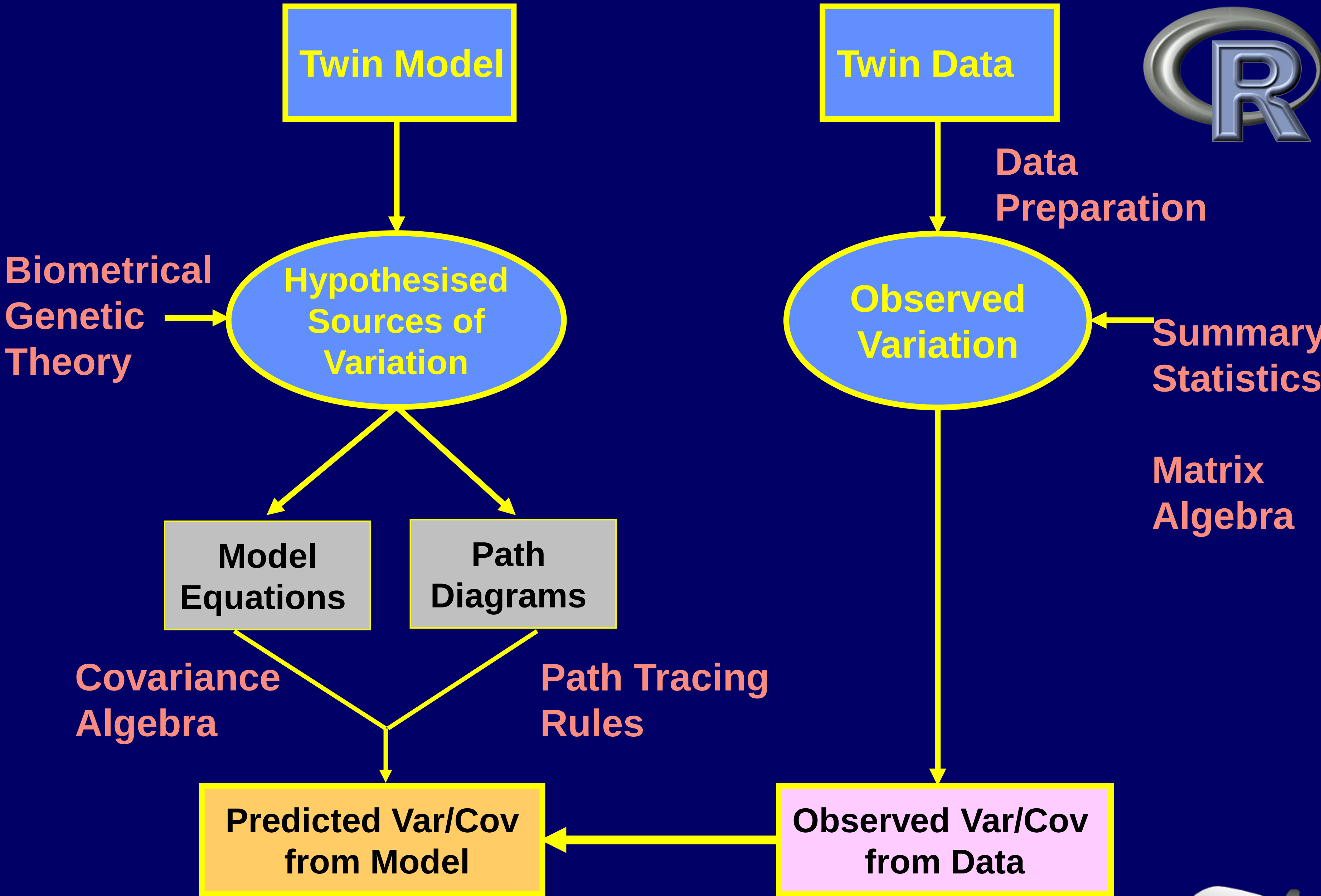
Path Analysis

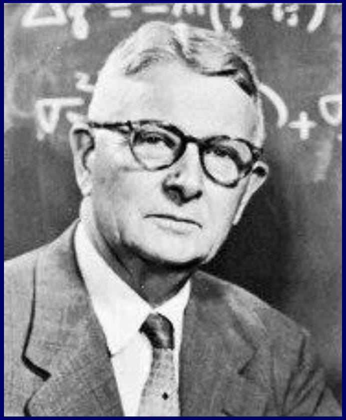
Frühling Rijsdijk

SGDP Centre
Institute of Psychiatry
King's College London, UK



Structural Equation Modelling (Maximum Likelihood)

Path Analysis



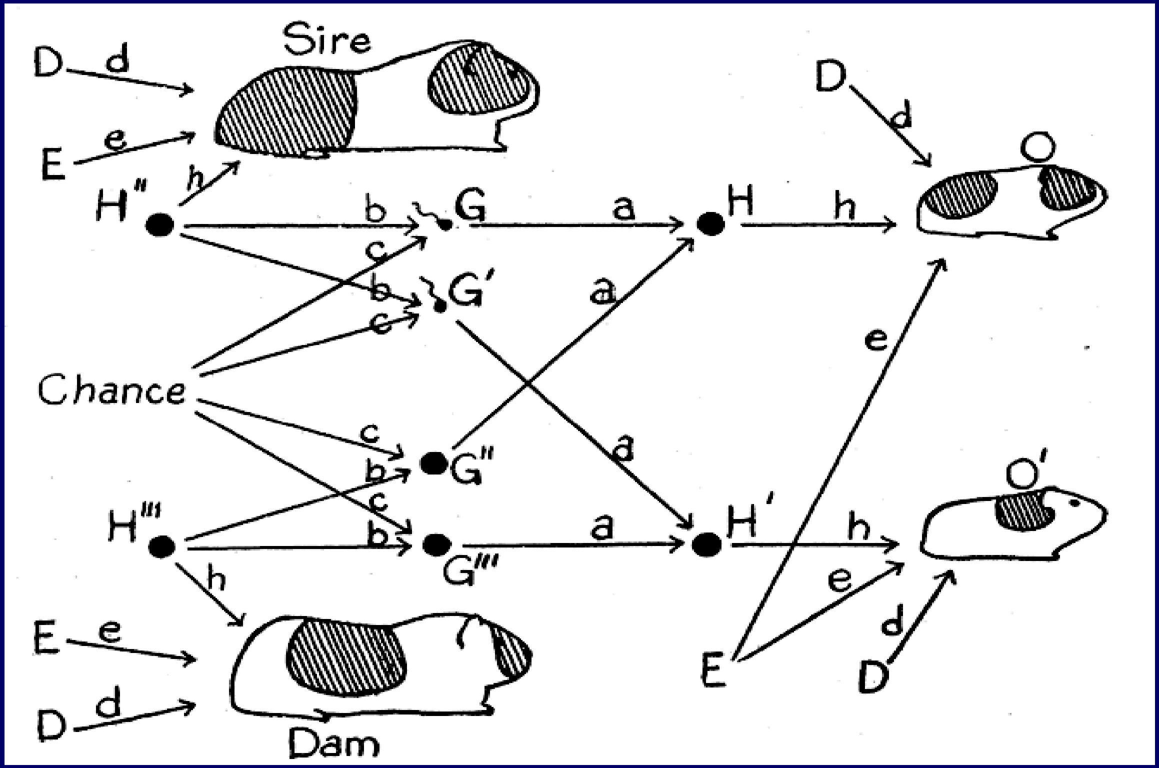
- Path analysis was developed around 1918 by Sewall Wright
- Combines knowledge we have with regard to causal relations with degree of observed correlations
- Guinea pigs: interrelationships of factors determining weight at birth and at weaning (33 days)

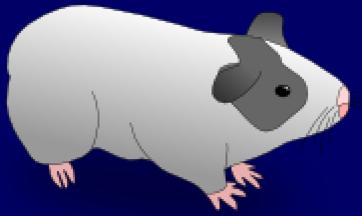
Birth weight
Early gain
Litter size
Gestation period

Environmental conditions
Health of dam
Heredity factors

Wright, S. (1921). "Correlation and causation". *J. Agricultural Research* **20**: 557–585

Path Diagram





Path Analysis

- Present linear relationships between variables by means of diagrams ; Derive predictions for the variances and covariances of the variables under the specified model

- The relationships can also be represented as structural equations and covariance matrices

- All three forms are mathematically complete, it is possible to translate from one to the other

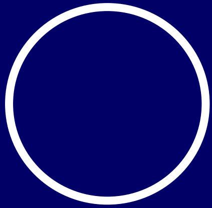
- Structural equation modelling (SEM) represents a unified platform for path analytic and variance components models

- In SEM models, expected relationships between observed variables are expressed by:
  - A system of linear model equations or
  - Path diagrams which allow the model to be represented in schematic form
- Both allow derivation of predicted variances and covariances of the variables under the specified model
- Aims of this session: Derivation of predicted Var-Cov Matrices using:

  (1) Path Tracing & (2) Covariance Algebra

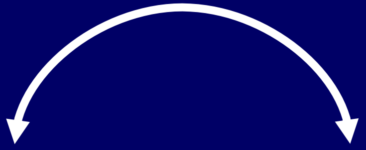
Path Diagram Conventions

Observed Variable

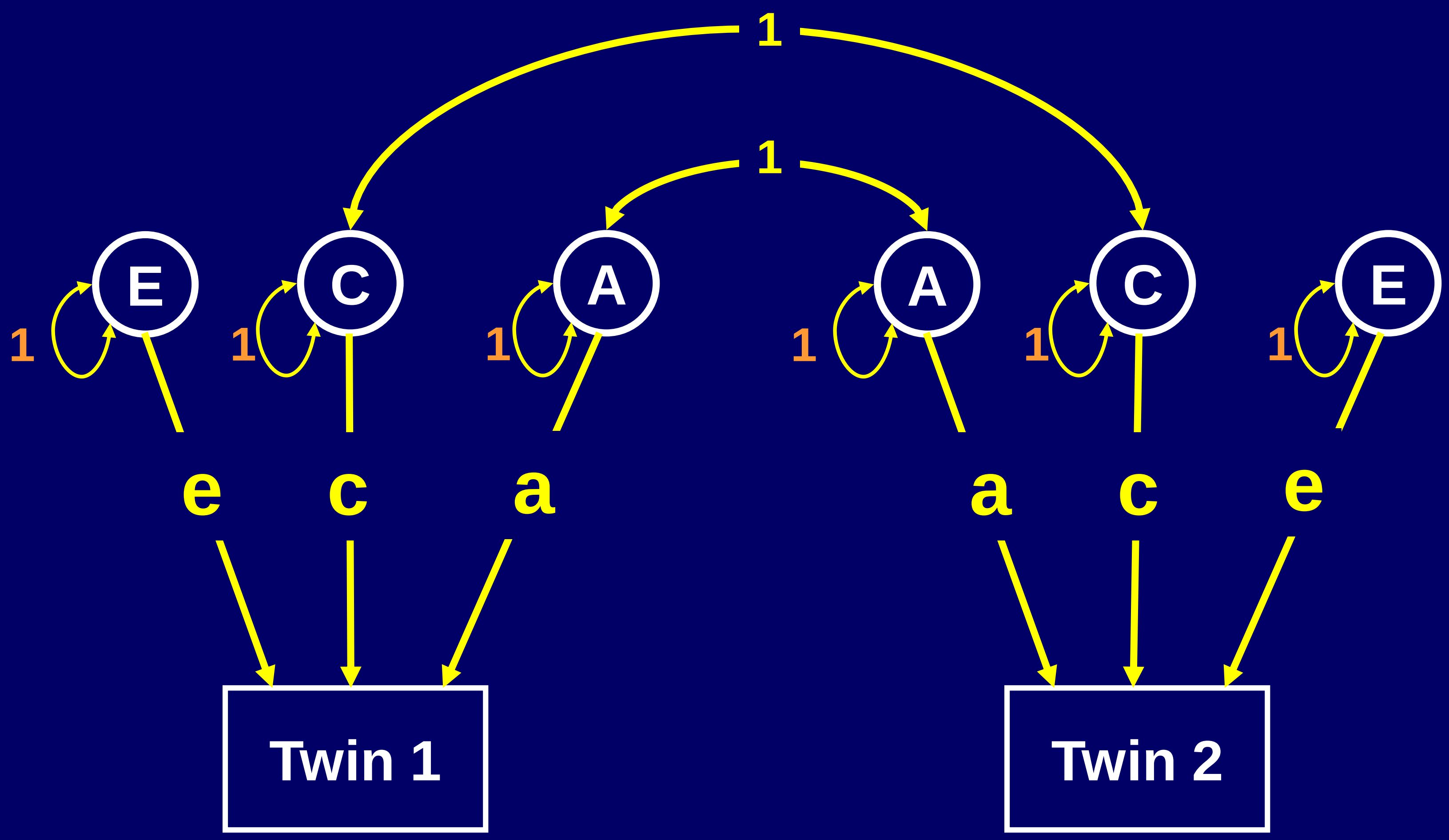


Latent Variable

Causal Path

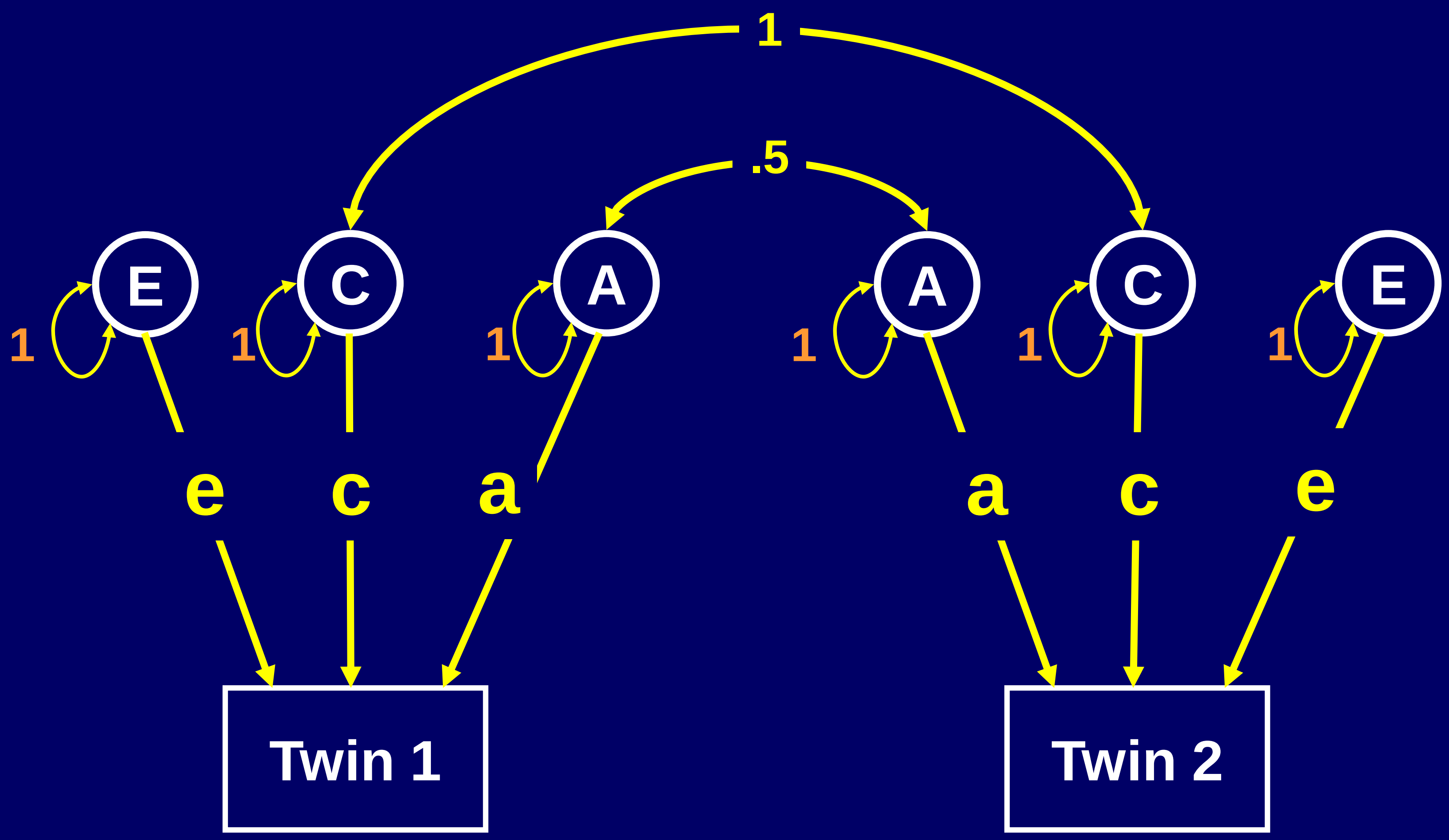


Covariance Path



Model for an MZ PAIR

Note: a, c and e are the same cross twins



Model for a DZ PAIR

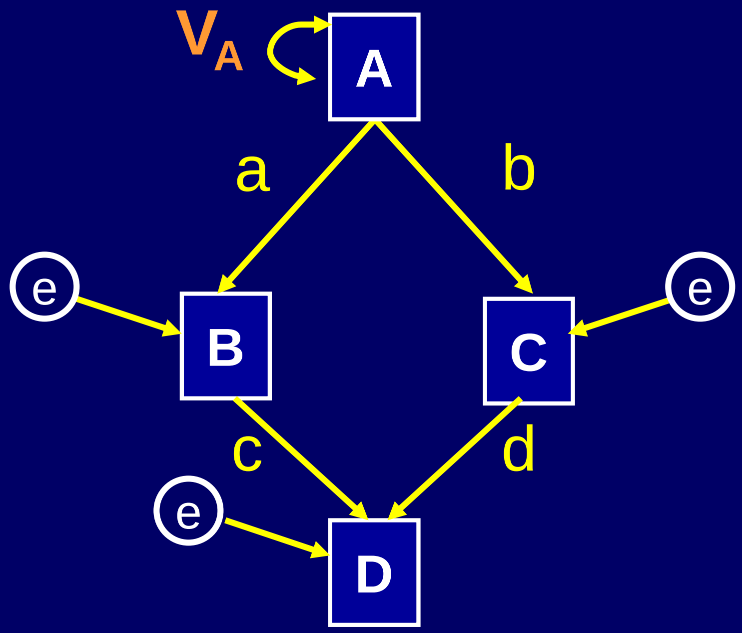
Note: a, c and e are also the same cross groups

(1) Path Tracing

- The covariance between any two variables is the sum of all legitimate chains connecting the variables

- The numerical value of a chain is the product of all traced path coefficients within the chain

- A legitimate chain is a path along arrows that follow 3 rules:



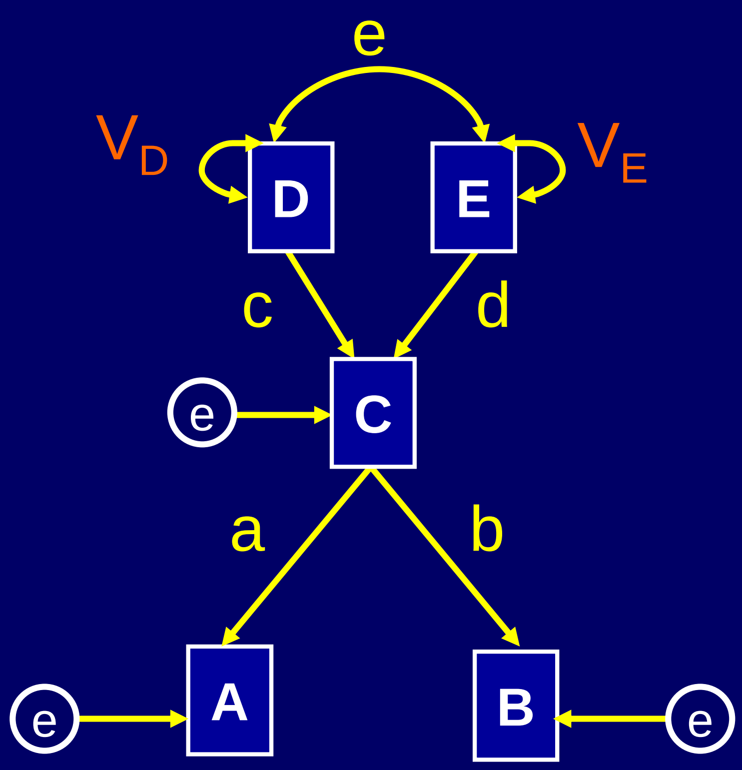
Cov_{BC} : $a*V_A*b$

NOT

$c*V_D*d$

(i) Trace backward, then forward, or simply forward from one variable to another. NEVER forward then backward. Include double-headed arrows from the independent variables to itself.

These variances will be 1 for standardized variables

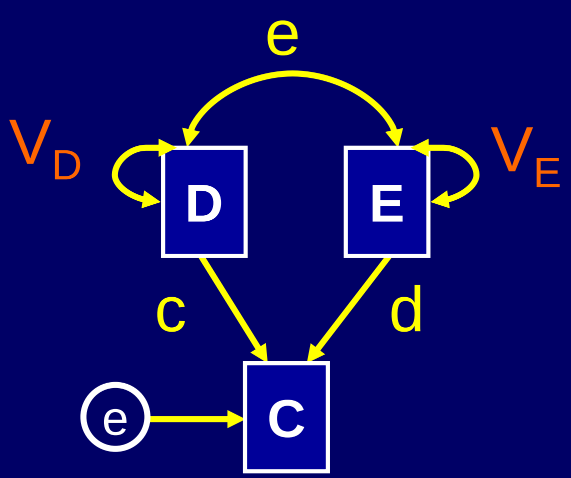


Cov_{AB} : $a*V_C*b$

NOT

$a* V_C *c*e*d* V_C$

(ii) Loops are not allowed, i.e. we can not trace twice through the same variable



Cov_{CD} : $c*V_D$ + $d*e$ NOT $d*V_E*e$

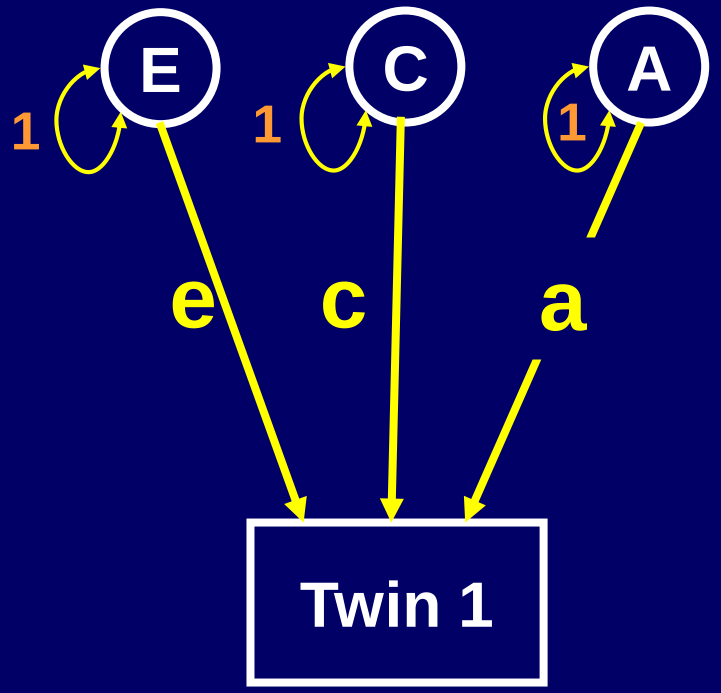
(iii) A maximum of one curved arrow per path.

So, the double-headed arrow from the independent variable to itself is included, unless the chain includes another double-headed arrow (e.g. a correlation path)

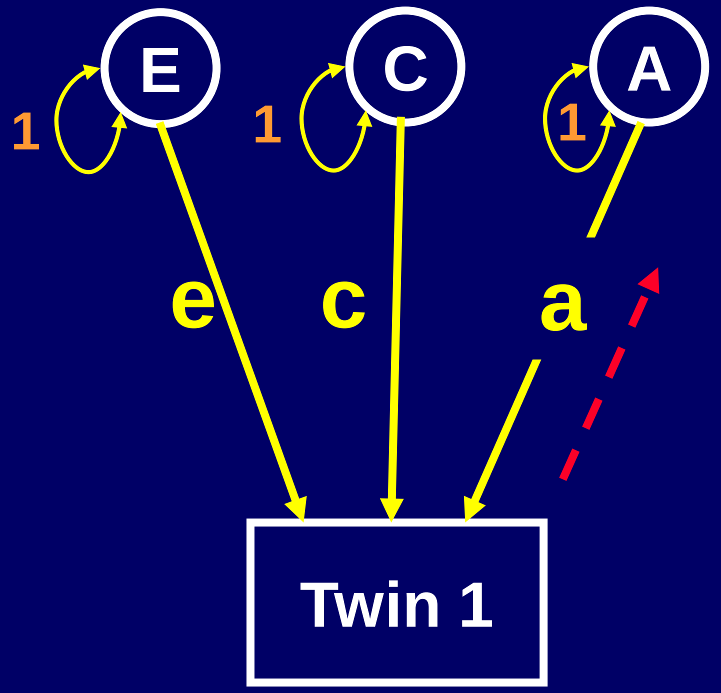
The Variance

Since the variance of a variable is the covariance of the variable with itself, the expected variance will be the sum of all paths from the variable to itself, which follow **Wright's rules**

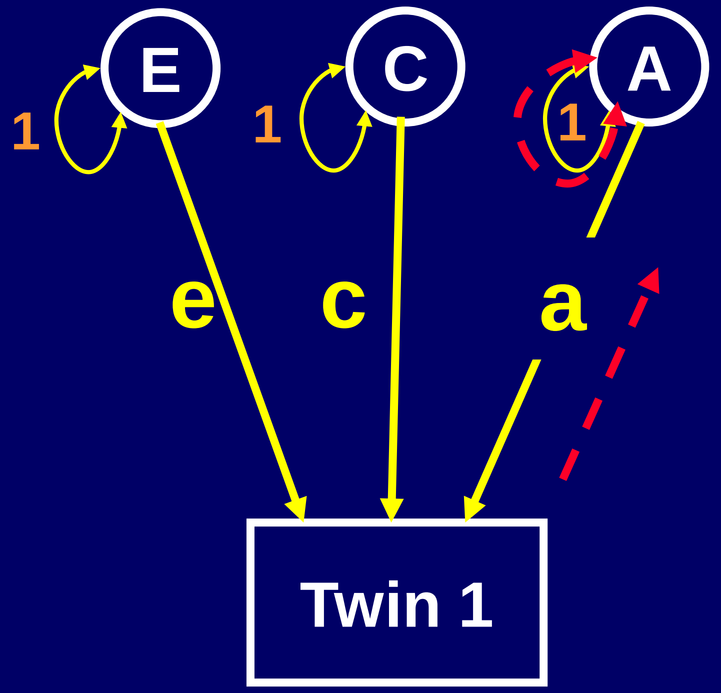
Variance of Twin 1 AND Twin 2 (for MZ and DZ pairs)



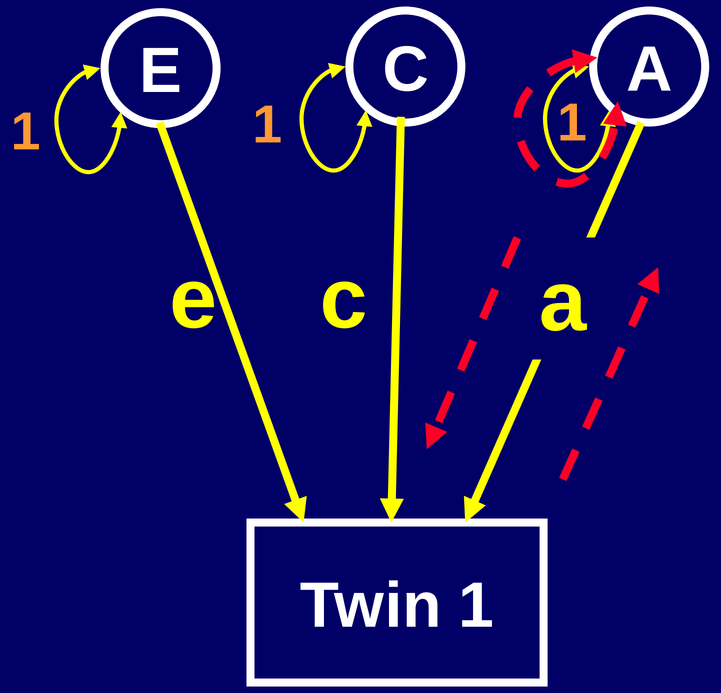
Variance of Twin 1 AND Twin 2 (for MZ and DZ pairs)



Variance of Twin 1 AND Twin 2 (for MZ and DZ pairs)



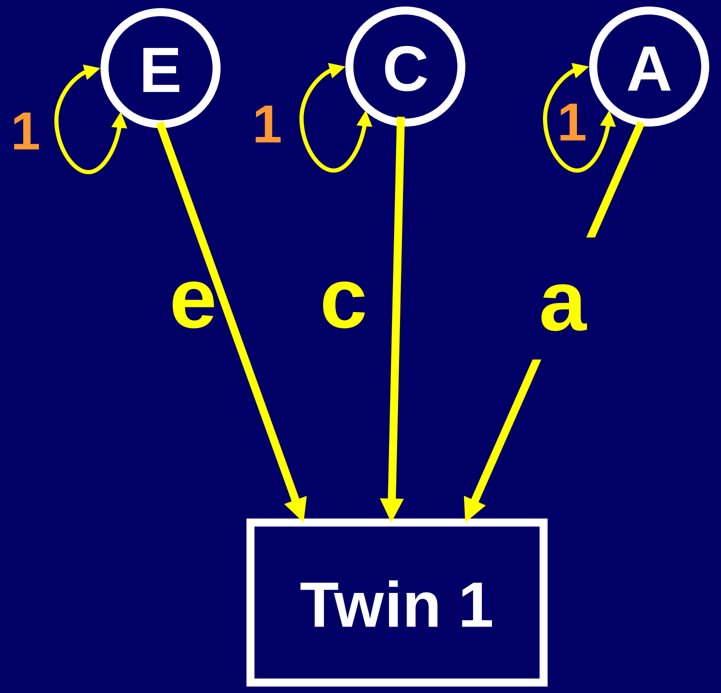
Variance of Twin 1 AND Twin 2 (for MZ and DZ pairs)



$$a*1*a = a^2$$

$+$

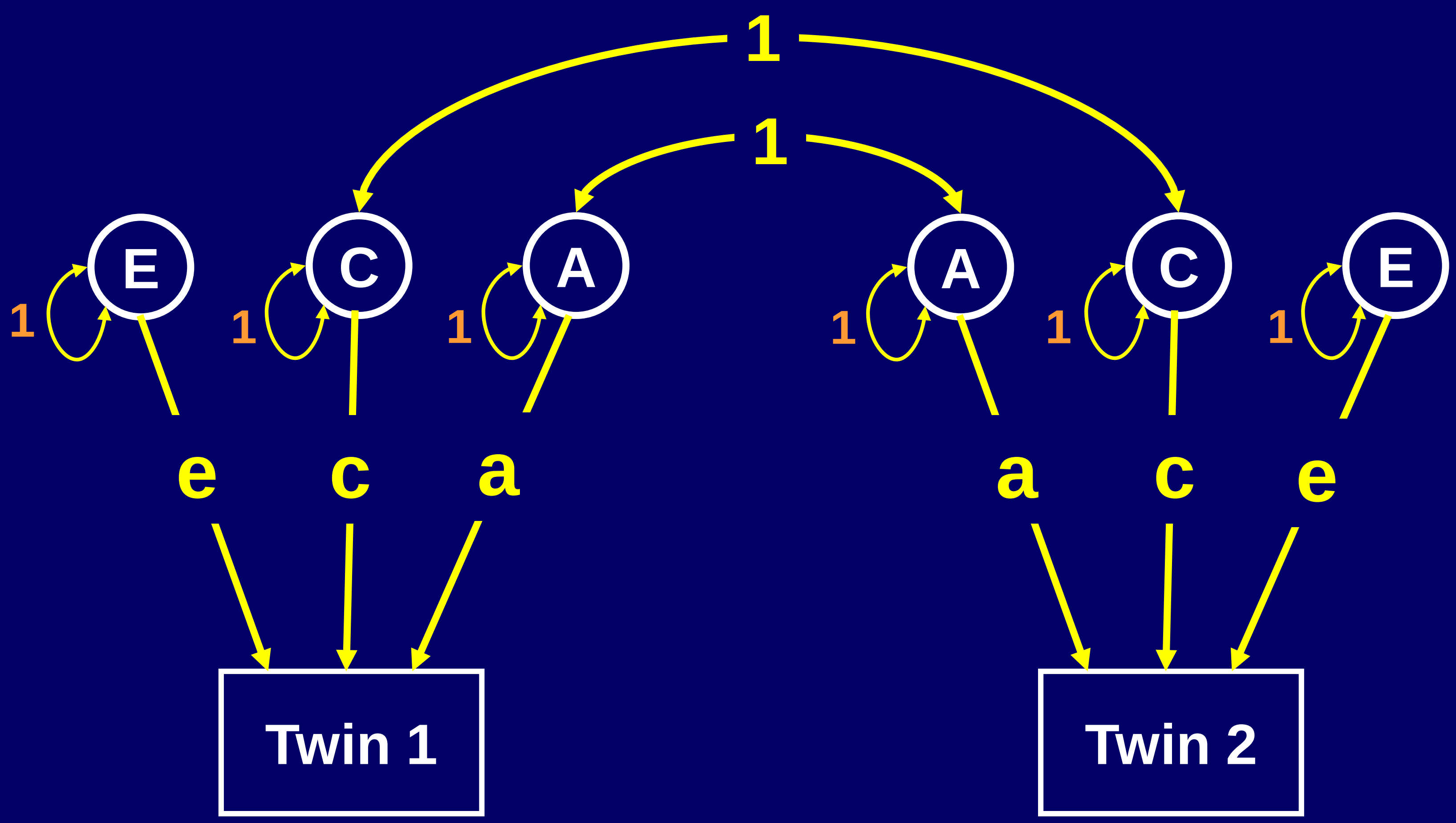
Variance of Twin 1 AND Twin 2 (for MZ and DZ pairs)



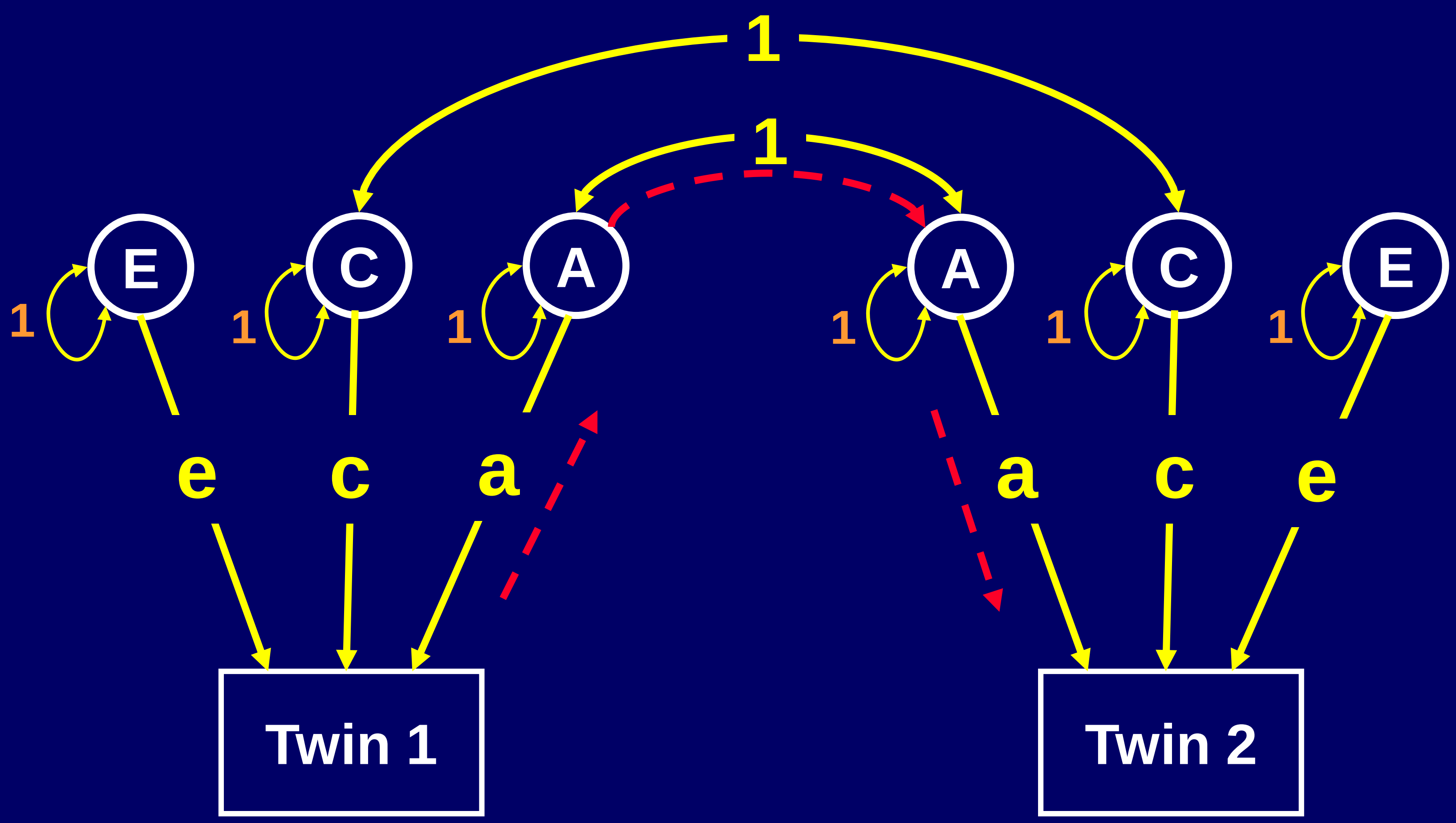
$$a*1*a = a^2$$
$$+$$
$$c*1*c = c^2$$
$$+$$
$$e*1*e = e^2$$

$$\text{Total Variance} = a^2 + c^2 + e^2$$

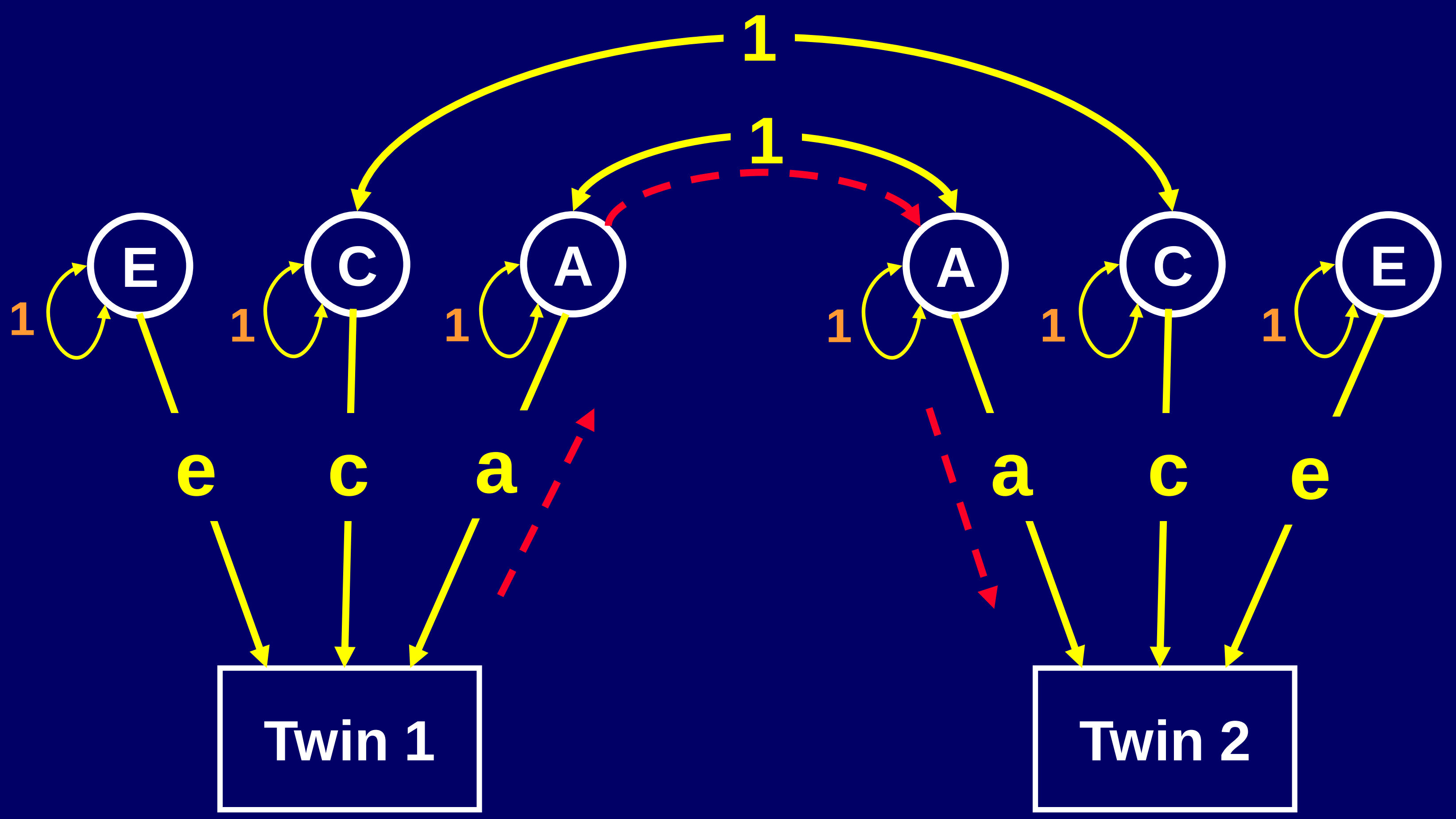
Covariance Twin 1-2: MZ pairs



Covariance Twin 1-2: MZ pairs

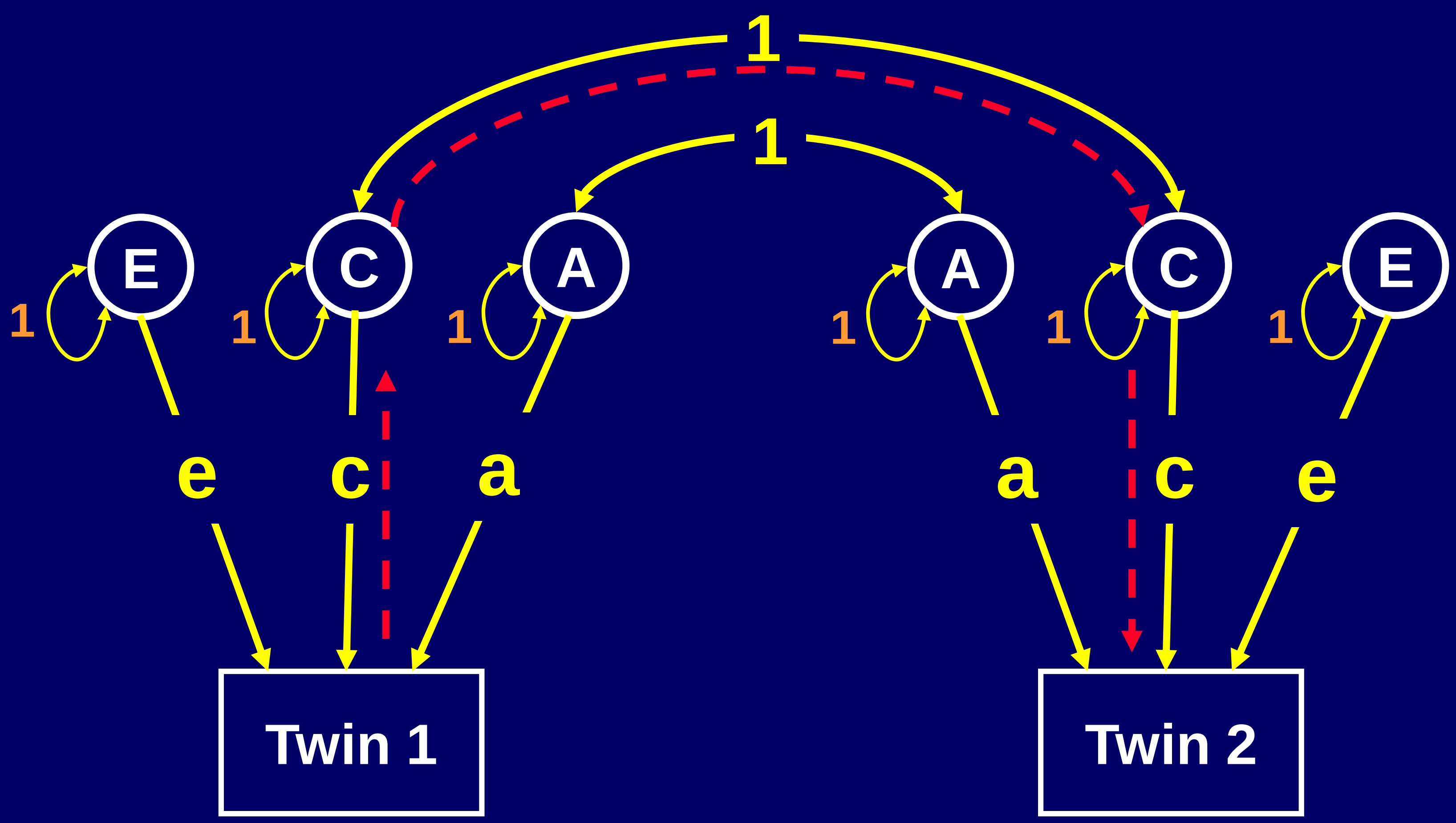


Covariance Twin 1-2: MZ pairs



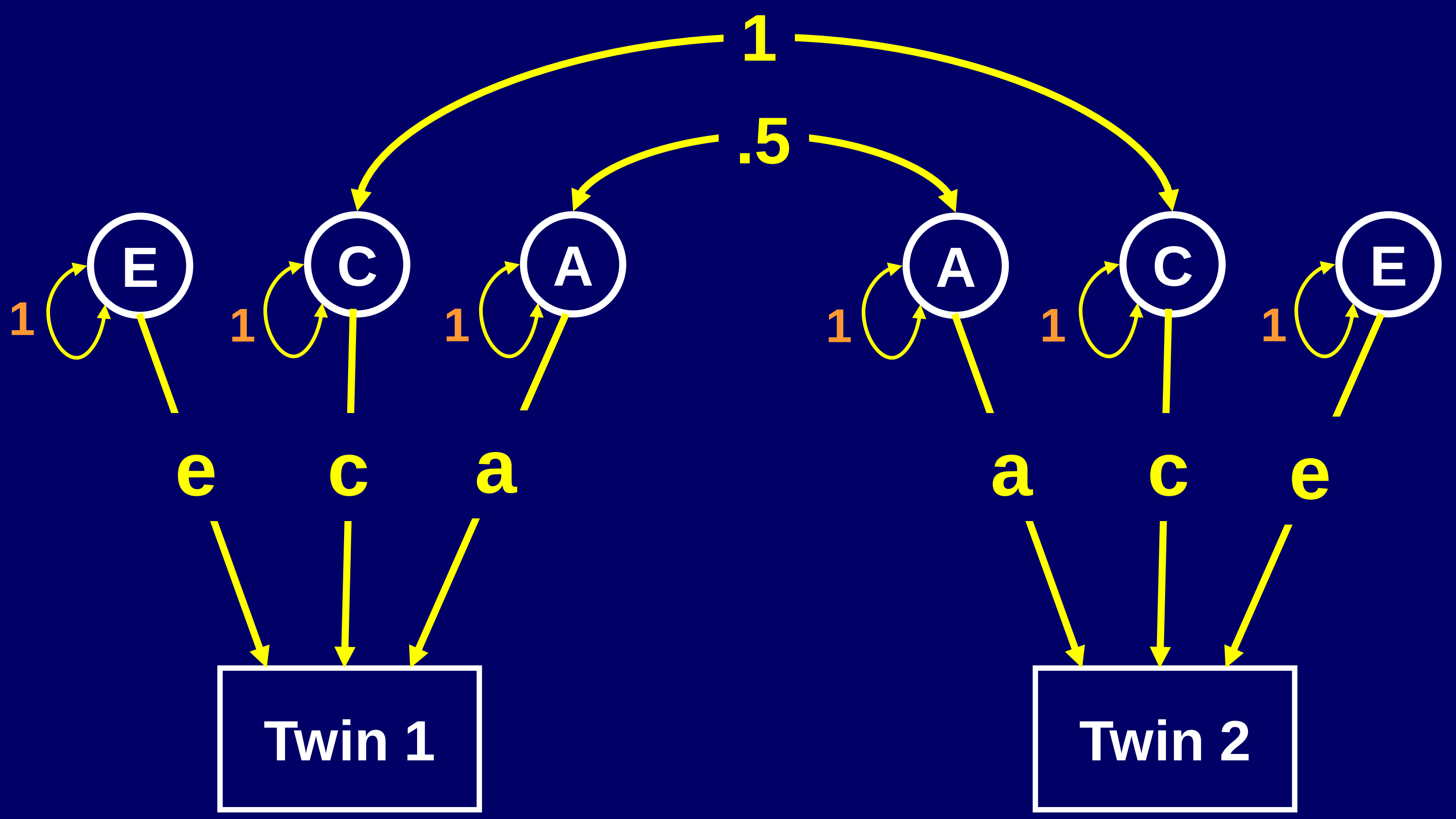
Total Covariance = a^2 +

Covariance Twin 1-2: MZ pairs

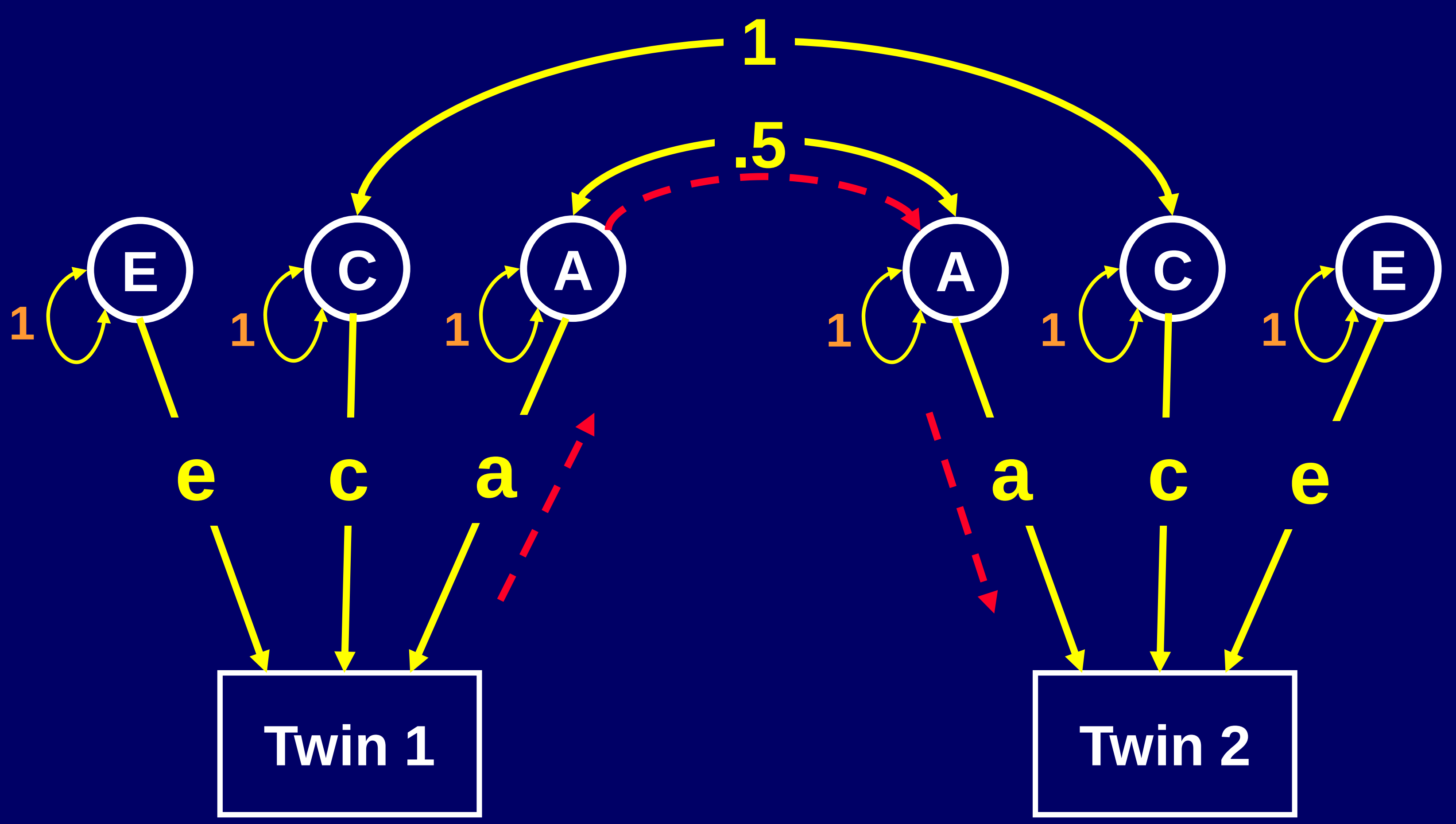


Total Covariance = $a^2 + c^2$

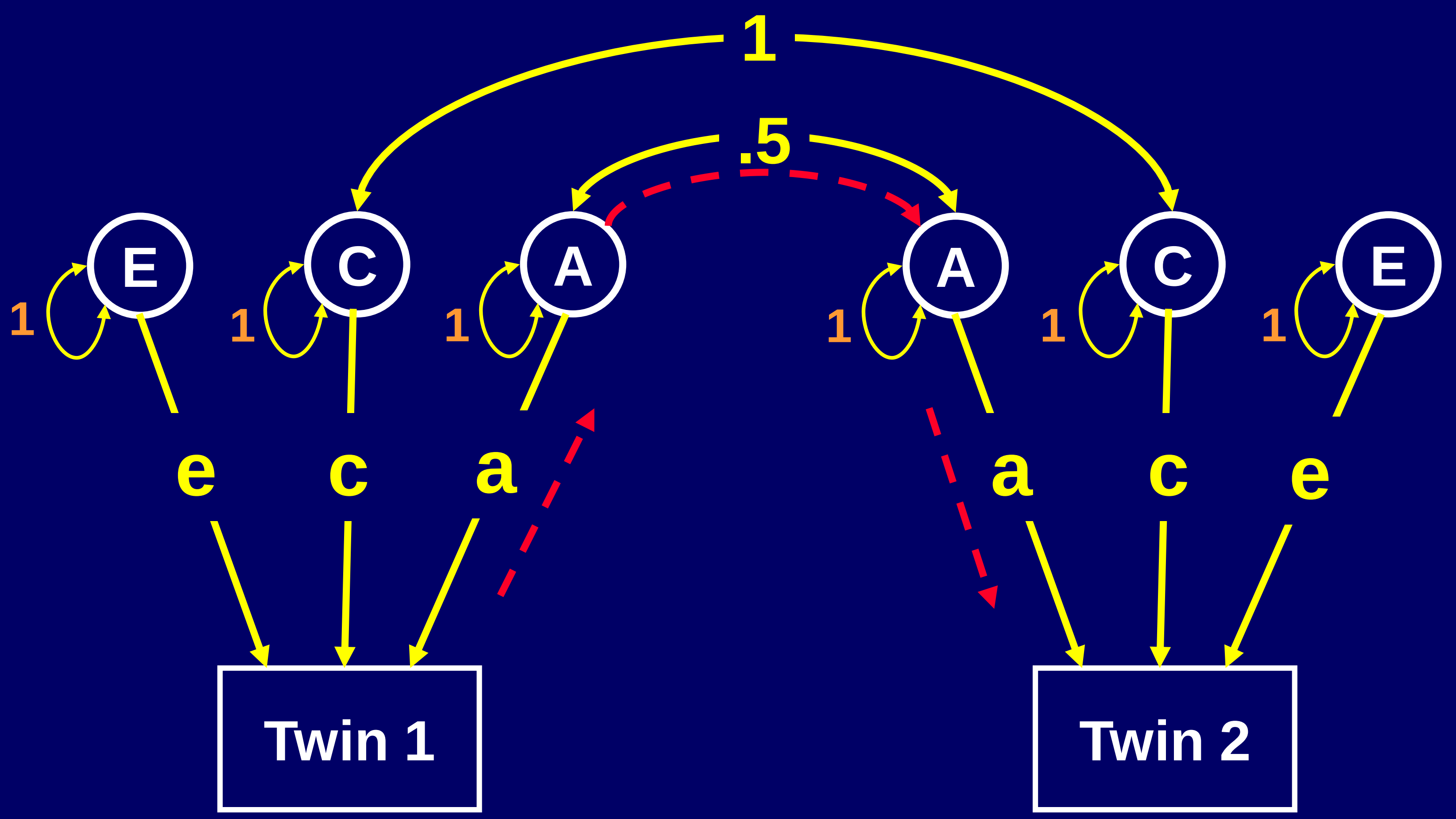
Covariance Twin 1-2: DZ pairs



Covariance Twin 1-2: MZ pairs

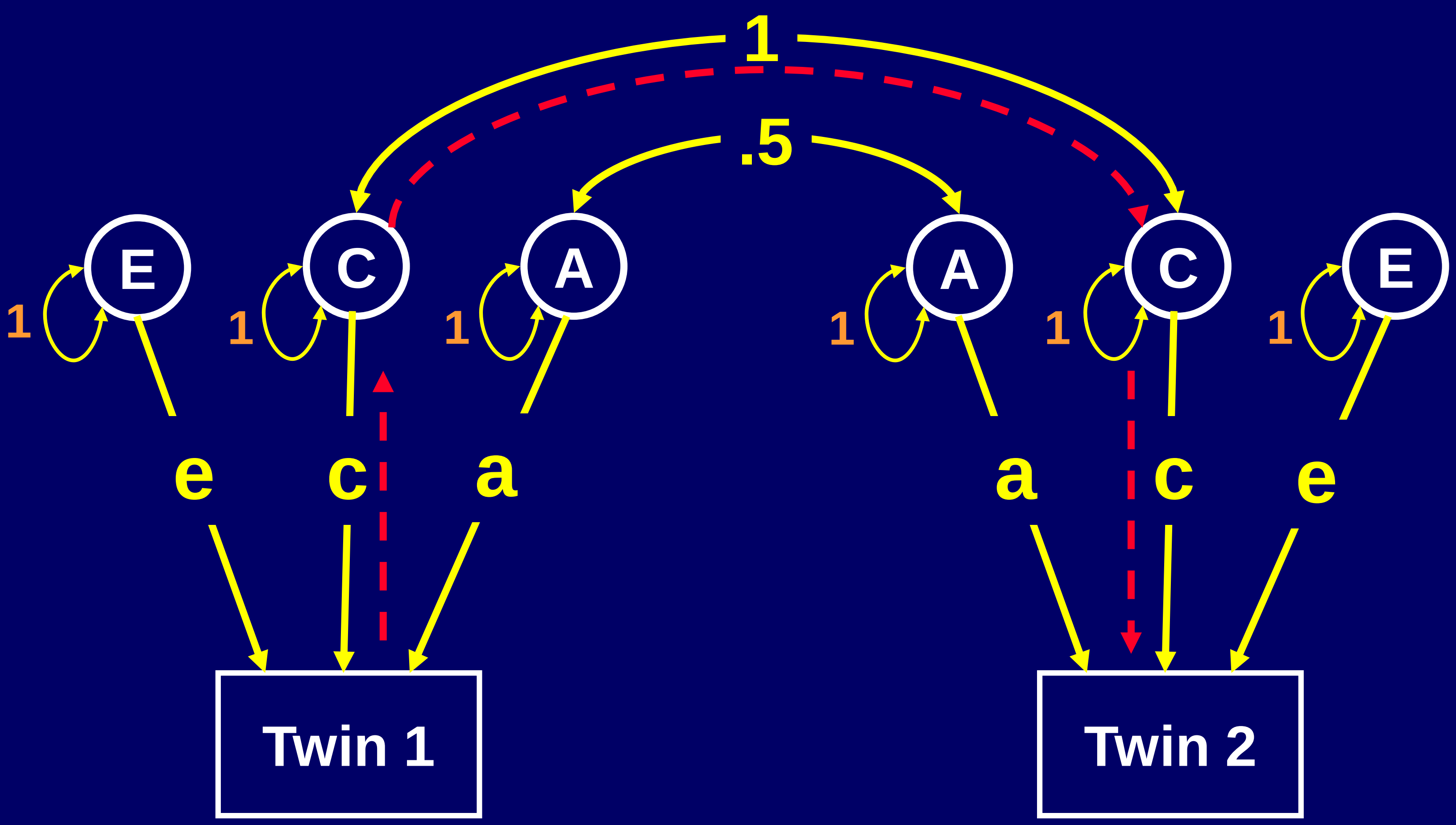


Covariance Twin 1-2: DZ pairs



Total Covariance = $.5a^2$ +

Covariance Twin 1-2: DZ pairs



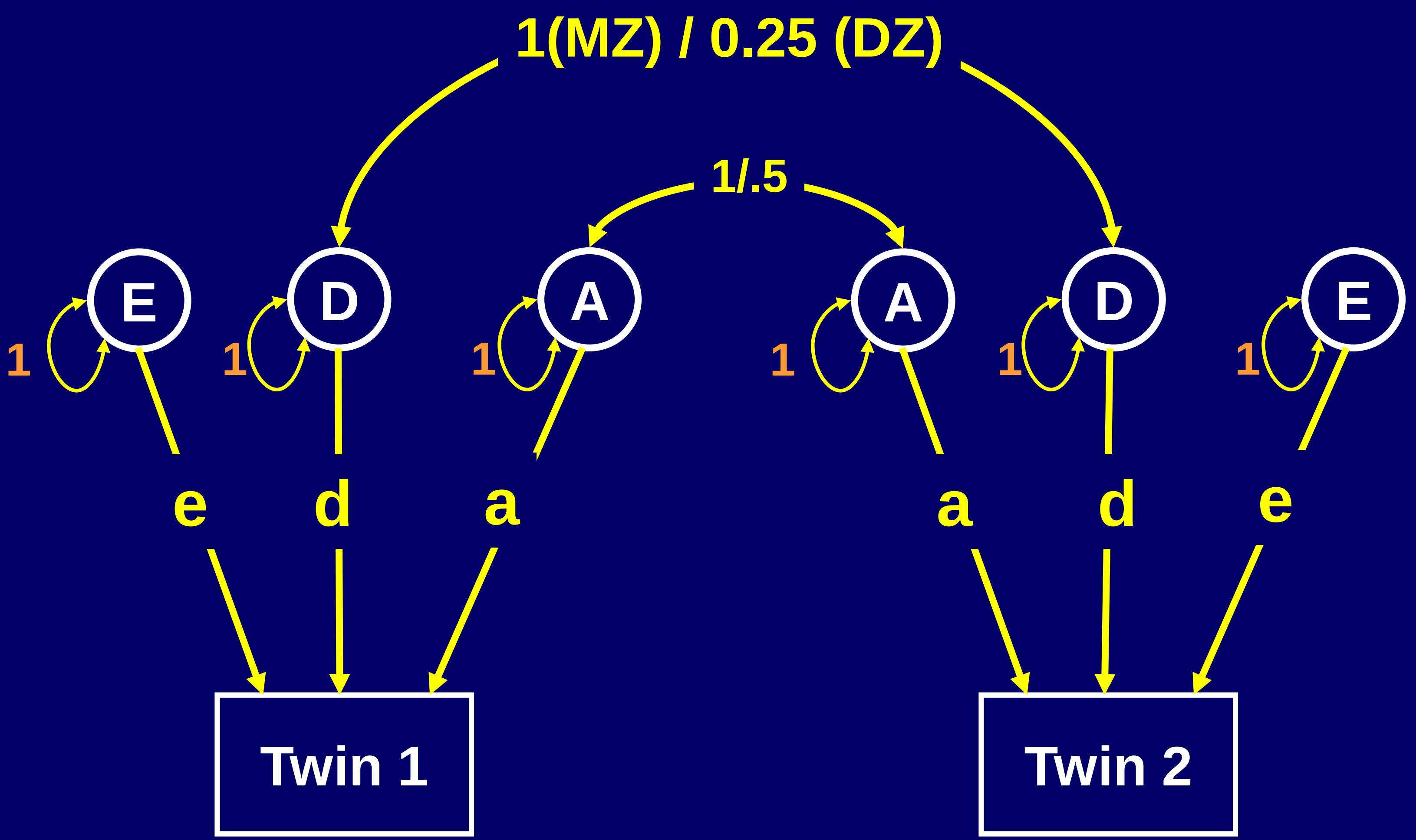
Total Covariance = $.5a^2 + c^2$

Predicted Var-Cov Matrices

$$Cov\,MZ = \begin{array}{c} \\ Tw1 \\ Tw2 \end{array} \begin{array}{cc} Tw1 & Tw2 \\ \left[\begin{array}{c} a^2 + c^2 + e^2 \\ a^2 + c^2 \end{array}\right. & \left.\begin{array}{c} a^2 + c^2 \\ a^2 + c^2 + e^2 \end{array}\right] \end{array}$$

$$Cov\,DZ = \begin{array}{c} \\ Tw1 \\ Tw2 \end{array} \begin{array}{cc} Tw1 & Tw2 \\ \left[\begin{array}{c} a^2 + c^2 + e^2 \\ \frac{1}{2}a^2 + c^2 \end{array}\right. & \left.\begin{array}{c} \frac{1}{2}a^2 + c^2 \\ a^2 + c^2 + e^2 \end{array}\right] \end{array}$$

ADE Model



Predicted Var-Cov Matrices

$$Cov\,MZ = \begin{array}{c|cc} & \text{Tw1} & \text{Tw2} \\ \hline \text{Tw1} & a^2+d^2+e^2 & a^2+d^2 \\ \text{Tw2} & a^2+d^2 & a^2+d^2+e^2 \end{array}$$

$$Cov\,DZ = \begin{array}{c|cc} & \text{Tw1} & \text{Tw2} \\ \hline \text{Tw1} & a^2+d^2+e^2 & \frac{1}{2}a^2+\frac{1}{4}d^2 \\ \text{Tw2} & \frac{1}{2}a^2+\frac{1}{4}d^2 & a^2+d^2+e^2 \end{array}$$

ACE or ADE

$Cov(mz) = a^2 + c^2$ or $a^2 + d^2$

$Cov(dz) = ½ a^2 + c^2$ or $½ a^2 + ¼ d^2$

$V_P = a^2 + c^2 + e^2$ or $a^2 + d^2 + e^2$

3 unknown parameters (a, c, e or a, d, e),
and only **3** distinctive predicted statistics:

Cov MZ, Cov DZ, Vp)

this model is **just identified**

Effects of C and D are confounded

The twin correlations indicate which of the two components is more likely to be present:

Cor(mz) = $a^2 + c^2$ or $a^2 + d^2$

Cor(dz) = $\frac{1}{2} a^2 + c^2$ or $\frac{1}{2} a^2 + \frac{1}{4} d^2$

If $a^2 = .40$, $c^2 = .20$

$r_{mz} = 0.60$

$r_{dz} = 0.40$

} **ACE**

If $a^2 = .40$, $d^2 = .20$

$r_{mz} = 0.60$

$r_{dz} = 0.25$

} **ADE**

(2) Covariance Algebra

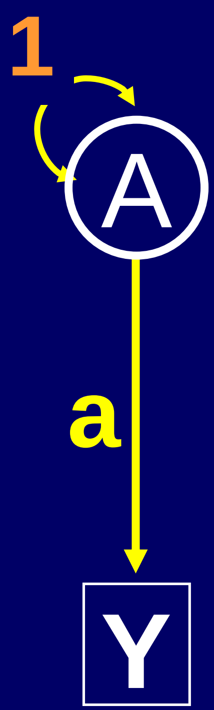
Three Fundamental Covariance Algebra Rules

$$Var(X) = Cov(X,X)$$

$$Cov(aX,bY) = ab\,Cov(X,Y)$$

$$Cov(X,Y+Z) = Cov(X,Y) + Cov(X,Z)$$

Example 1

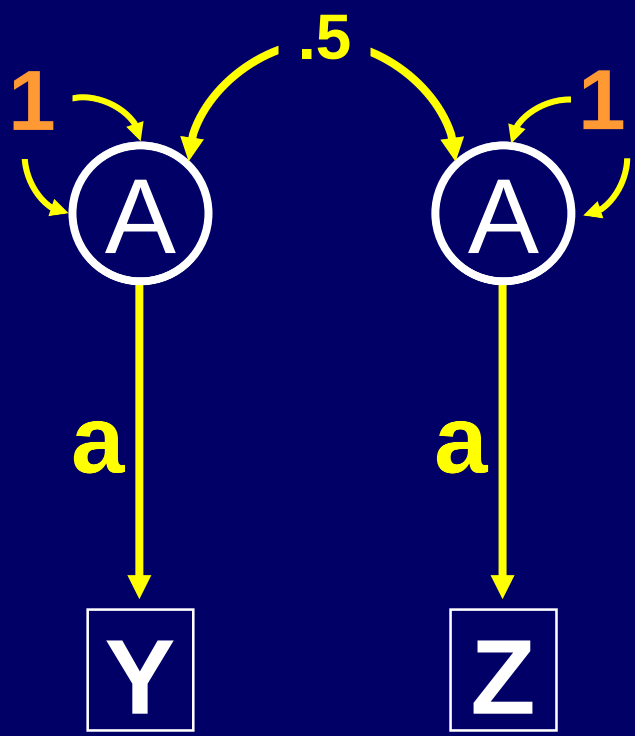


Y = aA

$$
\begin{aligned}
Var(Y) &= Var(aA) \\
&= Cov(aA, aA) \\
&= a^2 Cov(A, A) \\
&= a^2 Var(A) \\
&= a^2 * 1 \\
&= a^2
\end{aligned}
$$

The variance of a dependent variable (Y) caused by independent variable A, is the squared regression coefficient multiplied by the variance of the independent variable

Example 2



Y = aA **Z = aA**

$$Cov(Y,Z) = Cov(aA, aA)$$
$$= a^2 Cov(A,A)$$
$$= a^2 * .5$$

Summary

- Path Tracing and Covariance Algebra have the same aim:
  To work out the predicted variances and
  covariances of variables, given a specified model

- The Ultimate Goal:
  To fit predicted variances/covariances to
  observed variances/covariances of the data in order to estimate the model parameters:
  regression coefficients,correlations