Developmental Models: Latent Growth Models

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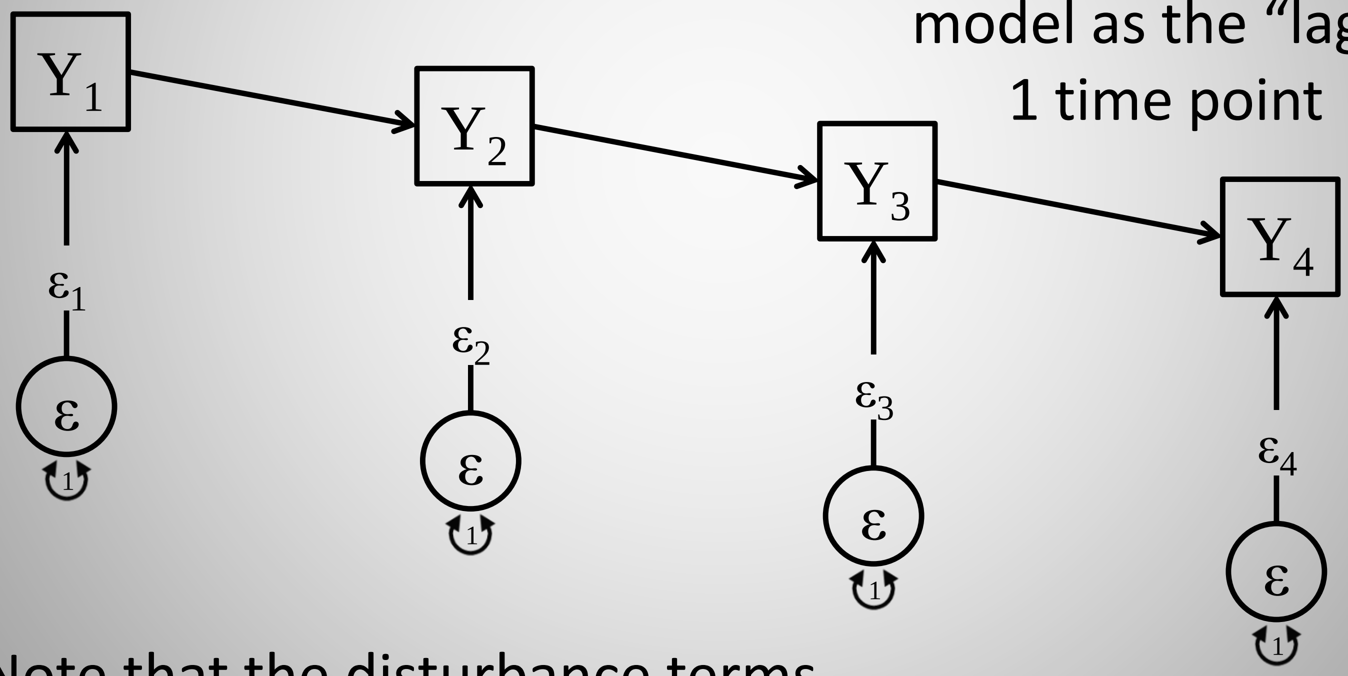
Two Broad Categories of Developmental Models

- Autoregressive Models:
  - The things that happened yesterday affect what happens today, which affect what happens tomorrow
  - Simplex Models
- Growth Models
  - Latent parameters are estimated for the level (stable) and the change over time (dynamic) components of the traits

The Univariate Simplex Model (in singletons)

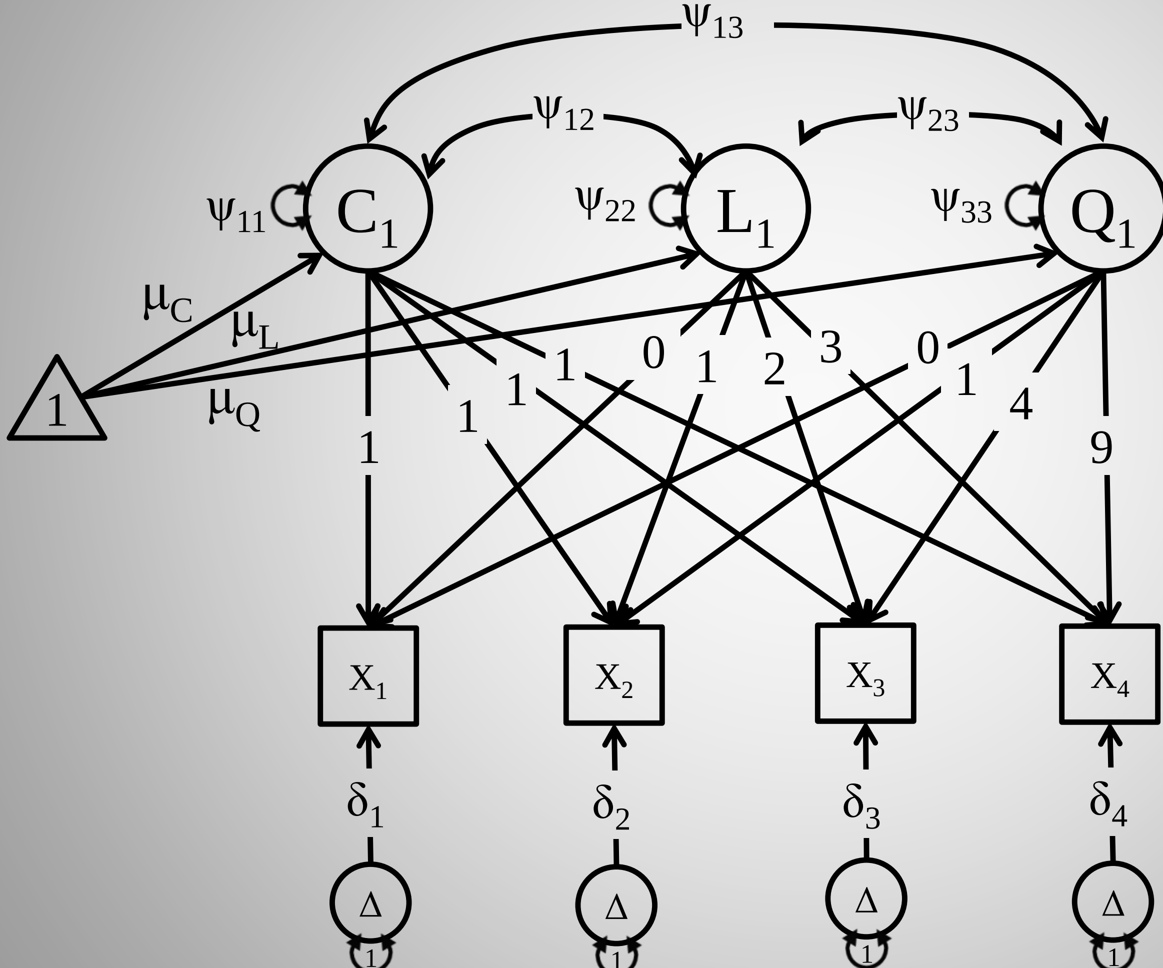
With a Univariate Simplex Model, the Y_t causes Y_{t+1} and is caused by Y_{t-1}

This is also called an AR1 model as the "lag" is only 1 time point



Note that the disturbance terms are uncorrelated

Latent Growth Model



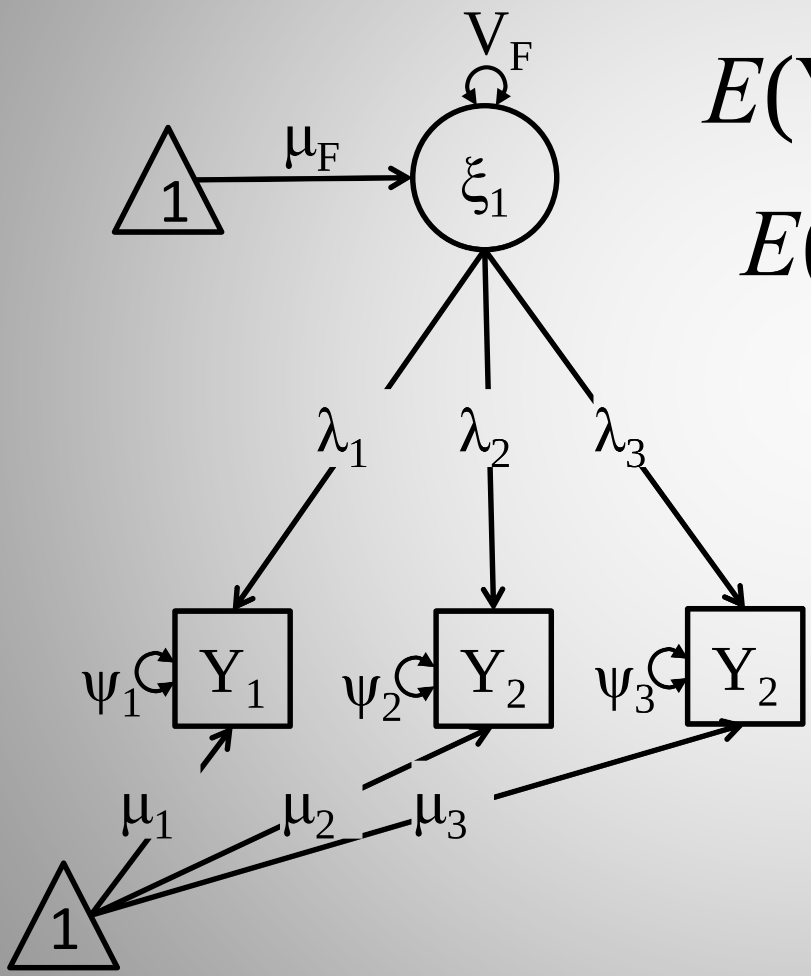
Note that the means for the latent variables are being estimated within the model

Identification of Mean Structures

- SEMs with Mean Structures must be identified both at the level of the Mean and at the level of the Covariance.
  - You can only estimate each mean once

- If your model is unidentified at either the mean or the covariance level, your model is unidentified
  - An overidentified covariance structure will not help identify the mean structure and vice versa.

Mean Structures in Factor Models



$$E(\mathbf{Y}) = \mathrm{L}E(x) \text{ or } E(\mathbf{Y}) = \mathrm{M}$$

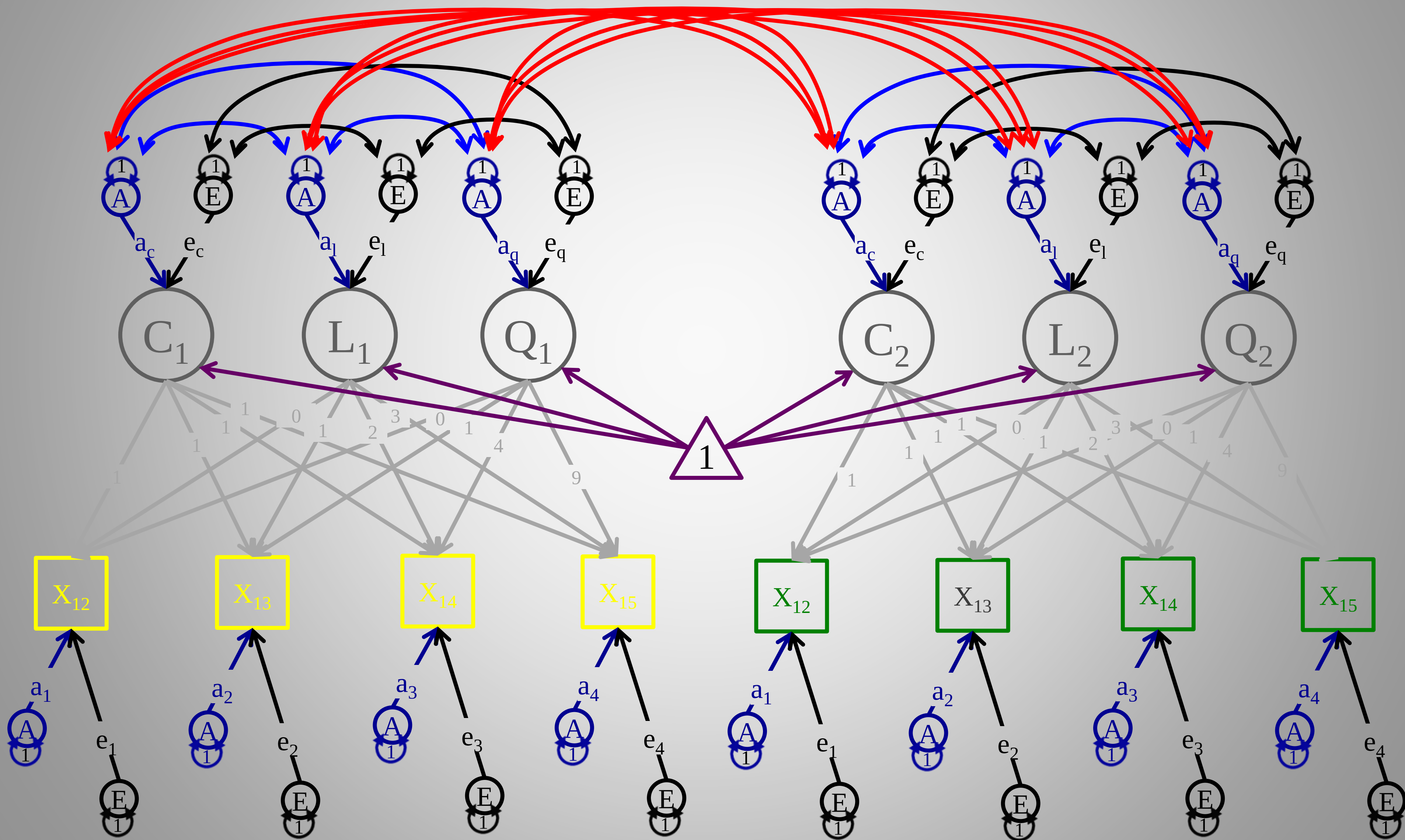
$$E(y_1) = l_1 m_F \text{ or } m_1$$

You must choose one of the other, as both ΛE(ξ) and M are not simultaneously identified

Latent Growth Models (LGM)

- Latent Growth Models are (probably) the most common SEM with mean structures in a single sample.
- Data requirements for LGM:
  1. Dependent Variables measured over time
  2. Scores have the same units and measure the same thing across time
     - Measurement Invariance can be assumed
  3. Data are time structured (tested at the same intervals)
     - The intervals do not have to be equal
       - 6 months, 9 months, 12 months, 18 months

Growth Model for Two Twins



Intuitive Understanding of the LGM

- Each time point is represented as an indicator of all of the latent growth parameters
  - The constant is analogous to the constant in a linear regression model.
  - All of the (unstandardized) loadings are fixed to 1.
- The linear effect is analogous to the regression of the observations on time (with loadings of 0, 1, …, t)
  - If latent slope loadings are set to -2,1,0,1,2,…,t , then the intercept will be at the third measurement occasion
- The quadratic increase is analogous to a non-linear effect of time on the observed variables and is interpreted in a very similar way to the linear effect.
- Cubic, Quartic and higher order time effects
  - Be cautious in your interpretation as they may only be relevant in your sample.
  - Interpretations of high order non-linear effects are difficult.

Interpretation of the Latent Growth Factors

- Residuals: The variance in the phenotype that is not explained by the latent growth structure
- Factor Loadings: The same as you would interpret loadings in any factor model (but they are typically not interpreted)
- Factor Means: The average effect of the intercept/linear/quadratic in the population (more on this next)
- Factor Covariance: Random Effects of the Latent Growth Parameters (more on this too)

Means of the Latent Parameters

- μ_C: Mean of the latent Constant
  - The average level of the latent phenotype when the linear effect is zero
- μ_L: Mean of the latent Linear slope
  - The average increase (decrease) over time
- μ_Q: Mean of the latent Quadratic slope
  - The average quadratic effect over time

Variances of the Latent Growth Parameters

- ψ_{ii}: Variance of the latent growth parameters
  - Dispersion of the values around the latent parameter
  - Large variances indicate more dispersion
    - Large variance on a Latent slope may indicate that the average parameter increase but some of the latent trajectories may be negative

- Typically the variance of higher order parameters are smaller than the variances of lower order parameters
  - $\psi_{11} > \psi_{22} > \psi_{33}$

Covariances between the Latent Growth Parameters

- ψ_{ij}: Covariance of the latent growth parameters
  - Generally expressed in terms of correlations
  - Important to keep in mind what the absolute variance in the constituent growth parameters are
    - E.G. if the variance of the linear increase is really small, the correlation may be very large as an artifact of the variance

- ψ_{12}: Covariance between the intercept and the linear increase
  - $\psi_{12} > 0$: the higher an individual starts, the faster they increase
  - $\psi_{12} < 0$: the higher an individual starts, the slower they increase

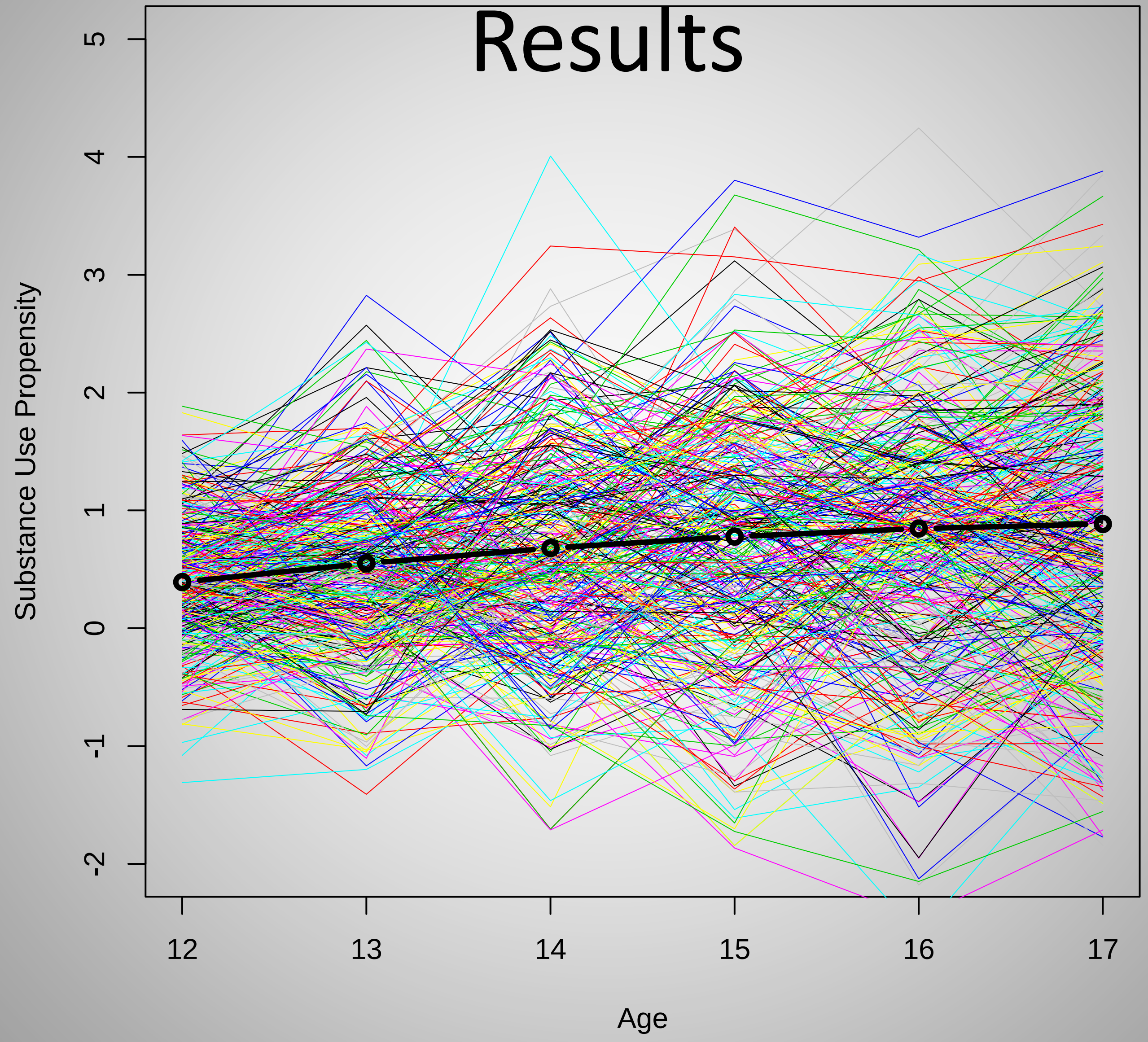
Presenting LGC Results

- The basic formula for the average effect of the growth parameters is very similar to the simple regression equation:

$$Y_t = m_c + m_L l_L + m_Q l_Q + \ldots + m_k l_k$$

- These expected values can easily be plotted against time.
- It is also possible to include either the standard errors of the parameters or the variances of the growth parameters in the graphs.
- Some people like to include the raw observations also.

Example Presentation of the LGC Results



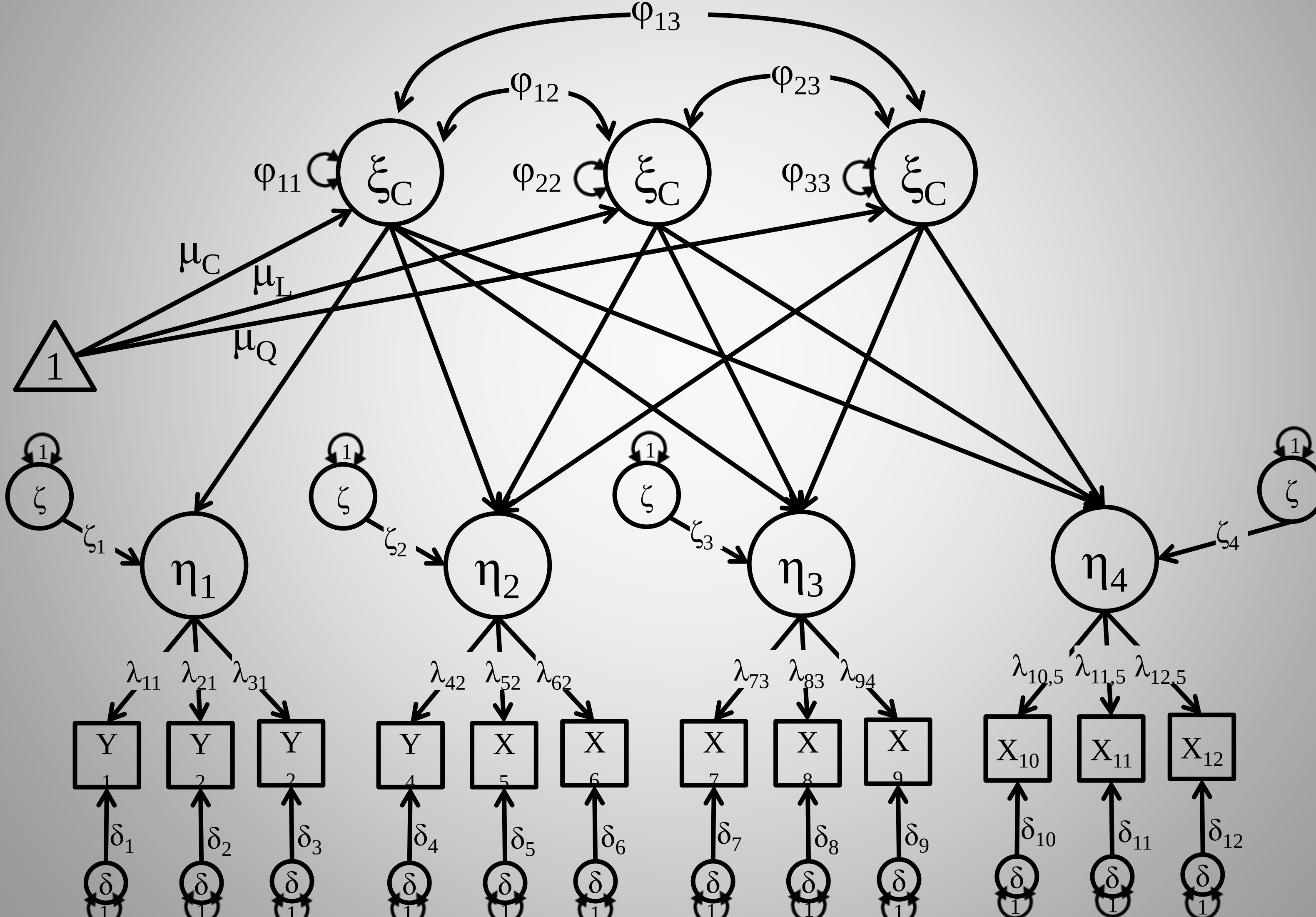
Alternative Specifications

- Instead of fixing the loadings to 0, 1, 2, 3, if we fix the loadings to -3, -1, 1, 3, it will reduced the (non-essential) multicolinearity
- Importantly, the model fit will not change!
- So what correlations are the right ones?

Caveat

- If the starting point is zero, then it might be best to pick a value in the middle of the range for the intercept, or generate an orthogonal set of contrasts
  – Just because you started your study when people were 8, 14, 22, doesn't mean that 8, 14 or 22 are meaningful starting points.
- If zero is a meaningful then it might be a good idea to keep that value as 0.
  – Critical Event
  – Treatment (with pretests and follow-up tests)

LGM on Latent Factors



Autoregressive LGM