Path Analysis

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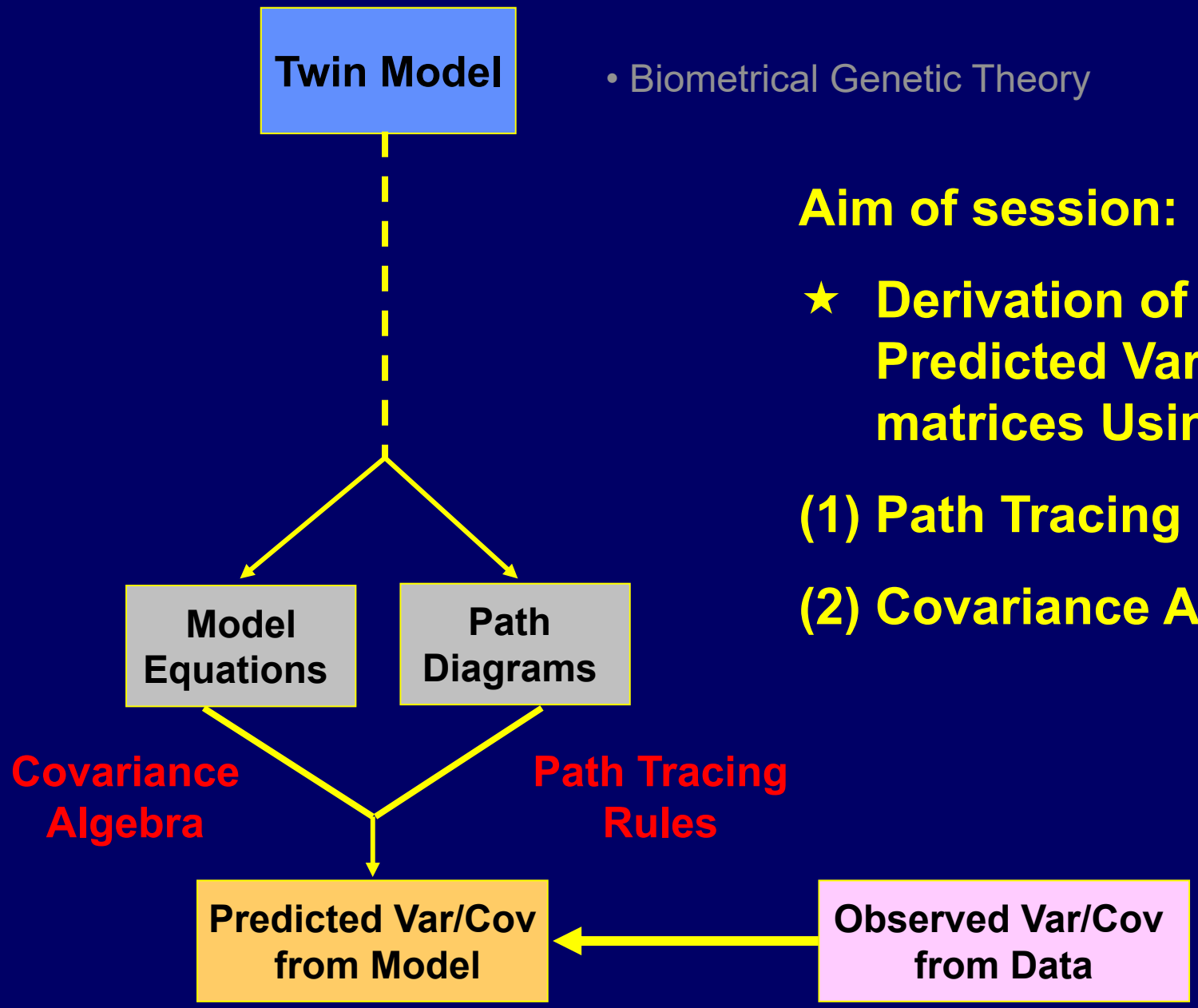


• Biometrical Genetic Theory

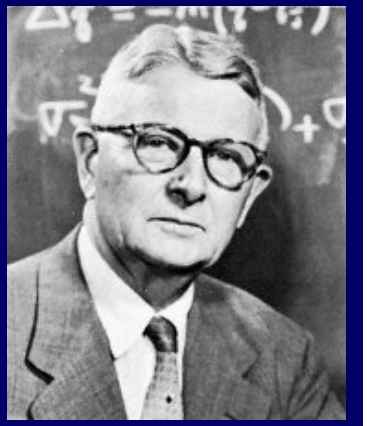
Aim of session:

★ **Derivation of Predicted Var/Cov matrices Using:**

(1) Path Tracing Rules

(2) Covariance Algebra

Path Analysis

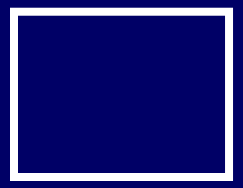


- Causal relationships <-> observed correlations
- Present linear relationships between variables in diagrams
- The relationships can also be represented as structural equations and covariance matrices
- All three forms are mathematically complete, it is possible to translate from one to the other
- Structural equation modelling (SEM) represents a unified platform for path analytic and variance components models

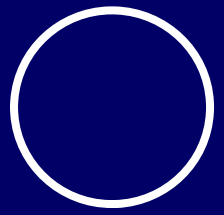
- In SEM expected relationships between observed variables are expressed by:
  - A system of linear model equations or
  - Path diagrams which allow the model to be represented in schematic form

- Both allow derivation of predicted variances and covariances of the variables under the specified model by using: (1) Path Tracing & (2) Covariance Algebra

Path Diagram Conventions

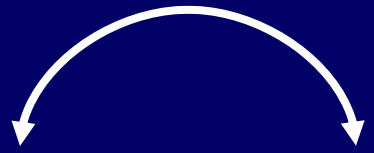


Observed Variables



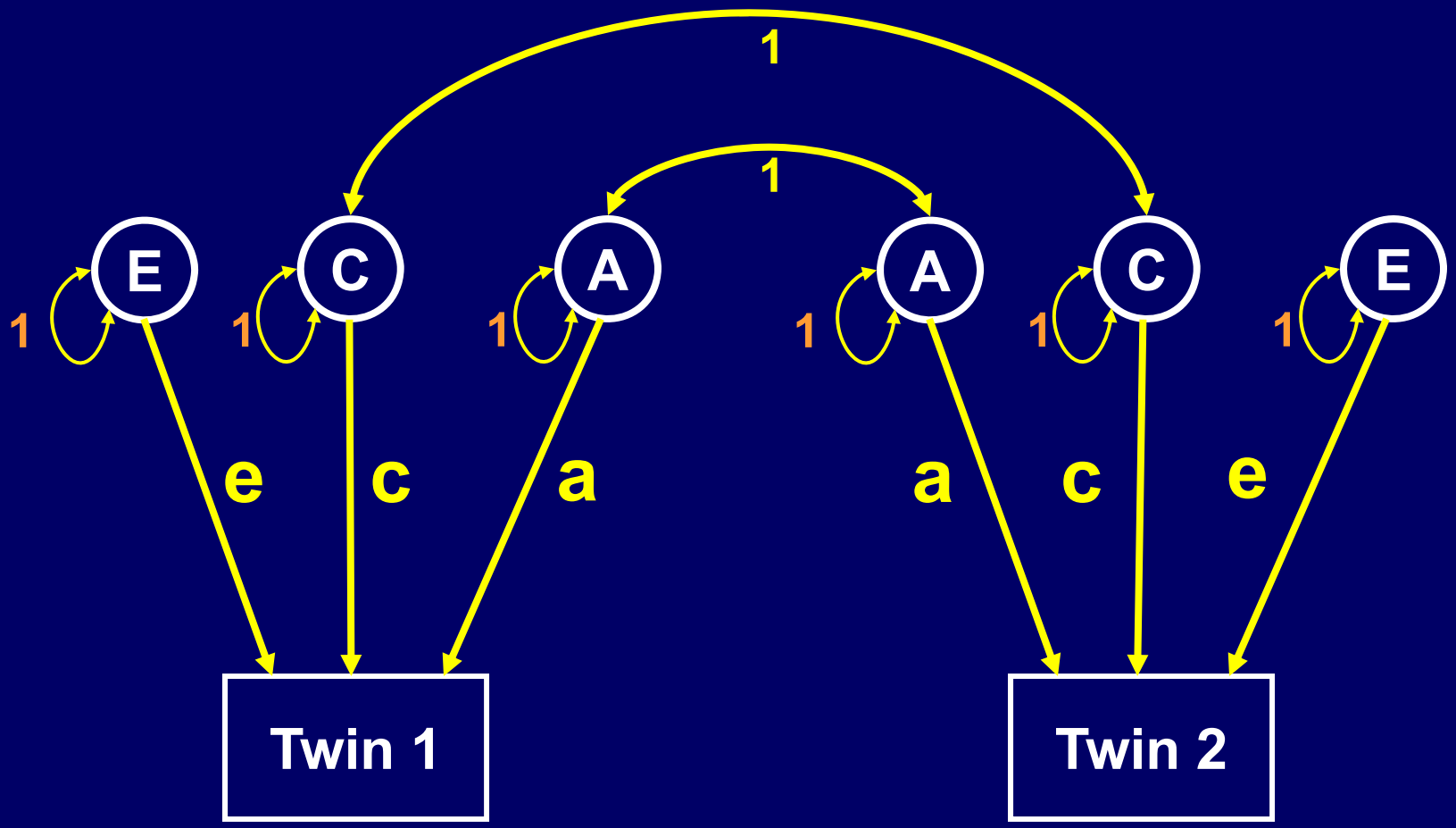
Latent Variables

Causal Paths



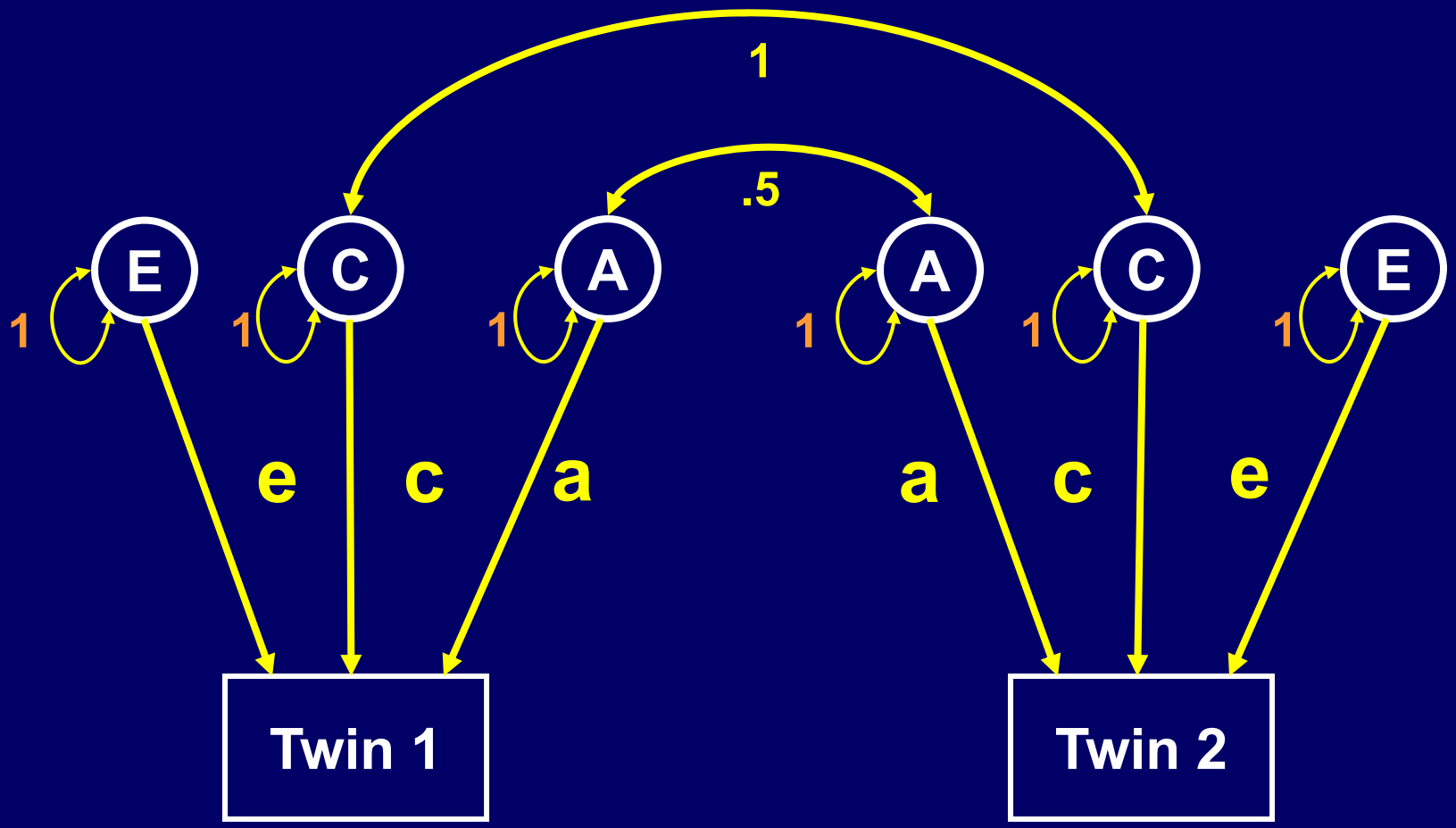
Covariance Paths

Path Diagrams for the Classical Twin Model



Model for an MZ PAIR

Note: a, c and e are the same cross twins



Model for a DZ PAIR

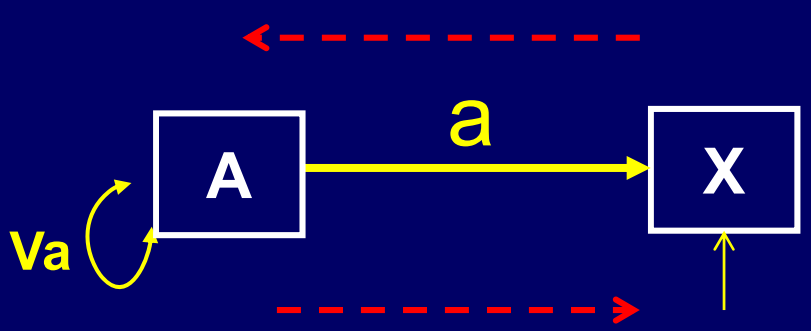
Note: a, c and e are also the same cross groups

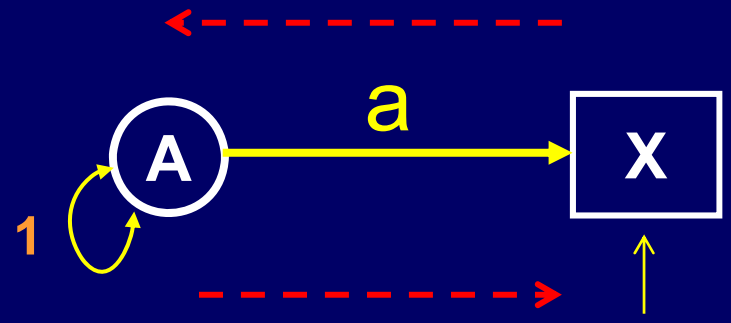
(1) Path Tracing

- The expected covariance between any two variables is the sum of all legitimate chains connecting the variables
  - Since the variance of a variable is the covariance of the variable with itself, the expected variance will be the sum of all legitimate chains from the variable to itself
- The numerical value of a chain is the product of all traced path coefficients within the chain
- A legitimate chain is a path along arrows that follow 3 rules: Wright's Rules

(I) Trace backward, then forward, or simply forward from variable to variable, but **NEVER** forward then backward!

- Include double-headed arrows from the independent variables to itself (the variance)
- These variances are **1** for **latent variables**

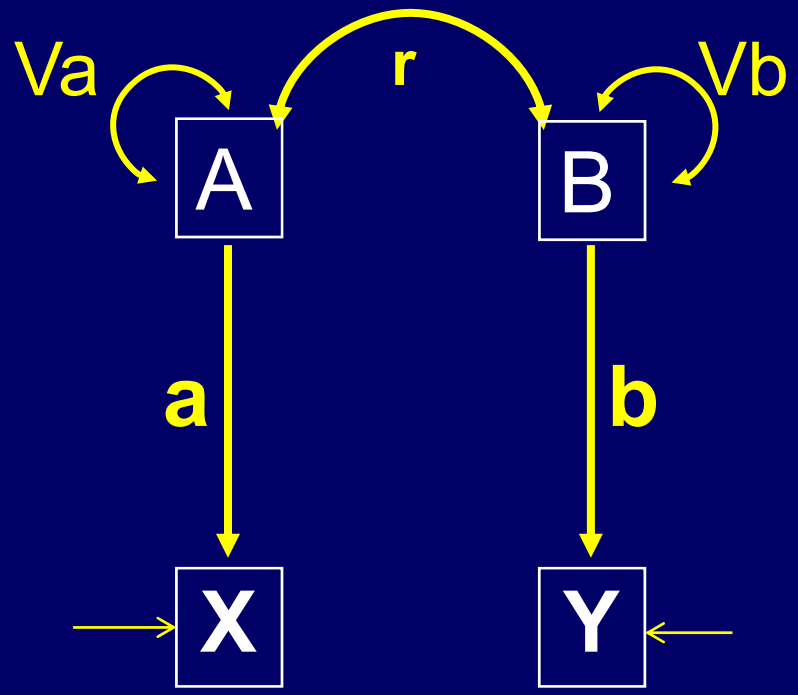




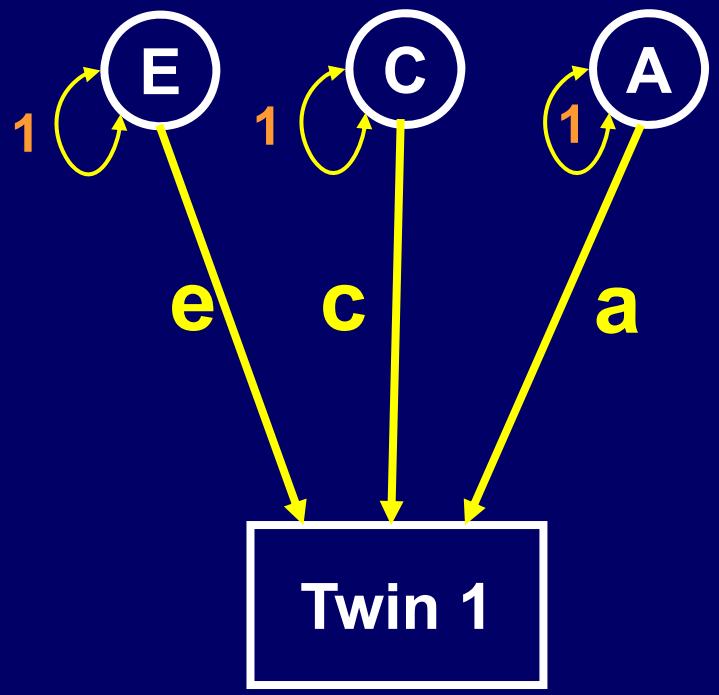
(II) You can pass through the same variable only once in a given chain of paths

(III) There is a maximum of one bi-directional path per chain.

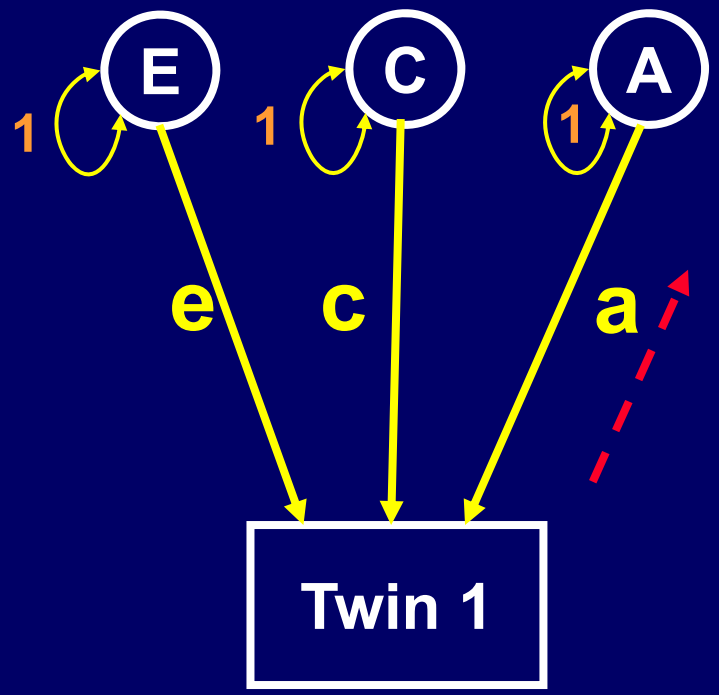
The double-headed arrow from the independent variable to itself is included, unless the chain includes another correlation path.



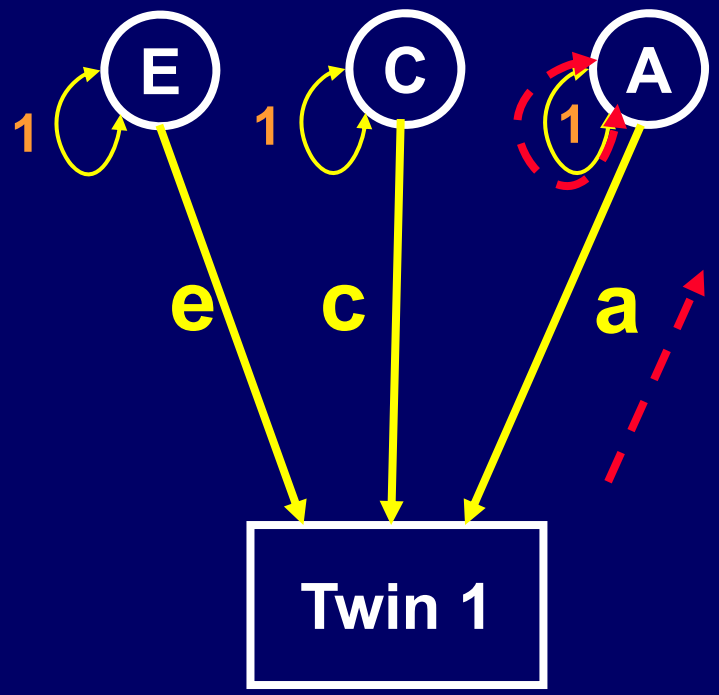
Variance of Twin 1 AND Twin 2 (for MZ and DZ pairs)



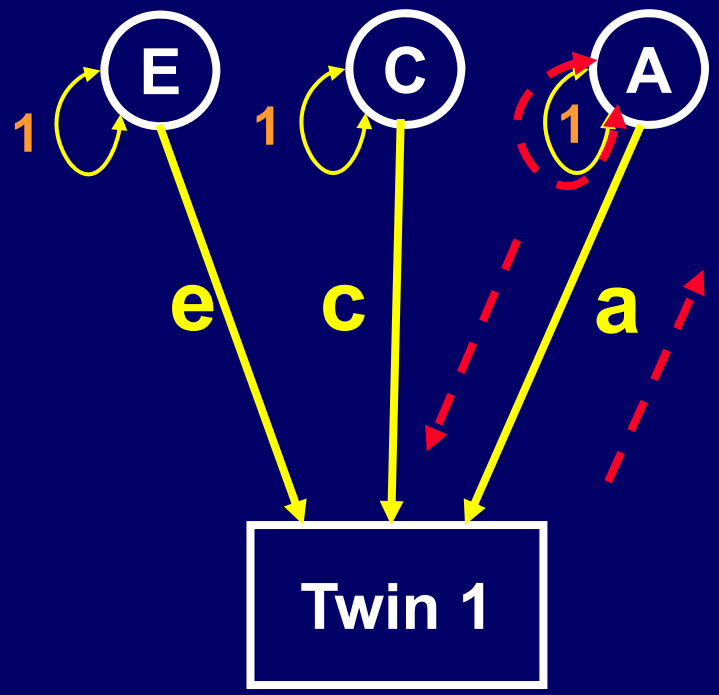
Variance of Twin 1 AND Twin 2 (for MZ and DZ pairs)



Variance of Twin 1 AND Twin 2 (for MZ and DZ pairs)



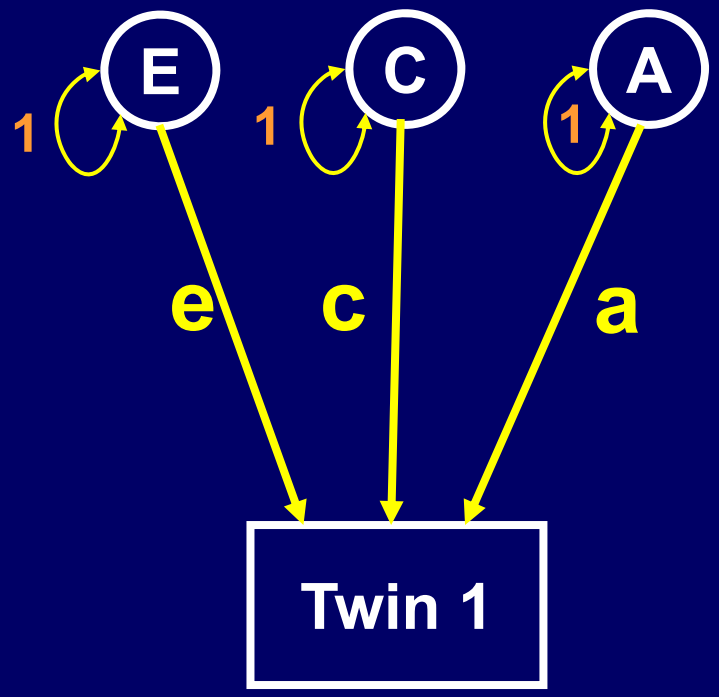
Variance of Twin 1 AND Twin 2 (for MZ and DZ pairs)



$a*1*a = a^2$

+

Variance of Twin 1 AND Twin 2 (for MZ and DZ pairs)



$a*1*a = a^2$

+

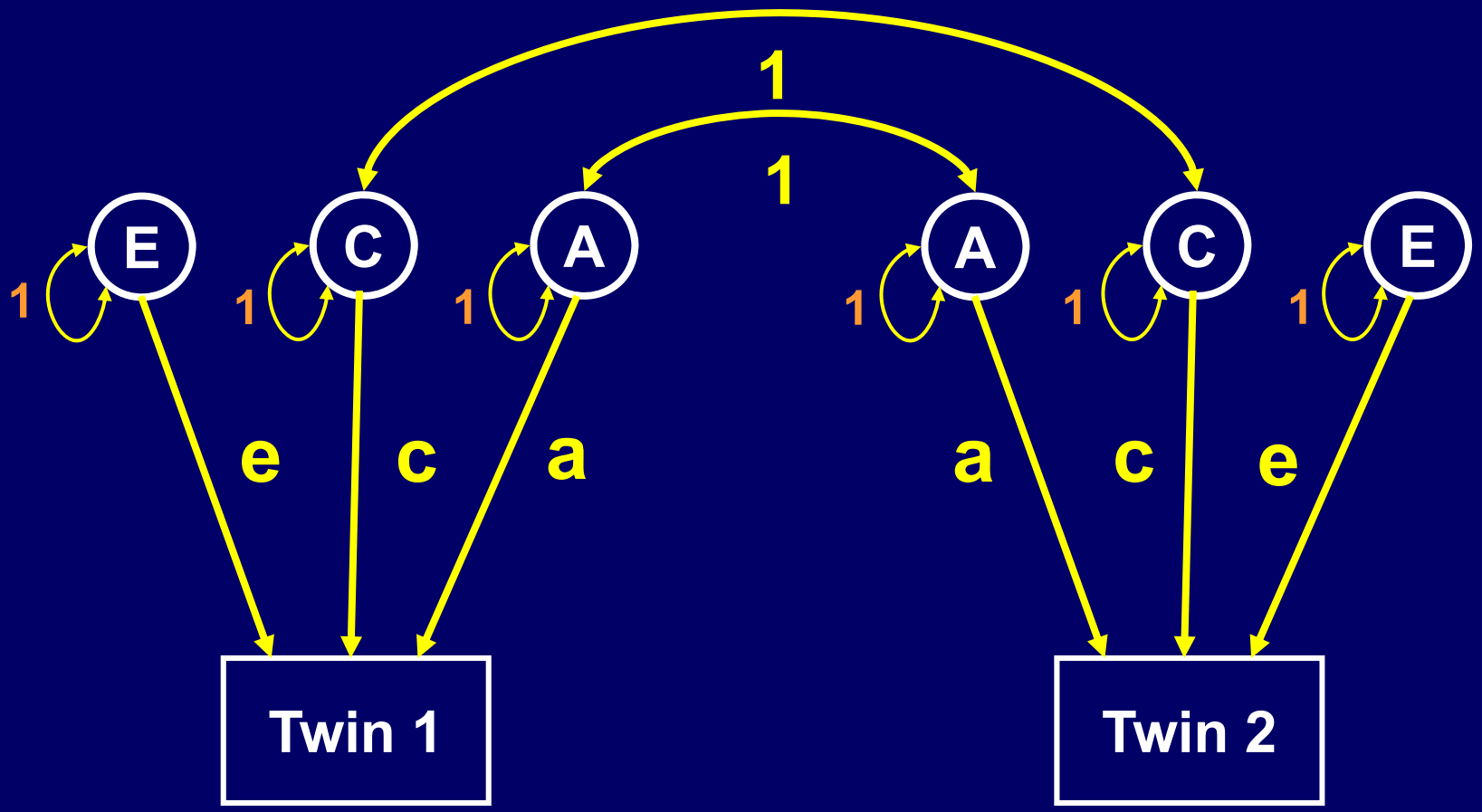
$c*1*c = c^2$

+

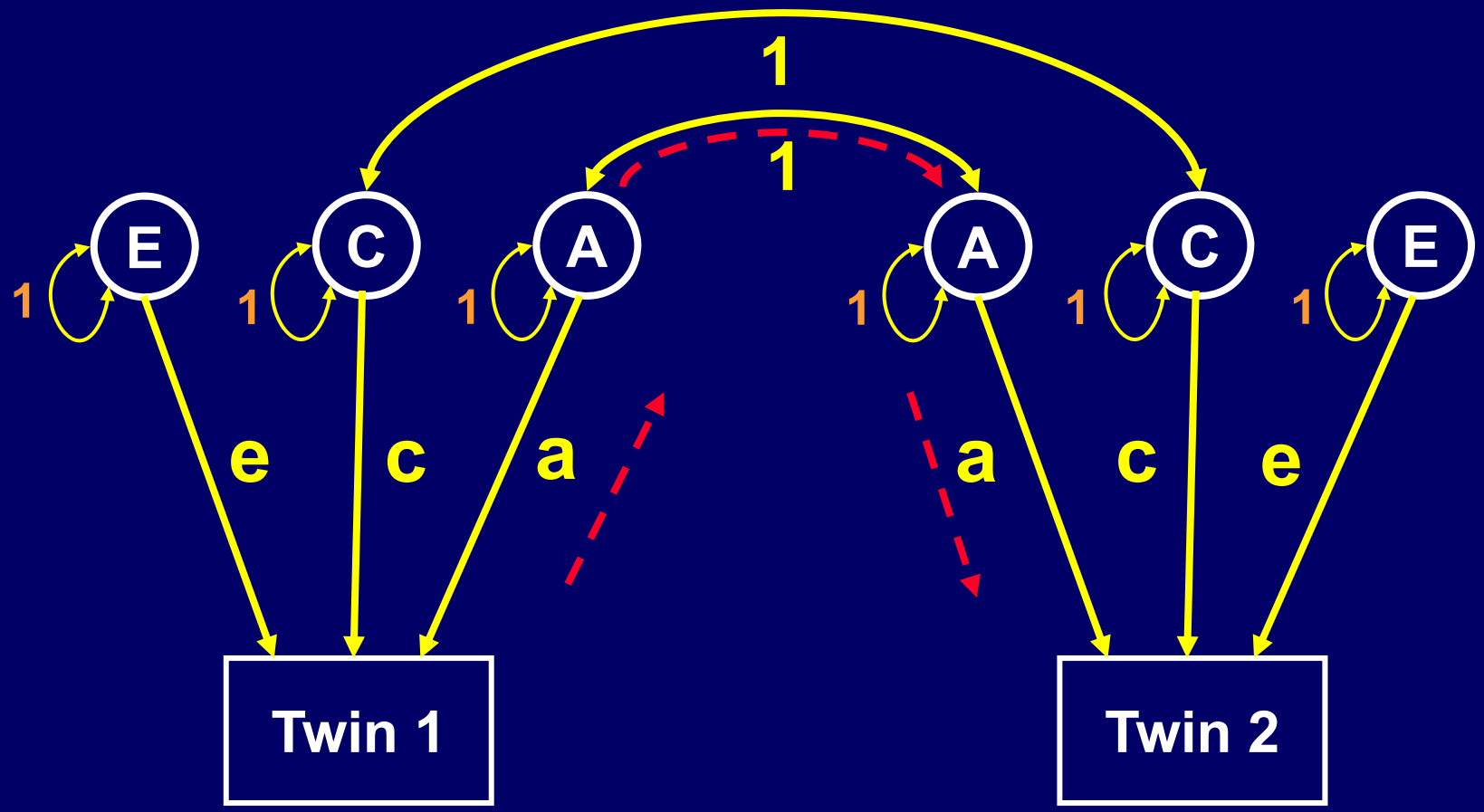
$e*1*e = e^2$

Total Variance = $a^2 + c^2 + e^2$

Covariance Twin 1-2: MZ pairs

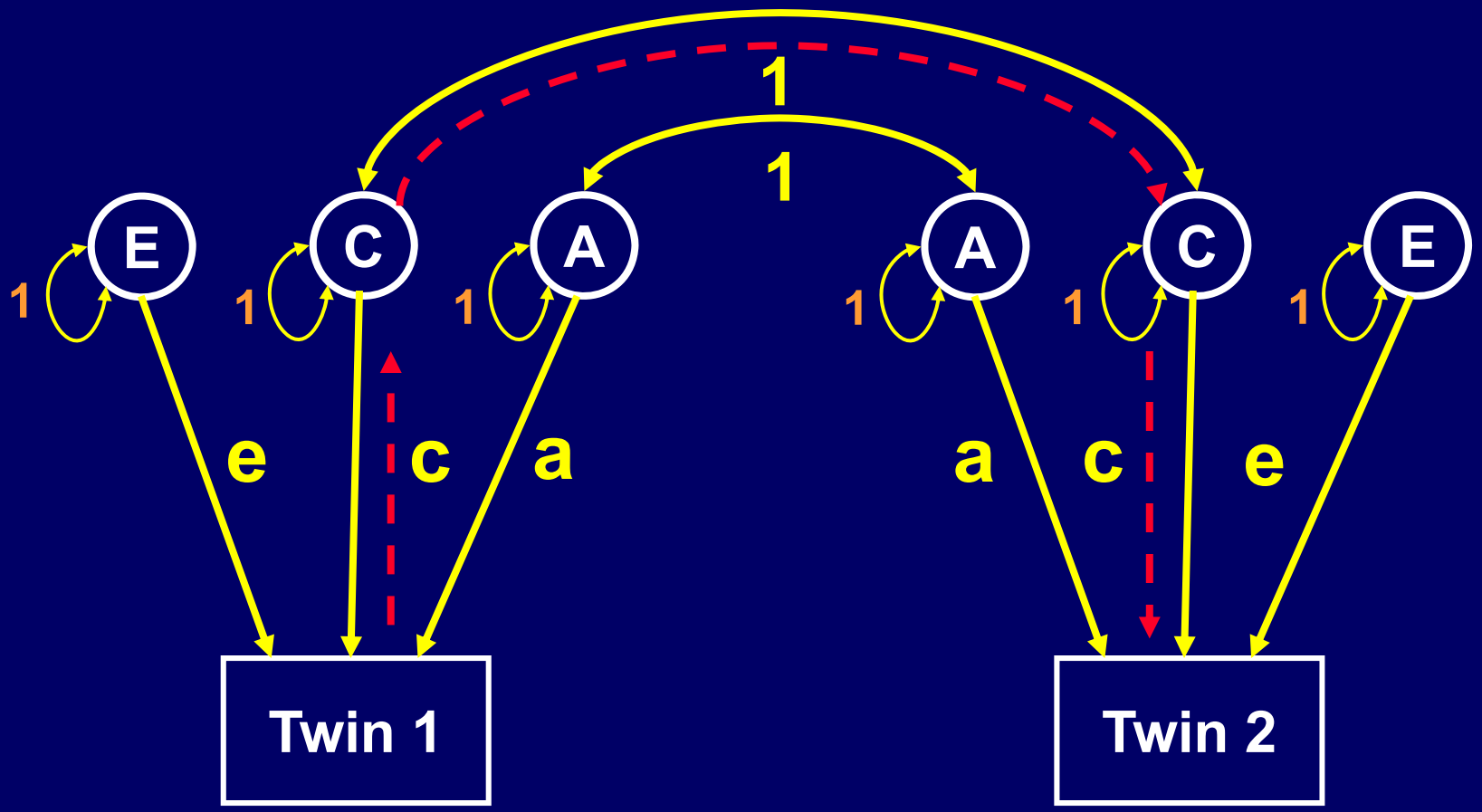


Covariance Twin 1-2: MZ pairs



Total Covariance = a^2 +

Covariance Twin 1-2: MZ pairs



Total Covariance = $a^2 + c^2$

Predicted Var-Cov Matrices

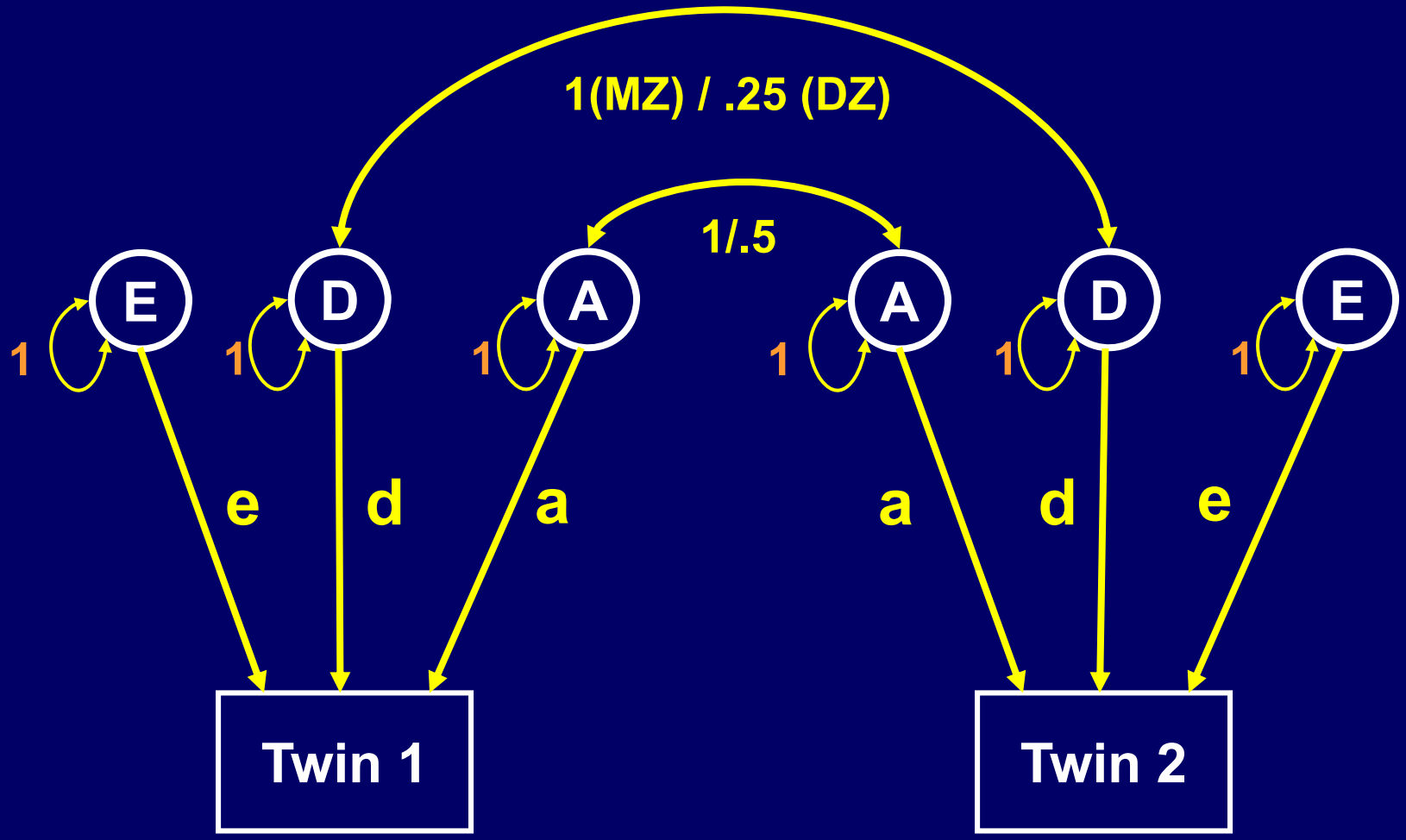
Cov MZ

	Tw1	Tw2
Tw1	$a^2+c^2+e^2$	a^2+c^2
Tw2	a^2+c^2	$a^2+c^2+e^2$

Cov DZ

	Tw1	Tw2
Tw1	$a^2+c^2+e^2$	$\frac{1}{2}a^2+c^2$
Tw2	$\frac{1}{2}a^2+c^2$	$a^2+c^2+e^2$

ADE Model



Predicted Var-Cov Matrices

Cov MZ

	Tw1	Tw2
Tw1	$a^2+d^2+e^2$	a^2+d^2
Tw2	a^2+d^2	$a^2+d^2+e^2$

Cov DZ

	Tw1	Tw2
Tw1	$a^2+d^2+e^2$	$\frac{1}{2}a^2+\frac{1}{4}d^2$
Tw2	$\frac{1}{2}a^2+\frac{1}{4}d^2$	$a^2+d^2+e^2$

(2) Covariance Algebra

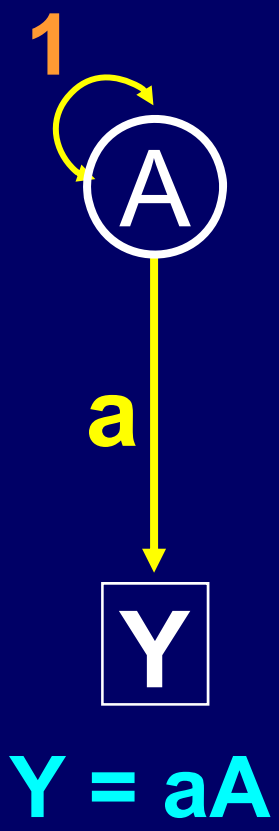
Three Fundamental Covariance Algebra Rules

$$\text{Var}(X) = \text{Cov}(X,X)$$

$$\text{Cov}(aX,bY) = ab\ \text{Cov}(X,Y)$$

$$\text{Cov}(X,Y+Z) = \text{Cov}(X,Y) + \text{Cov}(X,Z)$$

Example 1

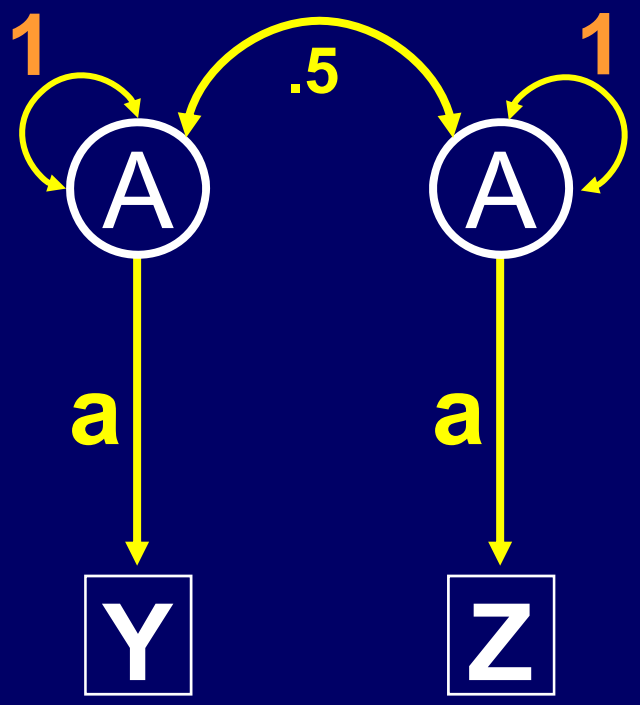


Y = aA

$$
\begin{aligned}
\text{Var}(Y) &= \text{Var}(aA) \\
&= \text{Cov}(aA,aA) \\
&= a^2\,\text{Cov}(A,A) \\
&= a^2\,\text{Var}(A) \\
&= a^2 * 1 \\
&= a^2
\end{aligned}
$$

The variance of a dependent variable (Y) caused by independent variable A, is the squared regression coefficient multiplied by the variance of the independent variable

Example 2



$Cov(Y,Z) = Cov(aA,aA)$

$= a^2\, Cov(A,A)$

$= a^2 * .5$

$= .5a^2$

Y = aA Z = aA