Factor model of Ordered-Categorical Measures: Measurement Invariance

Michel Nivard

Sanja Franic

Dorret Boomsma

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Conor Dolan

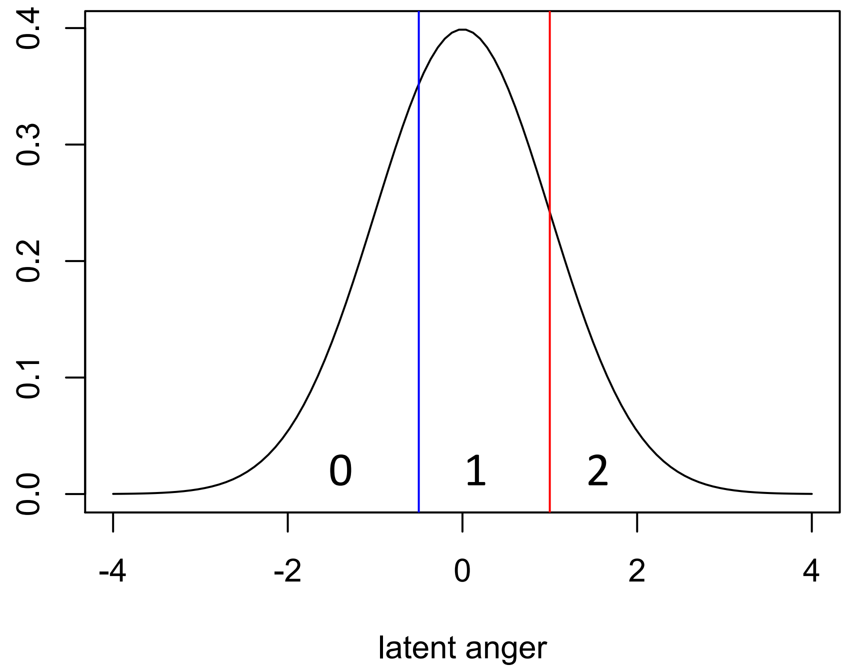
OpenMx “team”

Ordinal factor model

- This model works for ordered categorical indicators.
- It assumes these indicators are indicators of an underlying continuous (and normally distributed) trait.

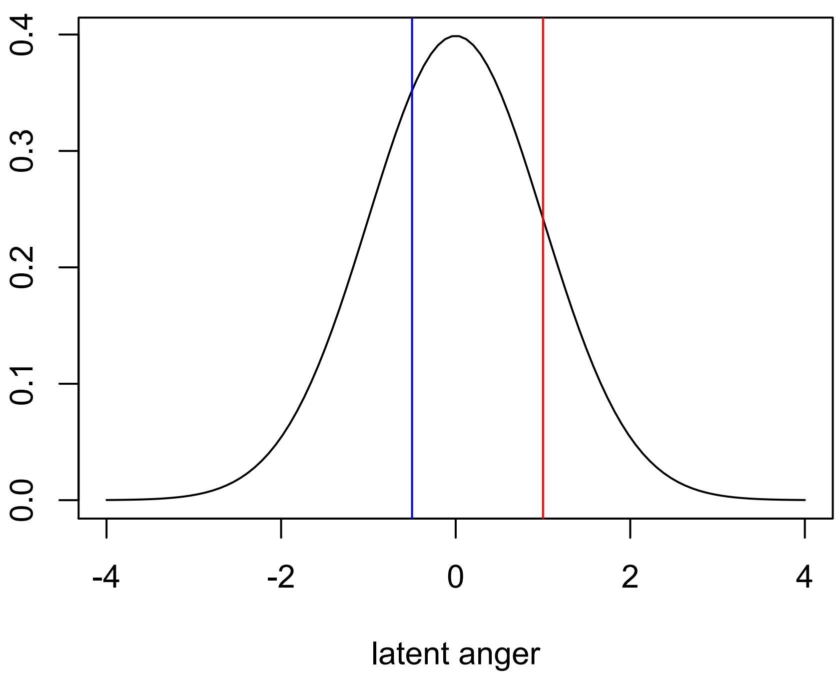
Ordinal indicators

- Q: do you get angry?
  - 0: Never
  - 1: Sometimes
  - 2: Often



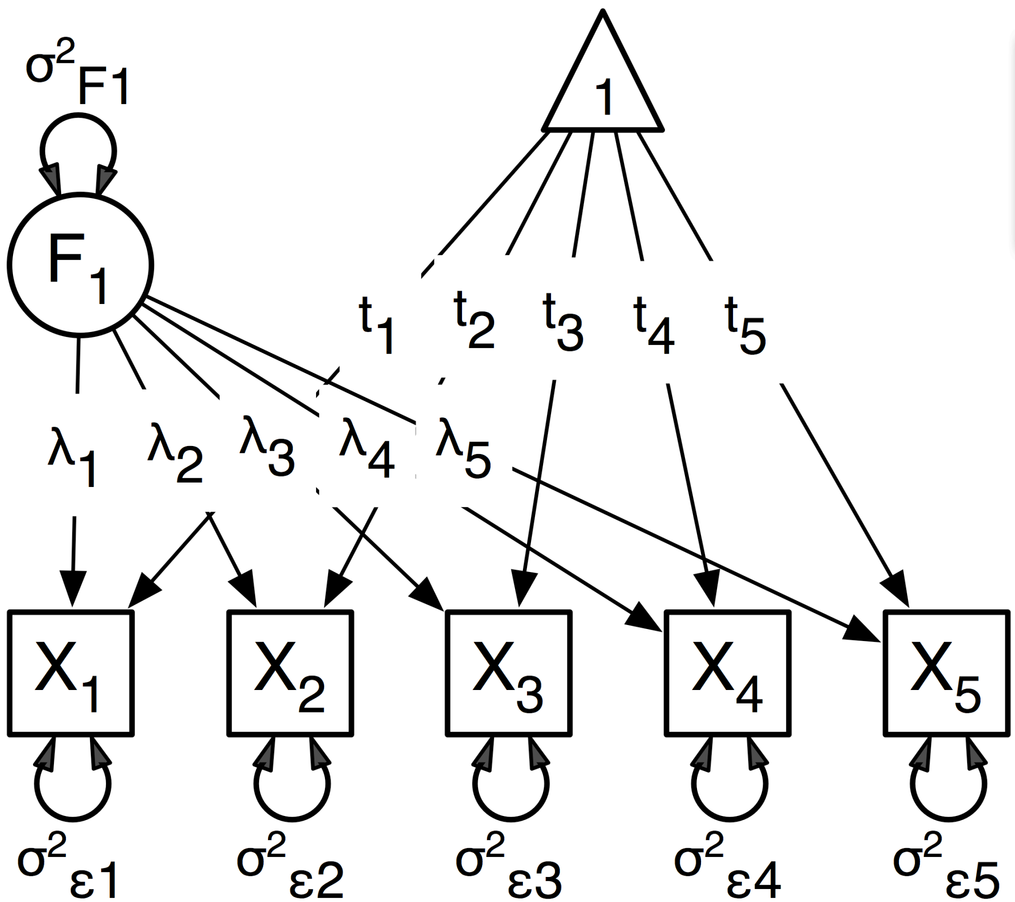
Ordinal indicators

- The latent distribution is restricted too mean=0 and variance = 1. Given these constraints we can estimate the the threshold parameters indicated in red and blue here.



Either:
μ and Σ
Or:
the Thresholds are identified.

Ordinal indicators factor model.



	x1	x2	x3	x4	x5
th1	?	?	?	?	?
th2	?	?	?	?	?
th3	?	?	?	?	?

Ordinal indicators factor model.

- Expected (polychoric) covariance:
  - $Cov(X_i) = \Sigma = \Lambda_i \Psi \Lambda_i^t + \Theta_i$
- Expected means/threshold:
  - $E(X_i) = \mu = \tau_i + \Lambda_i \kappa$

Λ = factor loadings
Ψ = factor (co)variance matrix
Θ = residual variances ε_1 to ε_n
τ = intercepts
κ = factor mean

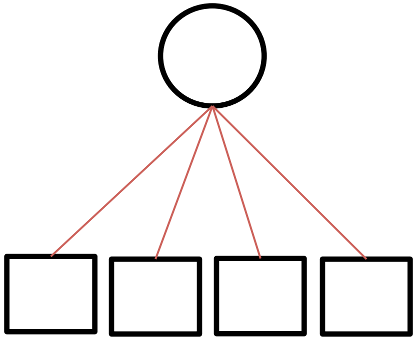
- Restrictions for identification (alternatives, Millsap 2004)
  1. $\Psi = 1$
  2. $diag(\Lambda_2 \Lambda_2 t + \Theta_2) = diag(I)$
  3. $\mu = 0 = \tau_i + \Lambda_i \kappa$

Model restrictions for measurement invariance.

- Configural invariance:
- Group 1:
  - $P_1 = \Lambda_1 \Psi_1 \Lambda_1 t + \Theta_1$
- Group 2:
  - $P_2 = \Lambda_2 \Psi_2 \Lambda_2 t + \Theta_2$
- Restrictions:
  - Thresholds are equal over groups.
  - $diag(\Lambda_1 \Lambda_1 t + \Theta_1) = \mathrm{diag}(\Lambda_2 \Lambda_2 t + \Theta_2) = \mathrm{diag}(I)$
  - Indicator means: Group1: 0 Group2: FREE
  - $\Psi_2 = \Psi_1 = 1$

Model restrictions for measurement invariance.

- metric invariance:
- Group 1:
  - $P_1 = \Lambda \Psi_1 \Lambda t + \Theta_1$
- Group 2:
  - $P_2 = \Lambda \Psi_2 \Lambda t + \Theta_2$
- Restrictions:



– Thresholds are equal over groups.
  - $diag(\Lambda_1 \Lambda_1 t + \Theta 1) = diag(\Lambda_2 \Lambda_2 t + \Theta_2) = diag(I)$
  - τ : group1: 0 group2: FREE
  - K: set to 0
  - $\Psi_1 = 1$ (however Ψ_2 = FREE)

Model restrictions for measurement invariance.

- Strong invariance:
  - This concerns the means model.
  - $E(X_i)= \mu_1 = 0$
  - $E(X_i)= \mu_2 = \Lambda\kappa$ and $\tau_2 = 0$

- Thresholds are equal over groups.
  - $diag(\Lambda_1 \Lambda_1 t + \Theta_1)=\text{diag}(\Lambda_2 \Lambda_2 t + \Theta_2)=\text{diag}(I)$
  - τ : group1: 0 group2: 0
  - K: set to 0, free in group 2.
  - $\Psi_1 = 1$ (however $\Psi_2 = \text{FREE}$)

Model restrictions for measurement invariance.

- Strict invariance:
  - Can we equate the matrix Θ over groups.
  - However at this point Θ is not a free parameter:
  - *diag*($\Lambda_1 \Lambda_1 t + \Theta 1$)=diag($\Lambda_2 \Lambda_2 t + \Theta_2$)=diag(I)
  - So we must chose arbitrary values for Θ that are equal for both groups.
  - Replace the residuals algebra (diag($\Lambda_2 \Lambda_2 t + \Theta_2$)=diag(I)) with a residuals matrix.