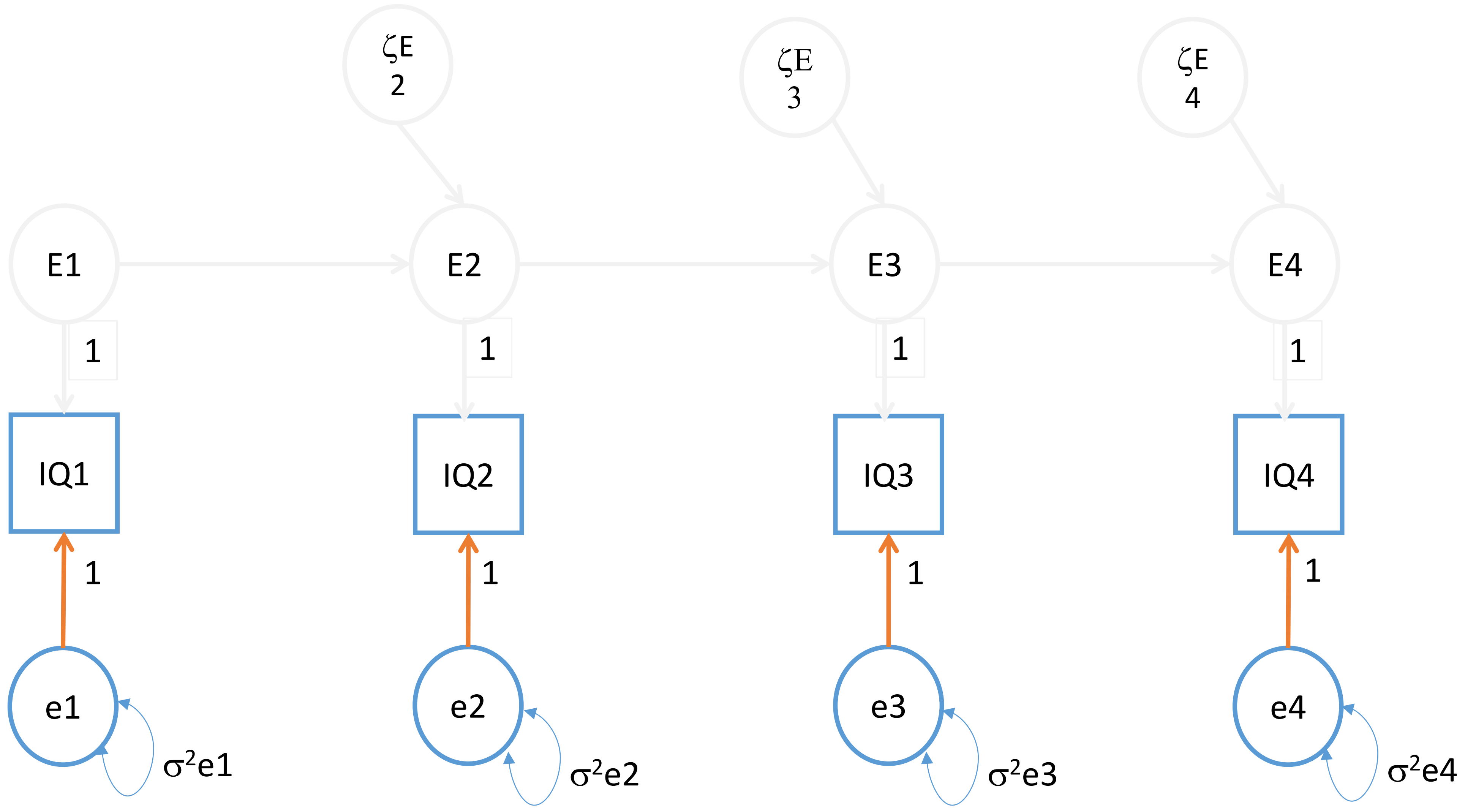
> RE_est

	iq5_T1	iq7_T1	iq10_T1	iq12_T1
iq5_T1	1.000	0.075	-0.061	0.022
iq7_T1	0.075	1.000	0.218	0.125
iq10_T1	-0.061	0.218	1.000	0.121
iq12_T1	0.022	0.125	0.121	1.000

-2LL = 21377.33


σ²ζA2
ζA2
σ²ζA3
ζA3
σ²ζA4
ζA4
σ²A1
A1
b A2,1
A2
b A3,2
A3
b A4,3
A4
1
1
1
1
IQ1
IQ2
IQ3
IQ4
1
1
1
1
a1
a2
a3
a4
σ²a1
σ²a1
σ²a1
σ²a1


σ²ζC2
ζC2
σ²ζC3
ζC3
σ²ζC4
ζC4
σ²C1
C1
b C2,1
C2
b C3,2
C3
b C4,3
C4
1
1
1
1
IQ1
IQ2
IQ3
IQ4
1
1
1
1
c1
c2
c3
a4
σ²c1
σ²c1
σ²c1
σ²c1


ζE 2
ζE 3
ζE 4
E1
E2
E3
E4
1
1
1
1
IQ1
IQ2
IQ3
IQ4
1
1
1
1
e1
e2
e3
e4
σ2e1
σ2e2
σ2e3
σ2e4

Model Comparison

```
         base comparison ep minus2LL   df      AIC   diffLL diffdf          p
1 twinmodel0       <NA> 76 21316.52 2724 15868.52       NA     NA         NA
2 twinmodel0 twinmodel1 34 21378.13 2766 15846.13 61.60317     42 0.02587147
```

Saturated vs direct (co)variance components

```
         base comparison ep minus2LL   df      AIC   diffLL diffdf          p
1 twinmodel0       <NA> 76 21316.52 2724 15868.52       NA     NA         NA
2 twinmodel0 twinmodel1 34 21377.33 2766 15845.33 60.80739     42 0.03022203
```

Direct (co)var components vs Simplex

```
         base comparison    ep minus2LL df         AIC  diffLL diffdf         p
1 twinmodel1        <NA>    34 21377.33 2766 15845.33       NA     NA        NA
2 twinmodel1  twinmodel2    24 21386.74 2776 15834.74 9.41163     10 0.4935339
```

Simplex Model

```
> TeA_est
        [,1]    [,2]    [,3]    [,4]
[1,] -26.699   0.000   0.000   0.000
[2,]   0.000 -26.699   0.000   0.000
[3,]   0.000   0.000 -26.699   0.000
[4,]   0.000   0.000   0.000 -26.699
>
> PsA_est
       [,1]   [,2] [,3]   [,4]
[1,] 87.654  0.000  0.0  0.000
[2,]  0.000 64.492  0.0  0.000
[3,]  0.000  0.000  1.5  0.000
[4,]  0.000  0.000  0.0 24.933

> BeA_est
      [,1]  [,2]  [,3] [,4]
[1,] 0.000 0.000 0.000    0
[2,] 0.812 0.000 0.000    0
[3,] 0.000 1.137 0.000    0
[4,] 0.000 0.000 0.847    0
```

```
> TeC_est
       [,1]   [,2]   [,3]   [,4]
[1,] 27.805  0.000  0.000  0.000
[2,]  0.000 27.805  0.000  0.000
[3,]  0.000  0.000 27.805  0.000
[4,]  0.000  0.000  0.000 27.805

 > BeC_est
     [,1]  [,2]  [,3] [,4]
[1,] 0.00 0.000 0.000    0
[2,] 0.66 0.000 0.000    0
[3,] 0.00 0.493 0.000    0
[4,] 0.00 0.000 1.041    0

> PsC_est
       [,1]   [,2]  [,3]   [,4]
[1,] 78.235  0.000  0.00  0.000
[2,]  0.000 -4.846  0.00  0.000
[3,]  0.000  0.000 18.08  0.000
[4,]  0.000  0.000  0.00 -7.237
```

```
> TeE_est
       [,1]   [,2]   [,3]   [,4]
[1,] 48.938  0.000  0.000  0.000
[2,]  0.000 63.023  0.000  0.000
[3,]  0.000  0.000 48.063  0.000
[4,]  0.000  0.000  0.000 45.668
```

Simplex Model

Submodel 1: No Θ_A

Θ_A (fixed)

[1,]	0	0	0	0
[2,]	0	0	0	0
[3,]	0	0	0	0
[4,]	0	0	0	0

$$\Sigma_A = (I-B_A)^{-1}\, \Psi_A\, (I-B_A)^{-1\,t} \, \cancel{+\,\Theta_A}$$

	base	comparison	ep	minus2LL	df	AIC	diffLL	diffdf	p
1	twinmodel2	<NA>	24	21386.74	2776	15834.74	NA	NA	NA
2	twinmodel2	twinmodel2	23	21389.54	2777	15835.54	2.799244	1	0.09430878

Simplex Model

Submodel 1: No Θ_A

```
> TeC_est
      [,1]   [,2]   [,3]   [,4]
[1,] 10.102  0.000  0.000  0.000
[2,]  0.000 10.102  0.000  0.000
[3,]  0.000  0.000 10.102  0.000
[4,]  0.000  0.000  0.000 10.102
```

```
> TeE_est
      [,1]   [,2]   [,3] [,4]
[1,] 48.969  0.000  0.000  0.0
[2,]  0.000 59.801  0.000  0.0
[3,]  0.000  0.000 43.966  0.0
[4,]  0.000  0.000  0.000 45.8
```

```
> PsA_est
    [,1]   [,2]    [,3]    [,4]
[1,] 61.1  0.000   0.000  0.000
[2,]  0.0 44.823   0.000  0.000
[3,]  0.0  0.000 -21.189  0.000
[4,]  0.0  0.000   0.000 -4.991
```

```
> PsC_est
      [,1]  [,2]   [,3]  [,4]
[1,] 95.642 0.000  0.000  0.00
[2,]  0.000 0.338  0.000  0.00
[3,]  0.000 0.000 27.944  0.00
[4,]  0.000 0.000  0.000 11.11
```

```
> BeA_est
      [,1]  [,2]  [,3] [,4]
[1,] 0.000 0.000 0.000    0
[2,] 1.056 0.000 0.000    0
[3,] 0.000 1.207 0.000    0
[4,] 0.000 0.000 0.887    0
```

```
> BeC_est
      [,1]  [,2]  [,3] [,4]
[1,] 0.000 0.000 0.000    0
[2,] 0.597 0.000 0.000    0
[3,] 0.000 0.484 0.000    0
[4,] 0.000 0.000 0.947    0
```

Simplex Model

Submodel 2: No Ψ_A at t2,3,4

Ψ_A (fixed)

```
[1,] s2(A1)   0    0    0
[2,]      0   0    0    0
[3,]      0   0    0    0
[4,]      0   0    0    0
```

$$\Sigma_A = (I-B_A)^{-1}\,\Psi_A\,(I-B_A)^{-1\,t}\ \sout{+\ \Theta_A} =$$

	base	comparison	ep	minus2LL	df	AIC	diffLL	diffdf	p
1	twinmodel2	<NA>	23	21389.54	2777	15835.54	NA	NA	NA
2	twinmodel2	twinmodel2	20	21392.01	2780	15832.01	2.464199	3	0.4817959

Simplex Model

Submodel 2: No Ψ_A at t2,3,4

```
> TeC_est
      [,1]  [,2]  [,3]  [,4]
[1,] 5.768 0.000 0.000 0.000
[2,] 0.000 5.768 0.000 0.000
[3,] 0.000 0.000 5.768 0.000
[4,] 0.000 0.000 0.000 5.768
```

```
> TeE_est
      [,1]   [,2]   [,3]   [,4]
[1,] 52.98  0.000  0.000  0.000
[2,]  0.00 56.394  0.000  0.000
[3,]  0.00  0.000 43.964  0.000
[4,]  0.00  0.000  0.000 45.071
```

```
> PsA_est
      [,1] [,2] [,3] [,4]
[1,] 46.65    0    0    0
[2,]  0.00    0    0    0
[3,]  0.00    0    0    0
[4,]  0.00    0    0    0
```

```
> PsC_est
       [,1]  [,2]   [,3]   [,4]
[1,] 109.96 0.000  0.000  0.000
[2,]   0.00 8.813  0.000  0.000
[3,]   0.00 0.000 27.389  0.000
[4,]   0.00 0.000  0.000 14.952
```

```
> BeA_est
      [,1]  [,2]  [,3] [,4]
[1,] 0.000 0.000 0.000    0
[2,] 1.637 0.000 0.000    0
[3,] 0.000 1.082 0.000    0
[4,] 0.000 0.000 0.886    0
```

```
> BeC_est
      [,1]  [,2]  [,3] [,4]
[1,] 0.000 0.000 0.000    0
[2,] 0.434 0.000 0.000    0
[3,] 0.000 0.533 0.000    0
[4,] 0.000 0.000 0.844    0
```

$$\Sigma_A = (I-B_A)^{-1}\,\Psi_A\,(I-B_A)^{-1\,t} + \Theta_A$$

Ψ_A

[1,]	46.65	0.000	0.000	0.000
[2,]	0.000	0.000	0.000	0.000
[3,]	0.000	0.000	0.000	0.000
[4,]	0.000	0.000	0.000	0.000

B_A

[1,]	0.000	0.000	0.000	0
[2,]	0.434	0.000	0.000	0
[3,]	0.000	0.533	0.000	0
[4,]	0.000	0.000	0.844	0

Θ_A **Fixed**

[1,]	0	0	0	0
[2,]	0	0	0	0
[3,]	0	0	0	0
[4,]	0	0	0	0

$$\Sigma_C = (I - B_C)^{-1} \Psi_C (I - B_C)^{-1\,t} + \Theta_C$$

Ψ_C

[1,]	109.960	0.000	0.00	0.000
[2,]	0.000	8.813	0.00	0.000
[3,]	0.000	0.000	27.389	0.000
[4,]	0.000	0.000	0.00	14.952

B_C

[1,]	0.000	0.000	0.000	0
[2,]	0.434	0.000	0.000	0
[3,]	0.000	0.533	0.000	0
[4,]	0.000	0.000	0.844	0

Θ_C

[1,]	5.768	0	0	0
[2,]	0	5.768	0	0
[3,]	0	0	5.768	0
[4,]	0	0	0	5.768

$$\Sigma_E = (I-B_E)^{-1} \Psi_E (I-B_E)^{-1\,t} + \Theta_E$$

Ψ_E **Fixed**

[1,]	0	0	0	0
[2,]	0	0	0	0
[3,]	0	0	0	0
[4,]	0	0	0	0

B_E **Fixed**

[1,]	0	0	0	0
[2,]	0	0	0	0
[3,]	0	0	0	0
[4,]	0	0	0	0

Θ_E

[1,]	52.98	0.000	0.000	0.000
[2,]	0.00	56.394	0.000	0.000
[3,]	0.00	0.000	43.964	0.000
[4,]	0.00	0.000	0.000	45.071

Heritability

```
> round(SA_est/Sph_est,3)
      [,1]  [,2]  [,3]  [,4]
[1,] 0.217 0.615 0.764 0.773
[2,] 0.615 0.577 0.896 0.900
[3,] 0.764 0.896 0.631 0.811
[4,] 0.773 0.900 0.811 0.557
```

```
> round(SC_est/Sph_est,3)
      [,1]  [,2]  [,3]  [,4]
[1,] 0.537 0.385 0.236 0.227
[2,] 0.385 0.163 0.104 0.100
[3,] 0.236 0.104 0.179 0.189
[4,] 0.227 0.100 0.189 0.224
```

```
> round(SE_est/Sph_est,3)
      [,1] [,2] [,3]  [,4]
[1,] 0.246 0.000 0.000 0.000
[2,] 0.000 0.260 0.000 0.000
[3,] 0.000 0.000 0.190 0.000
[4,] 0.000 0.000 0.000 0.219
```