# Introduction to OpenMx

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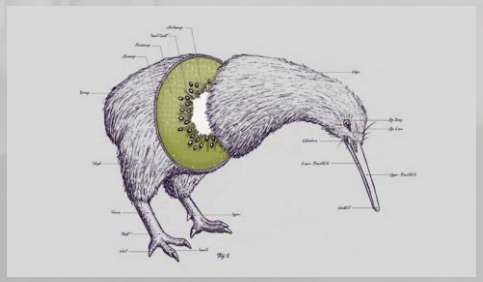


# What is OpenMx?

- Free, Open-source, full–featured SEM package
- Software which runs on Windows, Mac OSX, and Linux
- A package in the R statistical programing environment
- Two main approaches to writing OpenMx models – Path or Matrix Specification

- R is a functional language
  - Easy to define new functions
  - Items are stored as Objects
- OpenMx uses functions to build objects
  - Arguments to the function have an order
  - Order can be changed by naming arguments

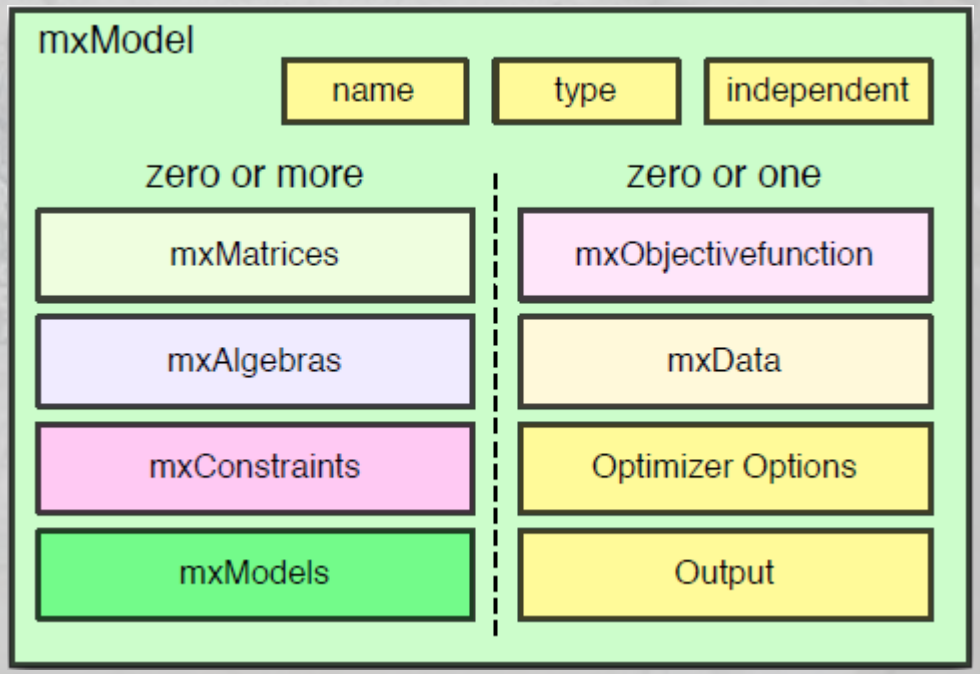
# Essential OpenMx functions

- mxModel()
- mxMatrix()
- mxAlgebra()
- mxData()
- mxFIMLObjective()
- mxRun()

# Essential OpenMx functions

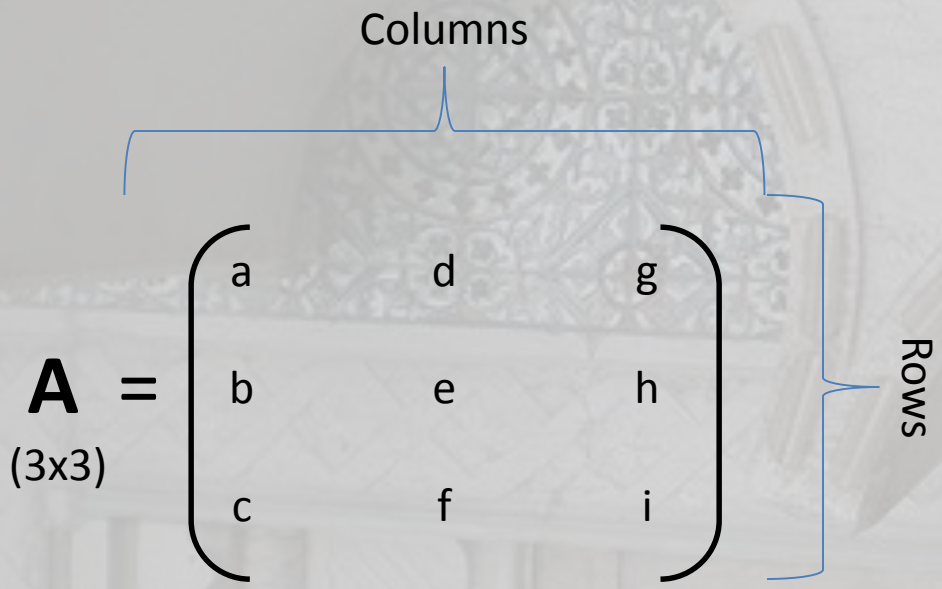
An MxModel Contains Objects (and potentially other MxModels)

- mxModel()
- mxMatrix()
- mxAlgebra()
- mxData()
- mxFIMLObjective()
- mxRun()



# Matrices are the building blocks

**Matrix**: a rectangular array of elements arranged in rows and columns.




The **order** or **dimension** of a matrix is defined by the number of row and columns in the matrix.

The order of a matrix is generally referred to as *M x N* (where M is the number of rows and N is the number of columns)

*Matrix A* is a 3 x 3 matrix.

Each element in the matrix is referred to by its placement in a row and column, where $a_{ij}$ is the element in *Matrix A* in the $i^{th}$ row and $j^{th}$ column.

Therefore, e is element $a_{(2,2)}$

# Matrices are the building blocks

mxMatrix( type="Lower", nrow=nv, ncol=nv, free=TRUE, values=.6, label="a11", name="a" ), # additive genetic path coefficients

- Many types eg. type="Lower"
- Denoted by names eg. name="a“
- Size eg. nrow=nv, ncol=nv
- Estimated parameters must be placed in a matrix & Mx must be told what type of matrix it is

# Matrices are the building blocks

```
mxMatrix( type="Zero", nrow=2, ncol=3, name="a" )
```

```
0 0 0
0 0 0
```

```
mxMatrix( type="Unit", nrow=2, ncol=3, name="a" )
```

```
1 1 1
1 1 1
```

```
mxMatrix( type="Ident", nrow=3, ncol=3, name="a" )
```

```
1 0 0
0 1 0
0 0 1
```

# Matrices are the building blocks

```
mxMatrix( type="Diag", nrow=3, ncol=3, free=TRUE, name="a" )
```

```
? 0 0
0 ? 0
0 0 ?
```

```
mxMatrix( type="Sdiag", nrow=3, ncol=3, free=TRUE, name="a" )
```

```
0 0 0
? 0 0
? ? 0
```

```
mxMatrix( type="Stand", nrow=3, ncol=3, free=TRUE, name="a" )
```

```
1 ? ?
? 1 ?
? ? 1
```

```
mxMatrix( type="Symm", nrow=3, ncol=3, free=TRUE, name="a" )
```

```
? ? ?
? ? ?
? ? ?
```

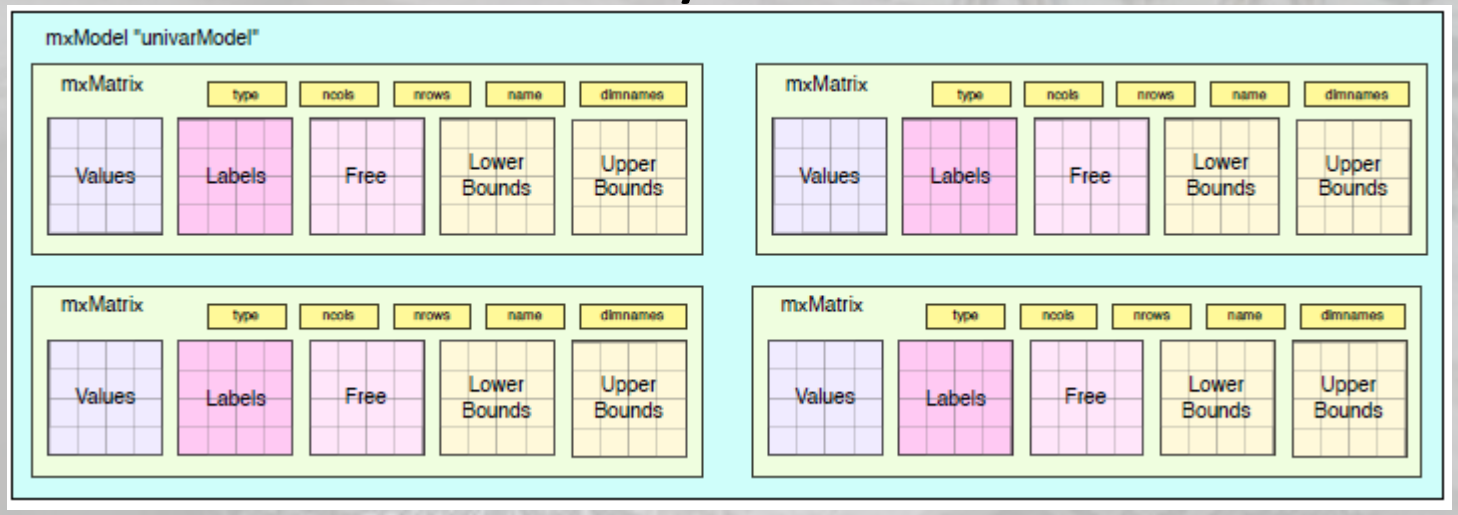
```
mxMatrix( type="Lower", nrow=3, ncol=3, free=TRUE, name="a" )
```

```
? 0 0
? ? 0
? ? ?
```

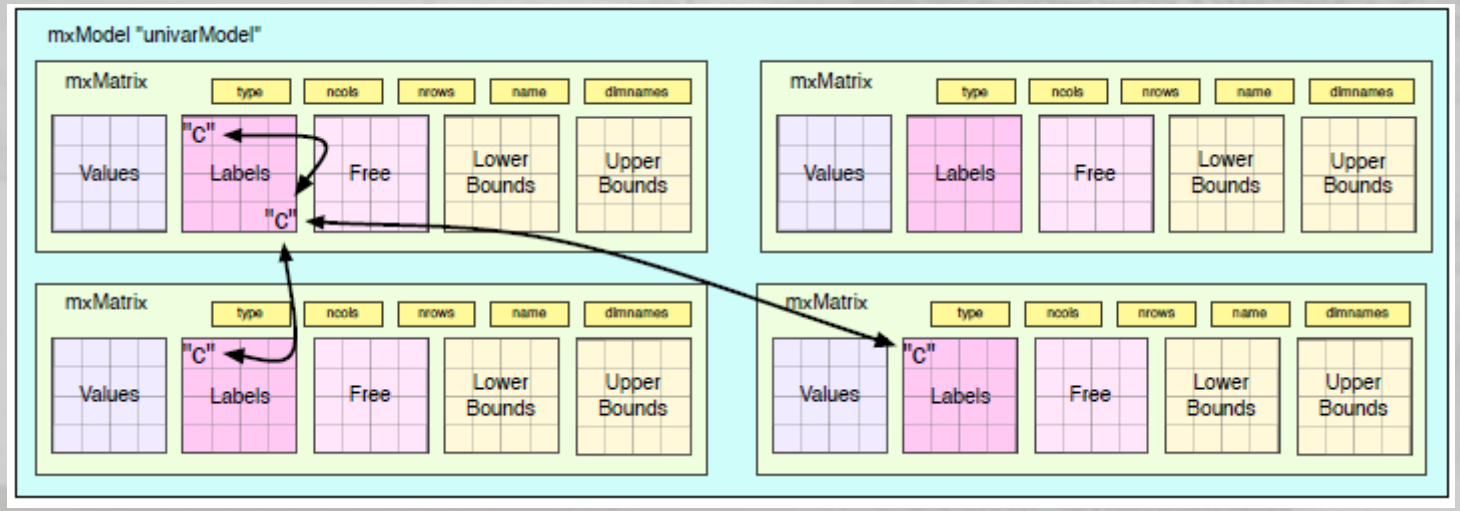
```
mxMatrix( type="Full", nrow=2, ncol=4, free=TRUE, name="a" )
```

```
? ? ? ?
? ? ? ?
```

# A model can have many matrices




# Equate parameters using labels



# Matrix Operations

## Matrix Addition and Subtraction:

- Matrices must be the same size

$$\begin{pmatrix} 2 & 1 \\ 3 & 5 \\ 6 & 2 \end{pmatrix} + \begin{pmatrix} 4 & 8 \\ 7 & 2 \\ 9 & 6 \end{pmatrix} = \begin{pmatrix} 2+4 & 1+8 \\ 3+7 & 5+2 \\ 6+9 & 2+6 \end{pmatrix} = \begin{pmatrix} 6 & 9 \\ 10 & 7 \\ 15 & 8 \end{pmatrix}$$

If the matrices are of different orders, it is impossible to add them

$$\begin{pmatrix} 2 & 1 \\ 3 & 5 \\ 6 & 2 \end{pmatrix} + \begin{pmatrix} 4 & 7 & 9 \\ 8 & 2 & 6 \end{pmatrix} = \text{Undefined}$$

Dot Product

Also known as the **element-wise product**

OpenMx symbol *

$$A \circ B = \begin{pmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{pmatrix} . \begin{pmatrix} B_{11} & B_{12} & B_{13} \\ B_{21} & B_{22} & B_{23} \\ B_{31} & B_{32} & B_{33} \end{pmatrix} = \begin{pmatrix} A_{11}B_{11} & A_{12}B_{12} & A_{13}B_{13} \\ A_{21}B_{21} & A_{22}B_{22} & A_{23}B_{23} \\ A_{31}B_{31} & A_{32}B_{32} & A_{33}B_{33} \end{pmatrix}$$

Matrix Multiplication (Star product)

Number of columns of the first matrix must equal the number of rows of the second matrix.

Product will have as many rows as the first matrix and as many columns as the second matrix.

OpenMx symbol %*%

$$\mathbf{C} = \mathbf{A} \times \mathbf{B}$$

$$\mathbf{C} = \begin{pmatrix} 3 & 4 & 7 \\ 5 & 6 & 1 \end{pmatrix} \times \begin{pmatrix} 2 & 1 \\ 3 & 5 \\ 6 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} 3*2 + 4*3 + 7*6 & 3*1 + 4*5 + 7*2 \\ 5*2 + 6*3 + 1*6 & 5*1 + 6*5 + 1*2 \end{pmatrix} = \begin{pmatrix} 60 & 37 \\ 34 & 37 \end{pmatrix}$$

# Kroneker Product

OpenMx symbol %x%

$$A \otimes B = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \otimes \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} = \begin{bmatrix} a_{11}B & a_{12}B \\ a_{21}B & a_{22}B \end{bmatrix}$$

$$= \begin{bmatrix} a_{11}b_{11} & a_{11}b_{12} & a_{12}b_{11} & a_{12}b_{12} \\ a_{11}b_{21} & a_{11}b_{22} & a_{12}b_{21} & a_{12}b_{22} \\ a_{21}b_{11} & a_{21}a_{12} & a_{22}b_{11} & a_{22}b_{12} \\ a_{21}b_{21} & a_{21}b_{22} & a_{22}b_{21} & a_{22}b_{22} \end{bmatrix}$$

# Quadratic Product

- The quadratic product is extremely useful in statistical analysis (particularly in Structural Equation Modeling)
- OpenMx symbol %&%

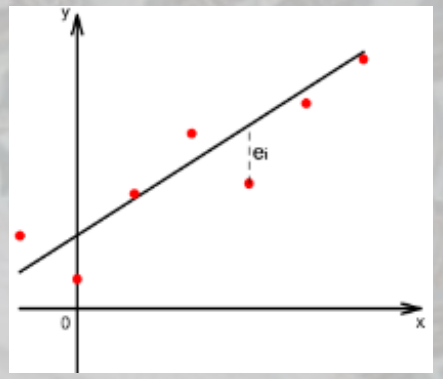
$$A\%\&\%B = ABAT =$$
$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \text{x} \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \text{x} \begin{bmatrix} a_{11} & a_{21} \\ a_{12} & a_{22} \end{bmatrix}$$

# Our first OpenMx script

- Linear regression ( aka simple regression, Ordinary Least Squares Regression, linear model)
- Raw data
- Describe the relationship between two variables, X and Y, as a straight line
- The regression of BMI on variable Age
  - Does age predict BMI?

BMI=weight in kg/ (height in m)$^2$

# Linear Regression

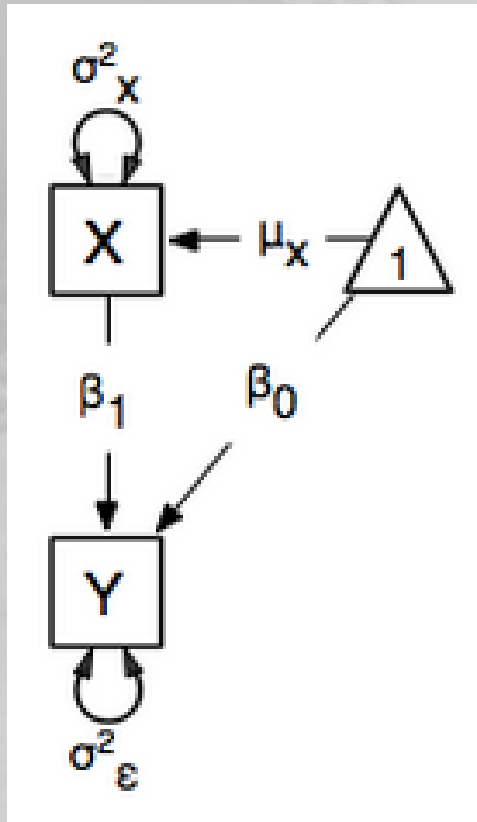


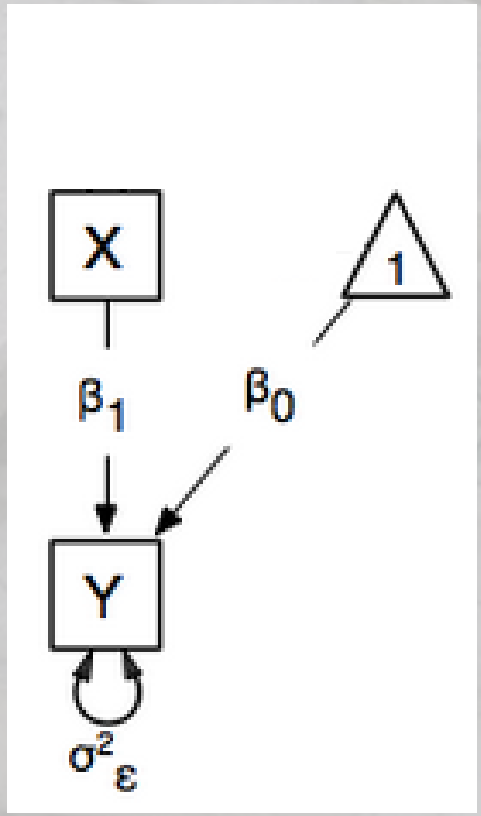

- The regression of variable Y on variable X is given by:

$$y_i = \beta_0 + \beta_1 x_i + \varepsilon_i \quad i = 1, \ldots, n$$

- where:
  - $\beta_0$ (Intercept) value of y when x=0
  - $\beta_1$ (Slope) increase in Y for each unit change in X
  - $\varepsilon_i$ (Random Error) $\varepsilon_i \sim N(0, \sigma^2)$
  - Linear Function: $\beta_0 + \beta_1 x_i$ = E ( Y | X = $x_i$ )

# As a path diagram



# Starting at the beginning…

- Data preparation
  - The algebra style used in OpenMx expects 1 line per case/family
  - (Almost) limitless number of families and variables
  - Data needs to be read into R before it can be analysed
    - (the commands to read the data can be nested within the R script)
    - Default missing code is now **NA**

# Getting your data into R

- Open RStudio
- Example data:ozbmi2.txt
- OZbmi<-read.table("ozbmi2.txt", header=T, na.strings = "NA")
- head(data)

```
> head(data)
  fam agecat  age zyg part wt1 wt2     ht1     ht2    htwt1    htwt2     bmi1     bmi2
1 115      0 0.21   1    2  58  57     1.7     1.7 20.0692 19.7232 20.9943 20.8726
2 121      0 0.24   1    2  54  53  1.6299  1.6299 20.3244 19.9481 21.0828 20.9519
3 158      0 0.21   1    2  55  50  1.6499  1.6799  20.202 17.7154 21.0405  20.121
4 172      0 0.21   1    2  66  76  1.5698  1.6499 26.7759 27.9155 23.0125 23.3043
5 182      0 0.19   1    2  50  48  1.6099  1.6299 19.2894 18.0662 20.7169 20.2583
6 199      0 0.26   1    2  60  60  1.5999  1.5698 23.4375 24.3418 22.0804 22.3454
```

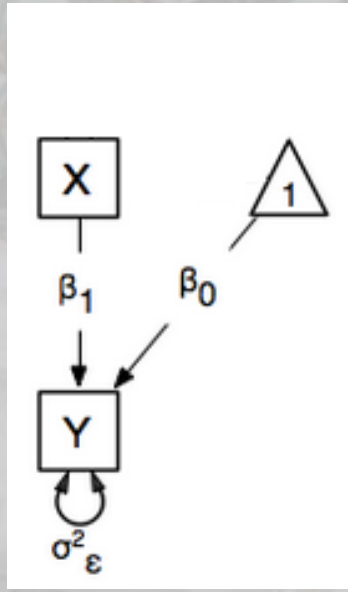
- # using subset function create new dataset without missing data
- OZbmi <- subset(data, age !="NA" , select=c(bmi1, age))

# Regression using lm

- BMIfit <- lm(bmi1 ~ age, data=OZbmi)
- summary(BMIfit) # show results
- coefficients(BMIfit) # model coefficients

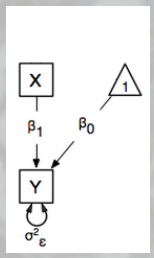
```
> #Running the regression using lm
> BMIfit <- lm(bmi1 ~ age, data=OZbmi)
> summary(BMIfit) # show results

Call:
lm(formula = bmi1 ~ age, data = OZbmi)

Residuals:
    Min      1Q  Median      3Q     Max
-3.4504 -0.5906 -0.0712  0.5188  4.3832

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 21.01355    0.03912  537.20   <2e-16 ***
age          2.03457    0.10483   19.41   <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.8944 on 3671 degrees of freedom
  (133 observations deleted due to missingness)
Multiple R-squared:  0.09305, Adjusted R-squared:  0.09281
F-statistic: 376.6 on 1 and 3671 DF,  p-value: < 2.2e-16

> coefficients(BMIfit) # model coefficients
(Intercept)         age
  21.013545    2.034573
```

# Regression using OpenMx



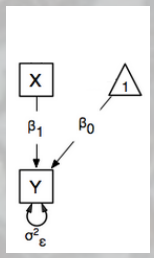

- Model contains:
  - 4 matrices and 3 estimated parameters
  - BMI
    - Free observed variable
    - $\beta_0$ and $\sigma^2{}_\varepsilon$ are estimated
  - Age
    - Fixed observed variable
  - Regression of BMI on Age
    - $\beta_1$ is estimated

# Regression using OpenMx




```
# Variance/Covariance matrix
Variance    <- mxMatrix(
                    type="Full",
                    nrow=1,
                    ncol=1,
                    free=TRUE,
                    values=11,
                    labels='resid',
                    name="residualVar" )
```

# Regression using OpenMx




```
require (OpenMx)
depVar <- 'bmi1‘

# Variance/Covariance matrix
Variance    <- mxMatrix( type="Full", nrow=1, ncol=1, free=TRUE,
                  values=11, labels='resid', name="residualVar" )

# Regression betas
b0          <-mxMatrix(type="Full", nrow=1, ncol=1, free=T, values=22,
                labels="beta0", name="Intercept" )
b1          <-mxMatrix(type="Full", nrow=1, ncol=1, free=T, values=0,
                labels="beta1", name="bAge" )

# Independent variable
x           <-mxMatrix(type="Full", nrow=1, ncol=1, free=F,
                labels="data.age", name="Age" )
```

```
> b0
FullMatrix 'Intercept'

@labels
     [,1]
[1,] "beta0"

@values
     [,1]
[1,]   22

@free
     [,1]
[1,] TRUE

@lbound: No lower bounds assigned.

@ubound: No upper bounds assigned.
```

```
> x
FullMatrix 'Age'

@labels
     [,1]
[1,] "data.age"

@values
     [,1]
[1,]    0

@free: No free parameters.

@lbound: No lower bounds assigned.

@ubound: No upper bounds assigned.
```

# Building the model $(y_i = \beta_0 + \beta_1 x_i)$

```
expMean     <- mxAlgebra(intercept + bAge%x%Age,
                         name="regress")
```

```
> expMean
mxAlgebra 'regress'
@formula:  Intercept + bAge * Age
@result: (not yet computed) <0 x 0 matrix>
dimnames: NULL
```

# Specify the data

```
regData     <- mxData( observed=OZbmi, type="raw" )
inclusions  <- list(Variance, b0,  b1, bAge, expMean)
```

# Define the objective

# NOTE- The matrix names defined in mxMatrix() statements are used here

# NOT the objects that have been defined.

```
regObj      <- mxFIMLObjective( covariance="VarCov",
                 means="regress", dimnames=depVar )
```

```
> regObj
MxFIMLObjective 'objective'
@covariance : 'VarCov'
@means : 'regress'
@vector : FALSE
@dims : 'bmi1'
@thresholds : NA
@threshnames : 'bmi1'
@info$likelihoods:  FALSE
@result: (not yet computed) <0 x 0 matrix>
```

#Build the model - specify the name of the model, the objects referenced, the data and the objective

```
regModel <- mxModel( "Regression101", inclusions,
                     regData, regObj )
```

```
> regModel
MxModel 'Regression101'
type : default
@matrices : 'residualVar', 'Intercept', 'bAge', and 'Age'
@algebras : 'regress'
@constraints :
@intervals :
@latentVars : none
@manifestVars : none
@data : 3808 x 13
@data means : NA
@data type: 'raw'
@submodels :
@objective : MxFIMLObjective
@independent : FALSE
@options :
@output : FALSE
```

# Run the model & summarize output

```
regFit      <- mxRun( regModel, intervals=FALSE )
regSum    <- summary( regFit )
```

```
> regFit       <- mxRun( regModel, intervals=FALSE )
Running Regression101
> ( regSum    <- summary( regFit ) )
data:
$Regression101.data
      bmi1             age
 Min.   :18.11   Min.   :0.1700
 1st Qu.:21.08   1st Qu.:0.2300
 Median :21.61   Median :0.3000
 Mean   :21.72   Mean   :0.3445
 3rd Qu.:22.29   3rd Qu.:0.4200
 Max.   :26.15   Max.   :0.8800
 NA's   :133

free parameters:
   name      matrix  row  col  Estimate  Std.Error lbound ubound
1 resid residualvar bmi1 bmi1  0.799574 0.01865790
2 beta0   Intercept    1    1 21.013549 0.03911028
3 beta1        bAge    1    1  2.034564 0.10481854

observed statistics:  3673
estimated parameters:  3
degrees of freedom:  3670
-2 log likelihood:  9601.954
saturated -2 log likelihood:  NA
number of observations:  3806
chi-square:  NA
p:  NA
Information Criteria:
     df Penalty Parameters Penalty Sample-Size Adjusted
AIC:   2261.954           9607.954                   NA
BIC: -20654.752           9626.687             9617.155
CFI: NA
TLI: NA
RMSEA:  NA
timestamp: 2014-03-03 20:37:46
frontend time: 0.378195 secs
backend time: 0.927716 secs
independent submodels time: 0 secs
wall clock time: 1.305911 secs
cpu time: 1.305911 secs
openmx version number: 1.4-3060
```

# Intuitive Logic of Optimization

1. Start with an arbitrary set of initial parameters values. *(Starting Values)*

2. Determine a direction of movement for the parameters (larger or smaller)

3. Determine a step length to move (how much larger or smaller)

4. Rinse and repeat until some termination criteria is reached and then stop.

```
#Looking at the optimization process

# ReRunning to look at optimization
regModel    <- mxOption(regModel,"Checkpoint Units",
                        "iterations")
regModel    <- mxOption(regModel,"Checkpoint Count", 1)
regFit    <- mxRun( regModel, intervals=FALSE, checkpoint=T )
```

# OpenMx vs lm

OpenMx

- regSum$parameters

```
> regSum$parameters
   name      matrix  row  col  Estimate  Std.Error lbound ubound lboundMet uboundMet
1 resid residualVar bmi1 bmi1  0.799574 0.01865790     NA     NA      FALSE      FALSE
2 beta0   Intercept    1    1 21.013549 0.03911028     NA     NA      FALSE      FALSE
3 beta1        bAge    1    1  2.034564 0.10481854     NA     NA      FALSE      FALSE
```

Lm

```
Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept) 21.01355    0.03912  537.20   <2e-16 ***
age          2.03457    0.10483   19.41   <2e-16 ***
```

- Is beta1 different from 0?

```
# Go back and pickup the model so that we can run significance tests
ageEfModel    <- regFit
#set beta 1 to 0
ageEfModel    <- omxSetParameters( ageEfModel, label="beta1",
                  free=FALSE, values=0 )
ageEfFit          <- mxRun(ageEfModel, intervals=FALSE)
(ageEfSumm    <- summary(ageEfFit))
```

```
# difference in fit
deltaLL <-ageEfSumm$Minus2LogLikelihood -
        regSum$Minus2LogLikelihood
# difference in df
deltaDF <-ageEfSumm$degreesOfFreedom -
        regSum$degreesOfFreedom
# significance test
pchisq(deltaLL, lower.tail=F, deltaDF)
```

# Questions?

Bus shelter on the road to Sintra (Portugal)