Faculty drive folder Michel

Multivariate analysis

Dorret I. Boomsma, Sanja Franic, Michel Nivard

Factor analysis (FA)
Measurement invariance (MI)

Structural equation models (SEM), e.g. twin models, longitudinal
Principal components analysis (PCA) & cholesky decompostion
Genetic structural equation modeling

Multivariate Analysis

- Yes: techniques used to analyze multivariate data that have been collected in *non-experimental* designs and that often involve *latent constructs* that are not directly observed.

- No: MANOVA, Regression, Discriminant analysis (experimental designs)

EXERCISES

1. Factor model IQ data (4 subtests): compare saturated & one factor model (Michel)

2. Use FA model to examine Measurement Invariance (MI): does an IQ test (continuous data) measure the same trait in men and women? (Sanja)

3. Use FA model to examine MI for Attention Problems (ordinal data) (Michel)

Example: depression

- I feel lonely
- I feel confused or in a fog
- I cry a lot
- I worry about my future.
- I am afraid I might think or do something bad
- I feel that I have to be perfect
- I feel that no one loves me
- I feel worthless or inferior
- I am nervous or tense
- I lack self confidence I am too fearful or anxious
- I feel too guilty
- I am self-conscious or easily embarrassed
- I am unhappy, sad or depressed
- I worry a lot
- I am too concerned about how I look
- I worry about my relations with the opposite sex

Is there a latent construct that underlies the observed variables (items) and that accounts for the inter-correlations between variables?

Factor analysis

Aims at accounting for covariances among observed variables / traits in terms of a smaller number of latent variates or common factors.

Factor Model: $y = \Lambda f + e$, where

y = observed variable(s) *such as depression items*
f = (unobserved) factor score(s) *such as depression*
e = unique factor / error
Λ = matrix of factor loadings

Factor analysis: Regression of observed variables (y) on latent variables (η)

Assume that the variance of the latent factor is 1.

What is the correlation between the first and second item?

One factor model: the latent factor could be depression and y_1 ..y_4 the items that assess depressive symptoms.

Factor analysis

Factor Model: $y = \Lambda f + e$,

With covariance matrix: $\Sigma = \Lambda \Psi \Lambda' + \Theta$

where Σ = covariance matrix (*sigma*)
Λ = matrix of factor loadings (*lambda*)
Ψ = correlation matrix of factor scores (*psi*)
Θ = (diagonal) matrix of unique variances (*theta*)

To estimate factor loadings we do not need to know the individual factor scores, as the expectation for Σ only consists of Λ, Ψ, and Θ.

•C. Spearman (1904): General intelligence, objectively determined and measured. American Journal of Psychology, 201-293
•L.L. Thurstone (1947): Multiple Factor Analysis, University of Chicago Press

Factor analysis

Factor Model: $y = \Lambda f + e$,

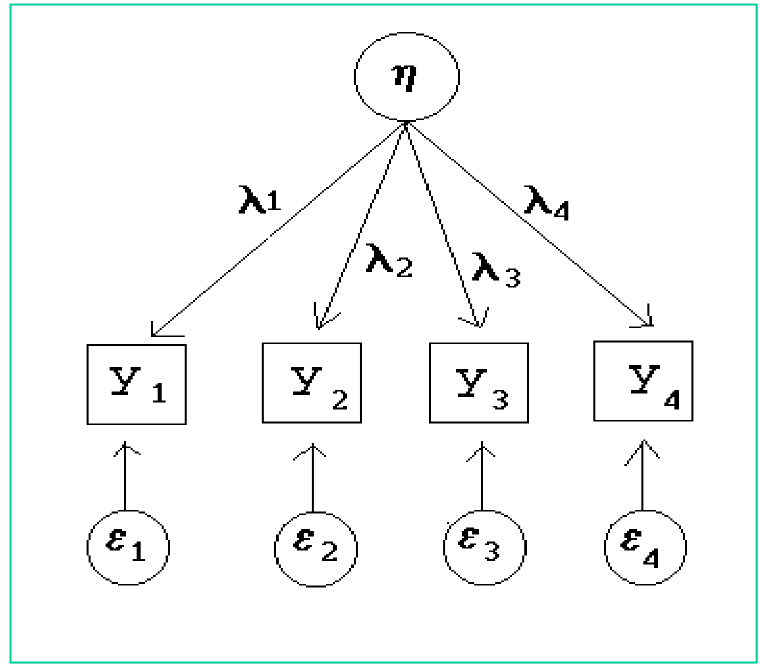
y is a value (observed) belonging to an individual.
Likewise, f and e are values (unobserved factor scores / errors) that characterize an individual.

Covariance matrix: $\Sigma = \Lambda \Psi \Lambda' + \Theta$

Σ, Λ, Ψ, Θ are population parameters [but can be different for e.g. men and women, or children and adults], they are estimated from the data.

Factor scores are not observed, but can be estimated

Estimates of factor loadings and unique variances can be used to construct individual factor scores: $\eta = A'P$, where A is a matrix with weights that is constant across subjects, depending on the factor loadings and the unique variances.



- R.P. McDonald, E.J. Burr (1967): A comparison of four methods of constructing factor scores. Psychometrika, 381-401
- W.E. Saris, M. dePijper, J. Mulder (1978): Optimal procedures for estimation of factor scores. Sociological Methods & Research, 85-106

Factor analysis

Covariance matrix: $\Sigma = \Lambda \Psi \Lambda' + \Theta$

Σ(pxp) = covariance matrix: p variables

Λ(qxp) = matrix of factor loadings: q factors (one or more)

Ψ(qxq) = correlation matrix of factor scores: the diagonal scales the variances of the latent factors; the off-diagonal elements specify correlations among latent variables

Θ(pxp) = (diagonal) matrix of unique variances.
If there are non-zero off-diagonal elements, then measurement errors might be correlated

Factor analysis

Factor Model: $y = \Lambda f + e$,
Covariance matrix: $\Sigma = \Lambda \Psi \Lambda' + \Theta$

Because the latent factors do not have a "natural" scale, the user needs to scale them. For example:

If $\Psi = I$: $\Sigma = \Lambda\Lambda' + \Theta$

- factors are standardized to have unit variance
- factors are independent

Another way to scale the latent factors would be to constrain one factor loading (so that latent factors have the same scale of measurement as the observed variable).

Confirmatory factor analysis

- a model is constructed in advance
- that specifies the number of (latent) factors
- that specifies the pattern of loadings on the factors
- that specifies the pattern of unique variances /measurement errors
- measurement errors may be correlated
- factor loadings can be constrained to be zero (or any other value)
- covariances among latent factors can be estimated or constrained
- multiple group analysis is possible

We can TEST if these constraints are consistent with the data.

Distinctions between exploratory (SPSS/SAS) and confirmatory factor analysis (LISREL/Mx)

In exploratory factor analysis:

• no model that specifies the number of latent factors
• no hypotheses about factor loadings (usually all variables load on all factors, factor loadings cannot be constrained)
• no hypotheses about interfactor correlations (either no correlations or all factors are correlated)
• unique factors must be uncorrelated
• all observed variables must have specific variances
• no multiple group analysis possible
• under-identification of parameters

Two common factor model

y_{ij}, i=1...P tests or items, j=1...N subjects

$y_{ij} = \lambda_{i1}\,\eta_{1j} + \lambda_{i2}\,\eta_{2j} + e_{ij}$

Λ matrix of factor loadings:

λ_{11}	λ_{12}
λ_{21}	λ_{22}
...	...
λ_{P1}	λ_{P2}

Factor loadings are invariant across subjects

Factor scores are subject specific

Identification

The factor model in which all variables load on all (2 or more) common factors is not identified. It is not possible in the present example to estimate all 6x2 loadings.

Identifying constraints

SPSS will produce a factor loading matrix with 6x2 loadings.

Spss automatically imposes the identifying constraint similar to:

$$\Lambda^t \Theta^{-1} \Lambda \text{ is diagonal,}$$

Where Λ is the matrix of factor loadings and Θ is the diagonal covariance matrix of the residuals (eij).

Other identifying constraint are possible

3 factors

λ_{11}	0	0
λ_{21}	λ_{22}	0
λ_{31}	λ_{32}	λ_{33}
…	…	…
λ_{P1}	λ_{P2}	λ_{P3}

2 factors

λ_{11}	0
λ_{21}	λ_{22}
λ_{31}	λ_{32}
…	…
λ_{P1}	λ_{P2}

Where you fix the zero is not important! Identical solutions.

Identical solutions, but different factor loadings! How to interpret?

Given more than 1 factor, raw factor loadings are not interpreted. They are usually subjected to a transformation called rotation:

$$\Lambda^* = \Lambda M$$

M is the rotation matrix, chosen to maximize "interpretability" of loadings

Rotation

Rotation increases ease of interpretation by making factor loading large or small.

The common factors can then be interpreted in terms of the observed variables that load on them.

Varimax – max/min factor loadings but keep common factors uncorrelated.

Promax – max/min factor loadings, but allow common factors to correlate.

Structural equation models (SEM)

Sometimes $x = \Lambda f + e$ is referred to as the measurement model.

The part of the model that specifies relations among latent factors is referred to as the covariance structure model, or the structural equation model.

Practical:
Fit a saturated and a 1-factor model.

- estimate the means and covariances of 4 IQ subscales. (Saturated model)
- Then we will fit a single factor model:
  - The expected covariance model is:
    - $\mathrm{Cov}(X_i) = \Sigma = \Lambda\Psi\Lambda^t + \Theta$
  - The expected means model is:
    - $E(X_i) = \mu = \tau + \Lambda\kappa$

Practical:
Fit a saturated and a 1-factor model.

- We start by estimating the saturated Model.
- In this model all means and (co) variances are estimated freely
- This model basically results in an covariance matrix and an means matrix of the data
- Means:
  - meanSat <- mxMatrix(type="Full”, nrow=1 , ncol=4, labels=c("m1","m2","m3","m4"), values=10,free=T,name="M")
- Covariances:
  - covSat <- mxMatrix(type="Symm", nrow=4, ncol=4, free=T, values=startcov, name="cov")

Practical:
Fit a saturated and a 1-factor model.

- You will be provided with all matrices, objects and code needed to fit the 1-factor model.
- You will have to write your own expression for the expected covariance

- $Cov(X_i) = \Lambda\Psi\Lambda^t + \Theta$
- facLoadings%*% facVariances %*% t(facLoadings) + resVariances

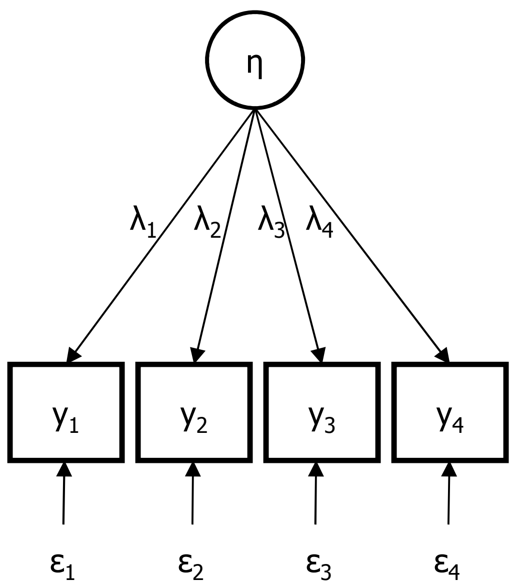
```
ExpCov <-mxAlgebra( expression= ?????????????,
                    name="expCov" )
```

Measurement invariance
in the linear factor model: practical

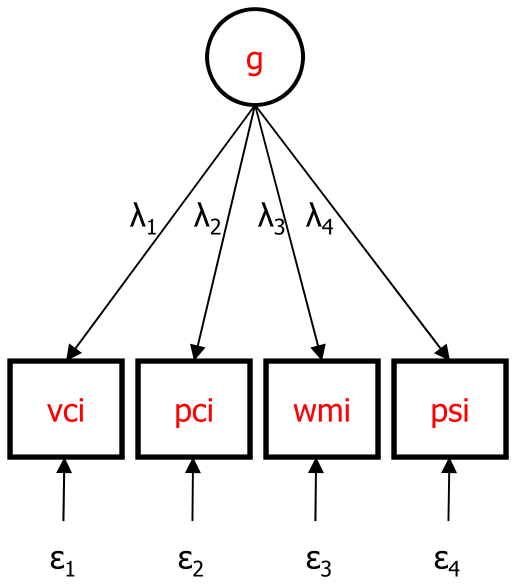
Measurement invariance in the linear factor model: practical

model that relates a continuous latent variable to continuous indicators

Linear factor model



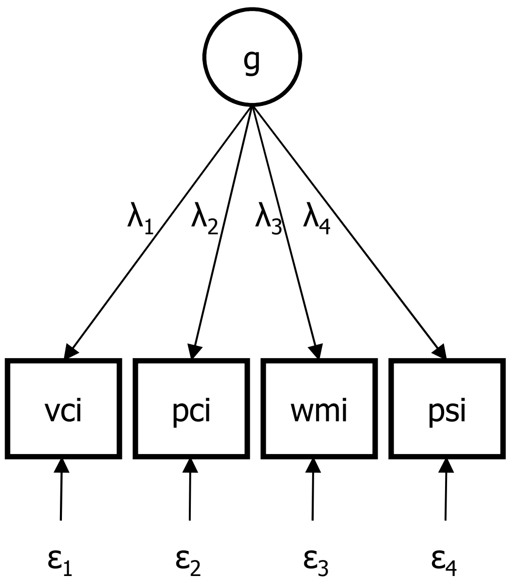
Linear factor model



IQ test (e.g. WAIS):

vci -- Verbal Comprehension Index
poi -- Perceptual Organization Index
wmi -- Working Memory Index
psi -- Processing Speed Index

Linear factor model

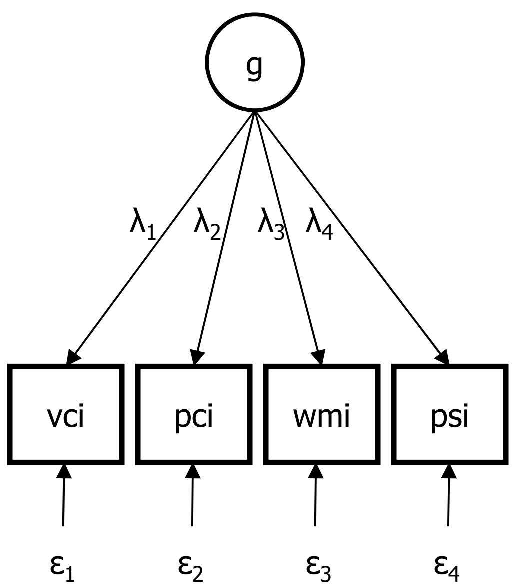


Do males, on average ,score differently than the females?

Men score significantly higher: MANOVA -> $p<.01$

Does this imply that women have a lower level of g?

Linear factor model



Do males, on average ,score differently than the females?

Men score significantly higher: MANOVA -> $p<.01$

Does this imply that women have a lower level of g?

Not necessarily.

It depends on whether the test measures the same construct in males as it does in females.

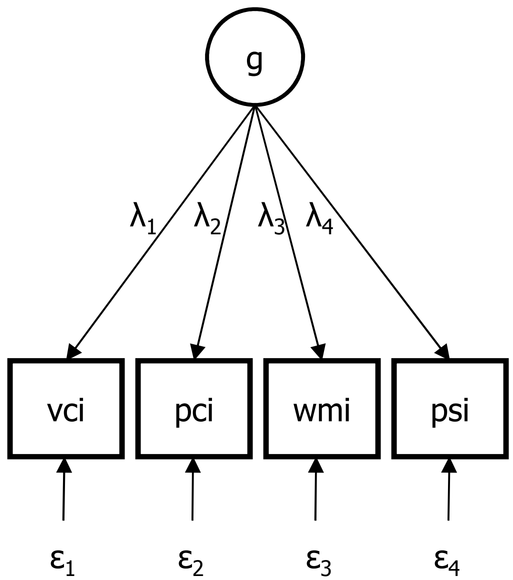
Linear factor model

Conditional distributions in 2 groups (conditional on a given value of η (η^*)):

$y_{1i} | \eta^* \sim N(T_1 + \Lambda_1 \eta^*, \Theta_1)$

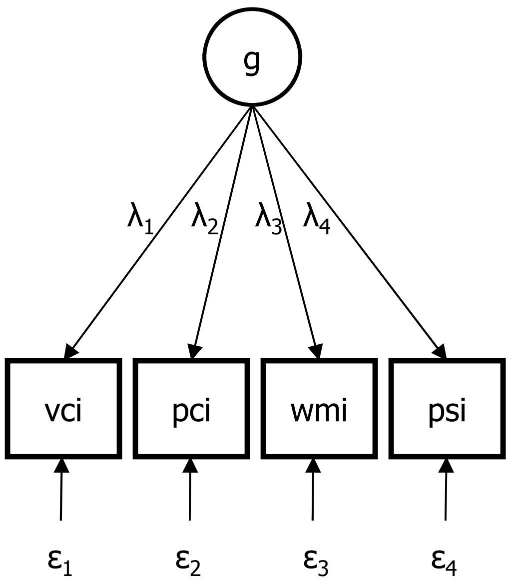
$y_{2i} | \eta^* \sim N(T_2 + \Lambda_2 \eta^*, \Theta_2)$

MI requires these distributions to be equal.



Linear factor model

$$\boldsymbol{\Sigma} = \boldsymbol{\Lambda}\,\boldsymbol{\Psi}\,\boldsymbol{\Lambda}^t + \boldsymbol{\Theta}$$



Conditional distributions in 2 groups (conditional on a given value of η (η*)):

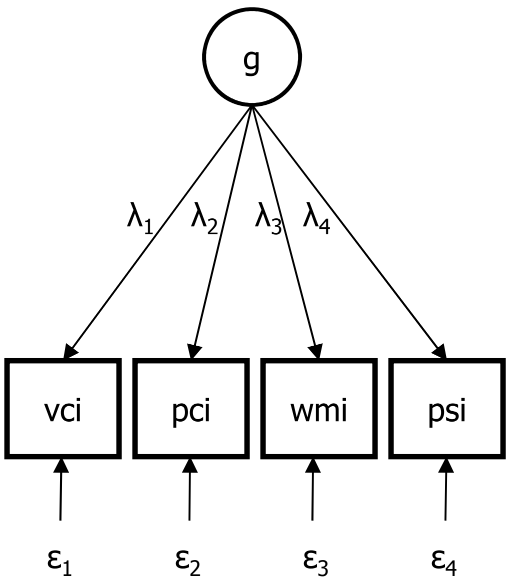
$y_{1i} | \, \eta^* \sim N\,(T_1 + \Lambda_1 \eta^*, \Theta_1)$

$y_{2i} | \, \eta^* \sim N\,(T_2 + \Lambda_2 \eta^*, \Theta_2)$

MI requires these distributions to be equal.

Linear factor model

$\boldsymbol{\Sigma = \Lambda \Psi \Lambda^t + \Theta}$



Conditional distributions in 2 groups (conditional on a given value of η (η*)):

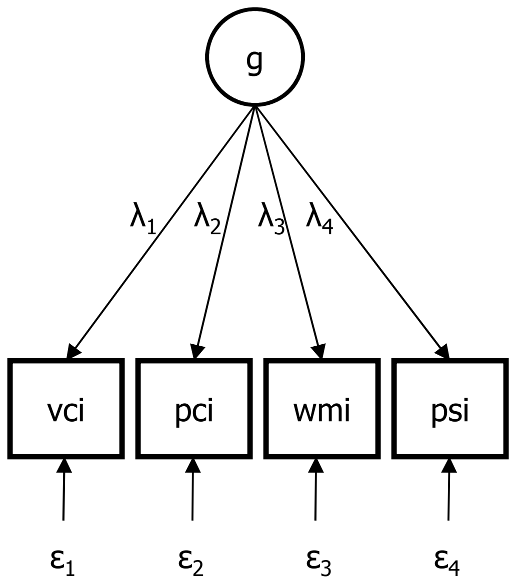
$\mathbf{y_{1i} | \eta^*} \sim N(T_1 + \Lambda_1 \eta^*, \Theta_1)$

$y_{2i} | \eta^* \sim N(T_2 + \Lambda_2 \eta^*, \Theta_2)$

MI requires these distributions to be equal.

Linear factor model

$$\boldsymbol{\Sigma = \Lambda \Psi \Lambda^t + \Theta}$$



Conditional distributions in 2 groups (conditional on a given value of η (η*)):

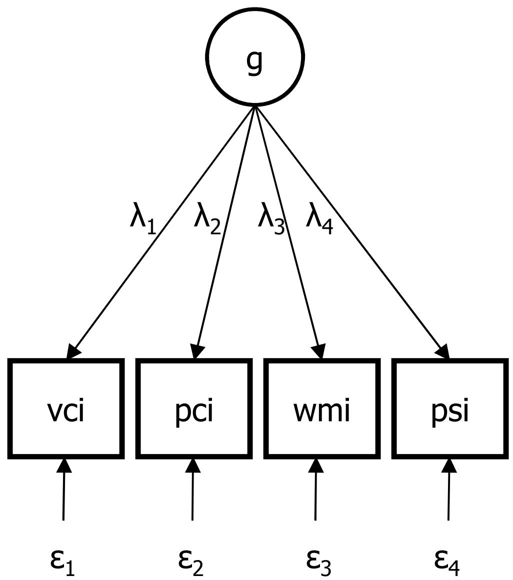
$\mathbf{y_{1i} | \eta^* \sim N}$ $(T_1 + \Lambda_1 \eta^*, \Theta_1)$

$y_{2i} | \eta^* \sim N (T_2 + \Lambda_2 \eta^*, \Theta_2)$

MI requires these distributions to be equal.

Linear factor model

$\boldsymbol{\Sigma = \Lambda \Psi \Lambda^t + \Theta}$



Conditional distributions in 2 groups (conditional on a given value of η (η*)):

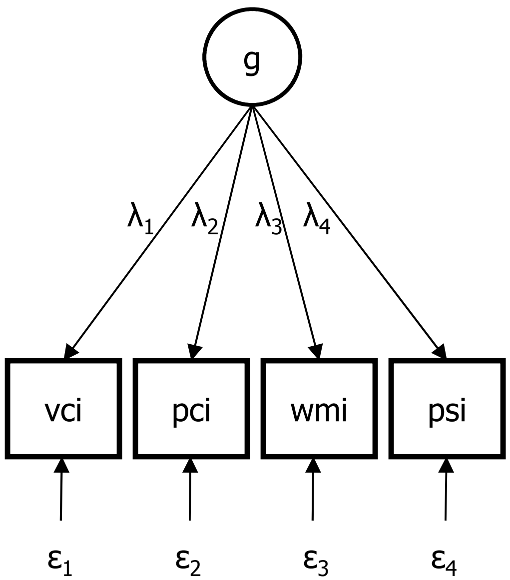
$\mathbf{y_{1i} | \eta^* \sim N} \ (\mathbf{T_1 + \Lambda_1 \eta^*}, \Theta_1)$

$y_{2i} | \eta^* \sim N \ (T_2 + \Lambda_2 \eta^*, \Theta_2)$

MI requires these distributions to be equal.

Linear factor model

$$\boldsymbol{\Sigma = \Lambda \Psi \Lambda^t + \Theta}$$



Conditional distributions in 2 groups (conditional on a given value of η (η*)):

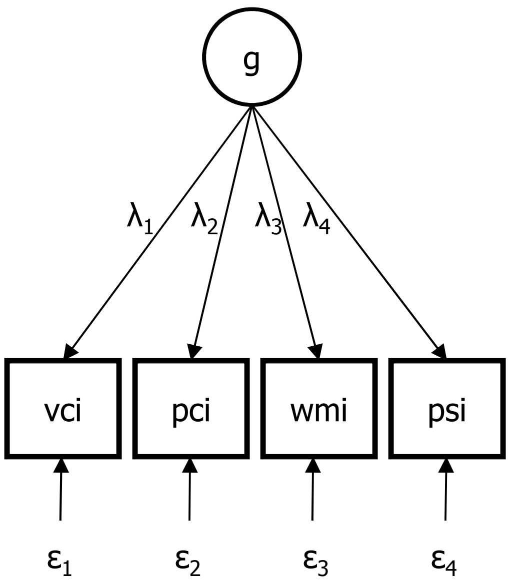
$\mathbf{y_{1i} | \eta^* \sim N (T_1 + \Lambda_1 \eta^*, \Theta_1)}$

$y_{2i} | \eta^* \sim N (T_2 + \Lambda_2 \eta^*, \Theta_2)$

MI requires these distributions to be equal.

Linear factor model

$\boldsymbol{\Sigma = \Lambda\ 0\ \Lambda^t + \Theta}$



Conditional distributions in 2 groups (conditional on a given value of η (η*)):

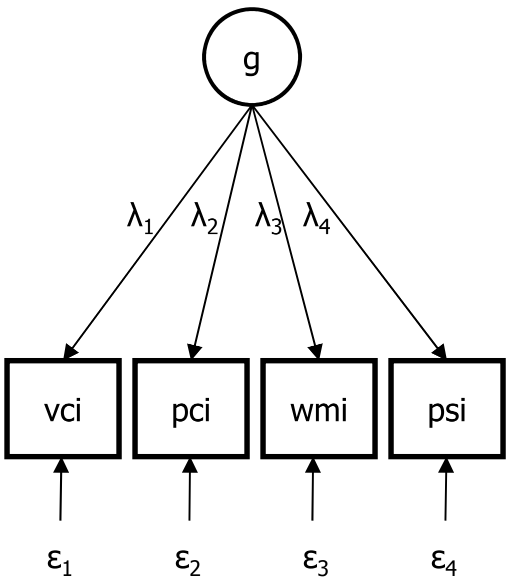
$\boldsymbol{y_{1i} | \eta^* \sim N(T_1 + \Lambda_1 \eta^*, \Theta_1)}$

$y_{2i} | \eta^* \sim N(T_2 + \Lambda_2 \eta^*, \Theta_2)$

MI requires these distributions to be equal.

Linear factor model

$\boldsymbol{\Sigma = \Lambda\ 0\ \Lambda^t + \Theta}$
$\boldsymbol{= \Theta}$



Conditional distributions in 2 groups (conditional on a given value of η (η*)):

$\boldsymbol{y_{1i} | \eta^* \sim N\ (T_1 + \Lambda_1\ \eta^*, \Theta_1)}$

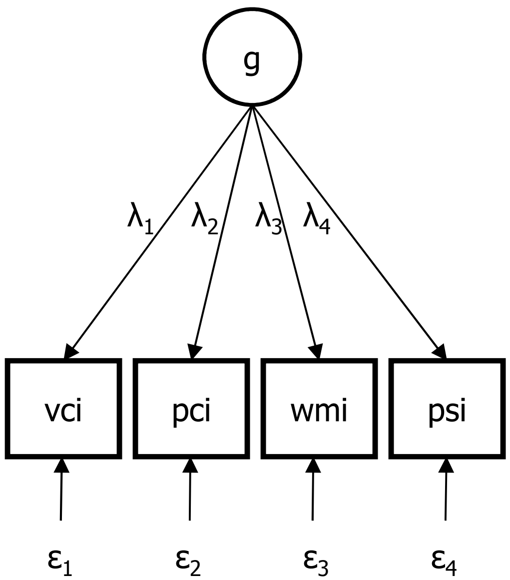
$y_{2i} | \eta^* \sim N\ (T_2 + \Lambda_2\ \eta^*, \Theta_2)$

MI requires these distributions to be equal.

Linear factor model

$\Sigma = \Lambda\ 0\ \Lambda^t + \Theta$
$= \Theta$

$E[y | \eta^*] = T + \Lambda \eta^*$



Conditional distributions in 2 groups (conditional on a given value of η (η*)):

$y_{1i} | \eta^* \sim N (T_1 + \Lambda_1 \eta^*, \Theta_1)$

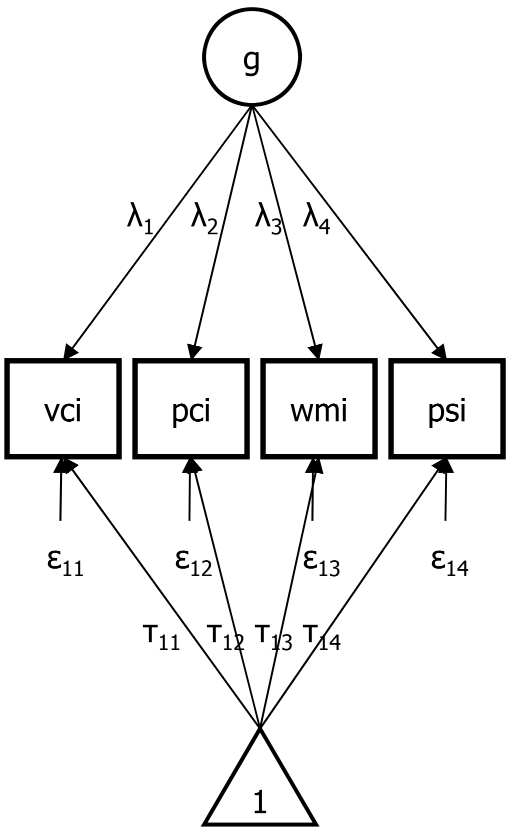
$y_{2i} | \eta^* \sim N (T_2 + \Lambda_2 \eta^*, \Theta_2)$

MI requires these distributions to be equal.

Linear factor model

$\Sigma = \Lambda\, 0\, \Lambda^t + \Theta$
$= \Theta$

$E[y \mid \eta^*] = T + \Lambda \eta^*$



Conditional distributions in 2 groups (conditional on a given value of η (η*)):

$y_{1i} \mid \eta^* \sim N(T_1 + \Lambda_1 \eta^*, \Theta_1)$

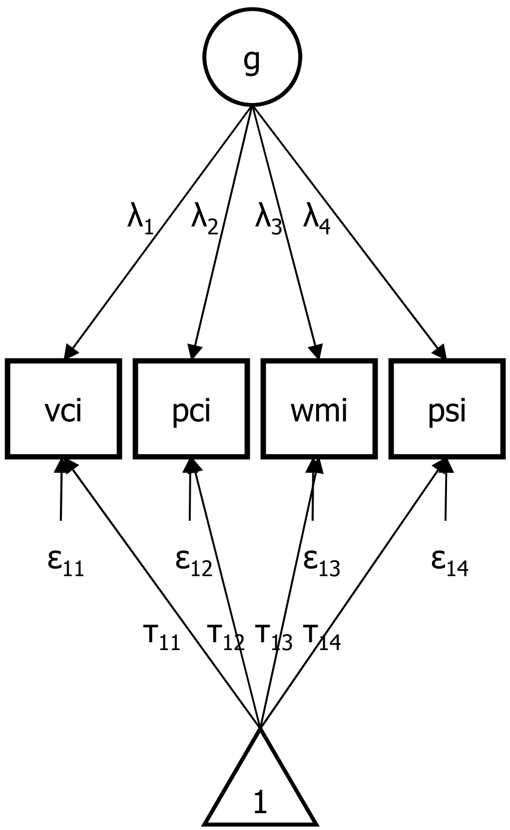
$y_{2i} \mid \eta^* \sim N(T_2 + \Lambda_2 \eta^*, \Theta_2)$

MI requires these distributions to be equal.

Linear factor model

$\Sigma = \Lambda\, 0\, \Lambda^t + \Theta$
$= \Theta$

$E[y|\eta^*] = T + \Lambda \eta^*$



Conditional distributions in 2 groups (conditional on a given value of η (η*)):

$y_{1i}| \eta^* \sim N(T_1 + \Lambda_1 \eta^*, \Theta_1)$ ←

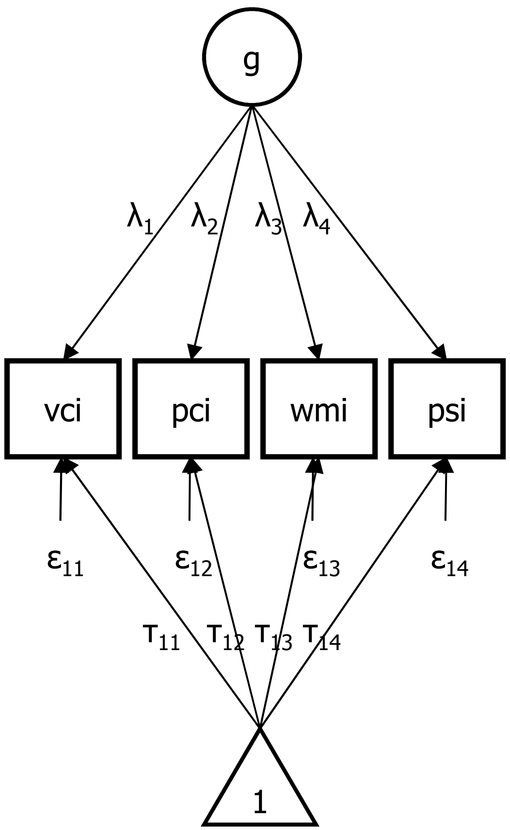
$y_{2i}| \eta^* \sim N(T_2 + \Lambda_2 \eta^*, \Theta_2)$

MI requires these distributions to be equal.

Linear factor model

$\boldsymbol{\Sigma = \Lambda\ 0\ \Lambda^t + \Theta}$
$\boldsymbol{= \Theta}$

$\mathbf{E\ [y\,|\,\eta\ ^*] = T + \Lambda\ \eta^*}$



Conditional distributions in 2 groups (conditional on a given value of η (η*)):

$\mathbf{y_{1i}\,|\ \eta^* \sim N\ (T_1 + \Lambda_1\, \eta^*, \Theta_1)}$

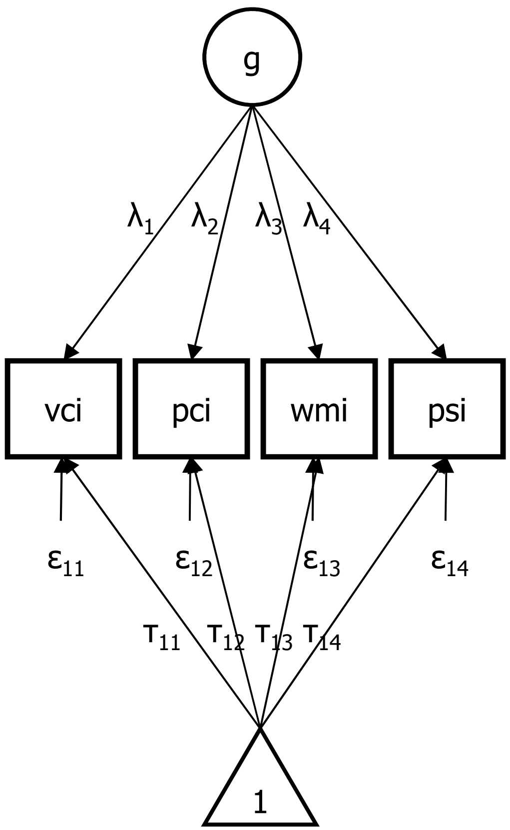
$y_{2i}|\ \eta^* \sim N\ (T_2 + \Lambda_2\, \eta^*, \Theta_2)$

MI requires these distributions to be equal.

Linear factor model

$\boldsymbol{\Sigma = \Lambda\, 0\, \Lambda^t + \Theta}$
$\boldsymbol{= \Theta}$

$\boldsymbol{E\,[y\,|\,\eta^*] = T + \Lambda\, \eta^*}$



Conditional distributions in 2 groups (conditional on a given value of η (η*)):

$\boldsymbol{y_{1i} \,|\, \eta^* \sim N\,(T_1 + \Lambda_1\, \eta^*, \Theta_1)}$

$y_{2i} \,|\, \eta^* \sim N\,(T_2 + \Lambda_2\, \eta^*, \Theta_2)$

MI requires these distributions to be equal.

This is the case if and only if:

$T_1 = T_2$

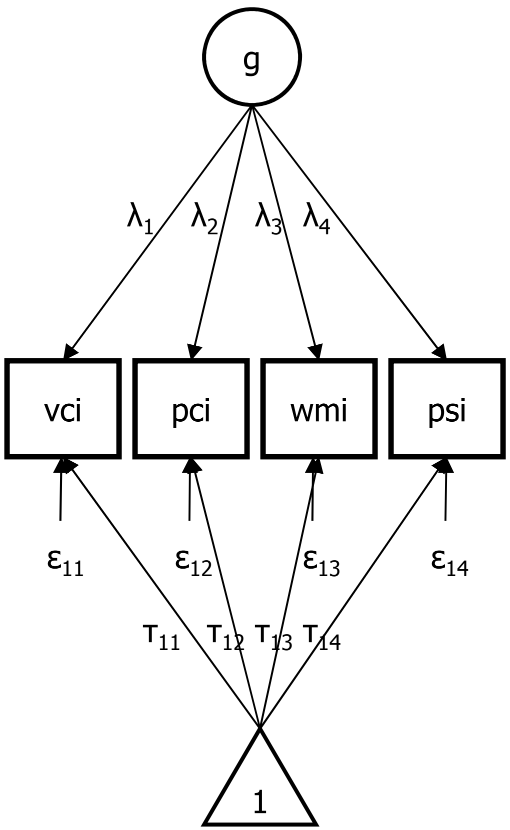
$\Lambda_1 = \Lambda_2$

$\Theta_1 = \Theta_2$

Linear factor model

$$\Sigma = \Lambda\, 0\, \Lambda^t + \Theta$$
$$= \Theta$$

$$E\,[y\,|\,\eta^*] = T + \Lambda\,\eta^*$$



Conditional distributions in 2 groups (conditional on a given value of η (η*)):

$y_{1i}|\ \eta^* \sim N\,(T_1 + \Lambda_1\,\eta^*,\ \Theta_1)$

$y_{2i}|\ \eta^* \sim N\,(T_2 + \Lambda_2\,\eta^*,\ \Theta_2)$

MI requires these distributions to be equal.

This is the case if and only if:

$T_1 = T_2$

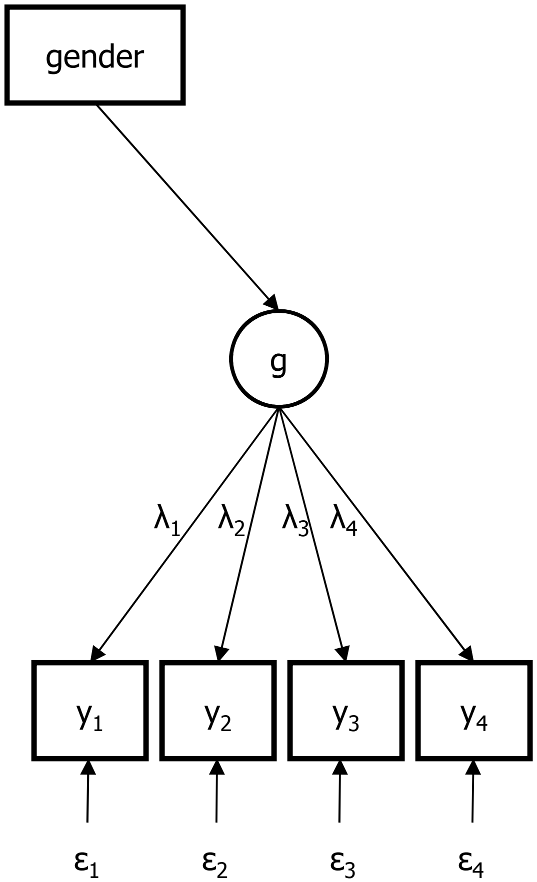
$\Lambda_1 = \Lambda_2$

$\Theta_1 = \Theta_2$

The test is MI with respect to group if the observed group differences in summary statistics (means and covariance matrix) are attributable to differences in the means and variance of the latent trait or common factor (Ψ_k and α_k).

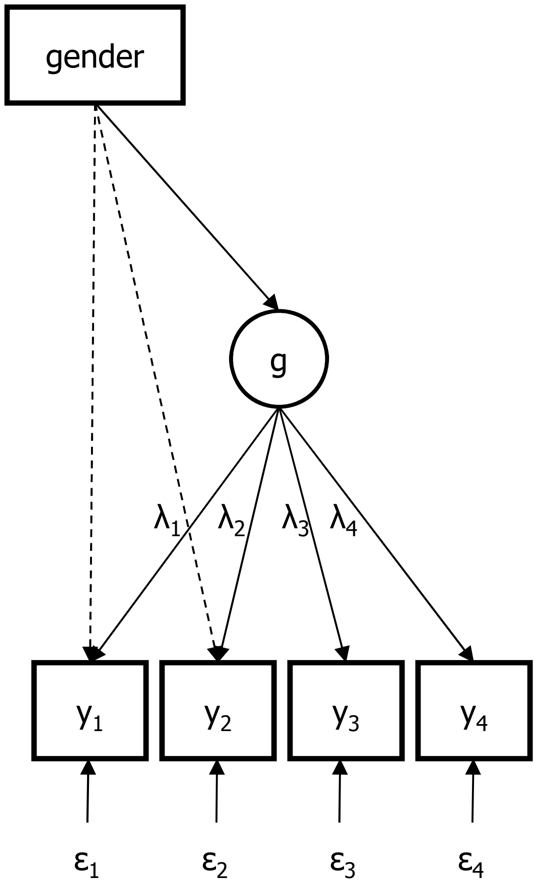
-> if the test measures the same latent variable in the two groups, then that latent variable should be the only source of differences between the groups.

Linear factor model



Measurement invariance

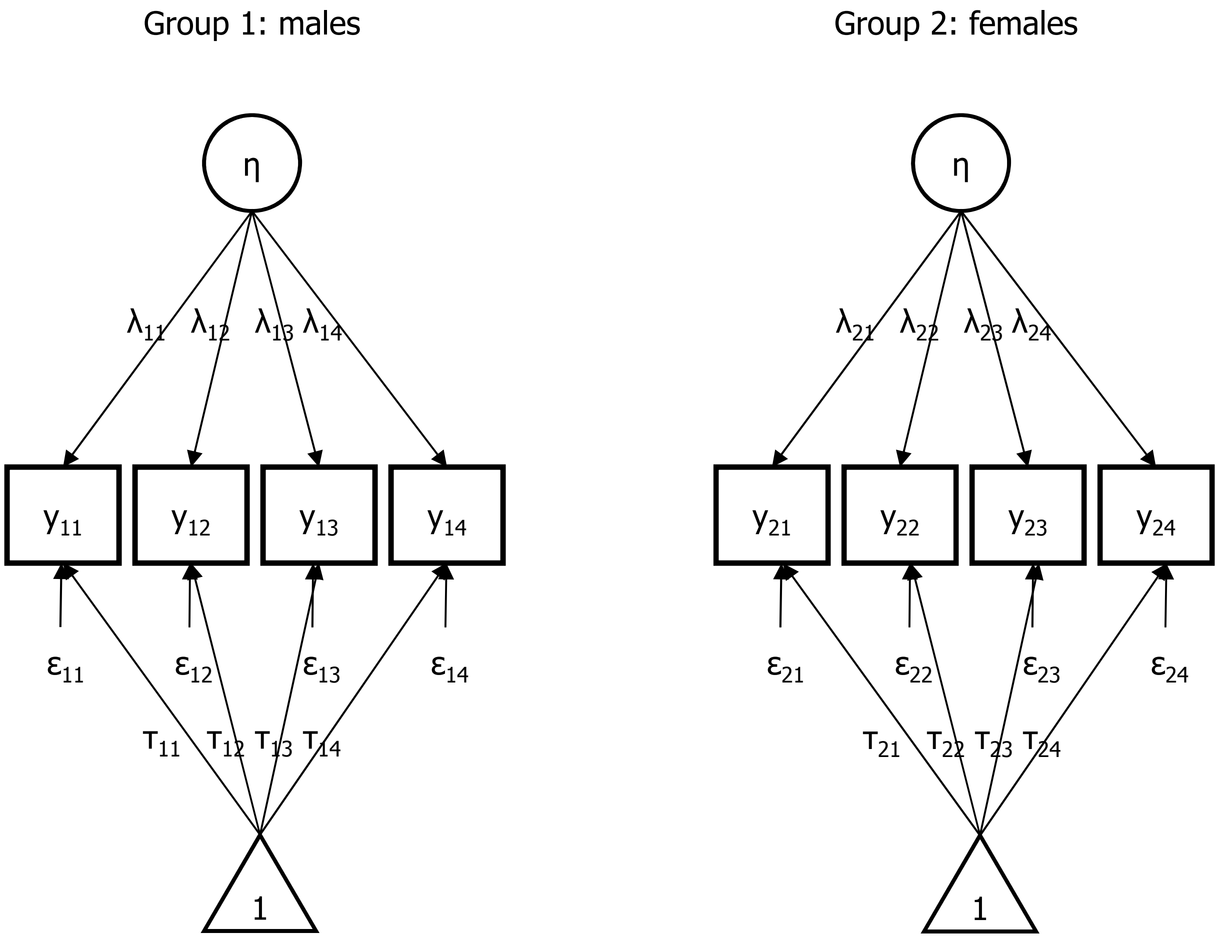
Linear factor model



Lack of measurement invariance

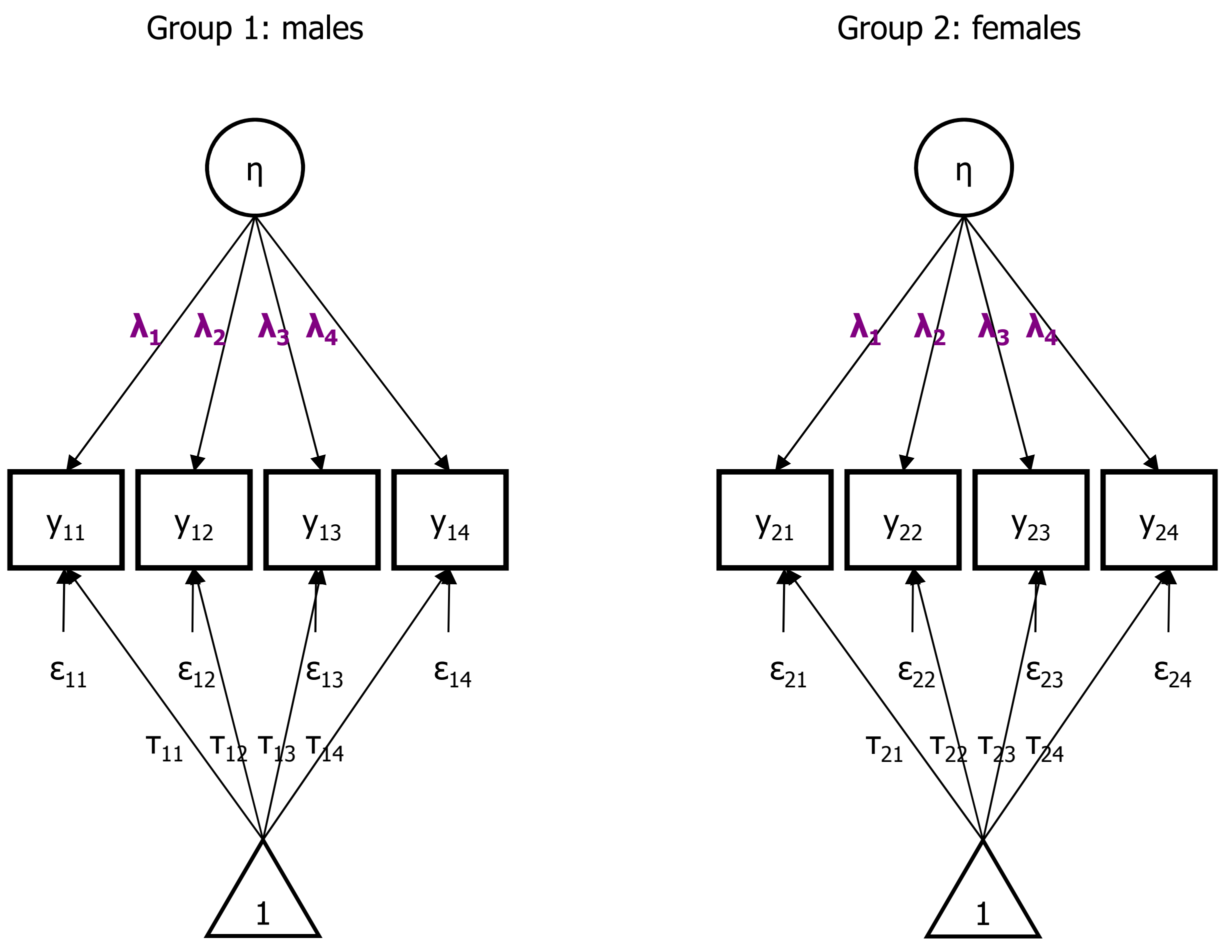
Establishing MI: testing a number of increasingly restrictive models

MODEL 1: Configural invariance -> in the 2 groups the same indicators load on the same factors
(i.e., the pattern or configuration of Λ and Θ are the same over groups)



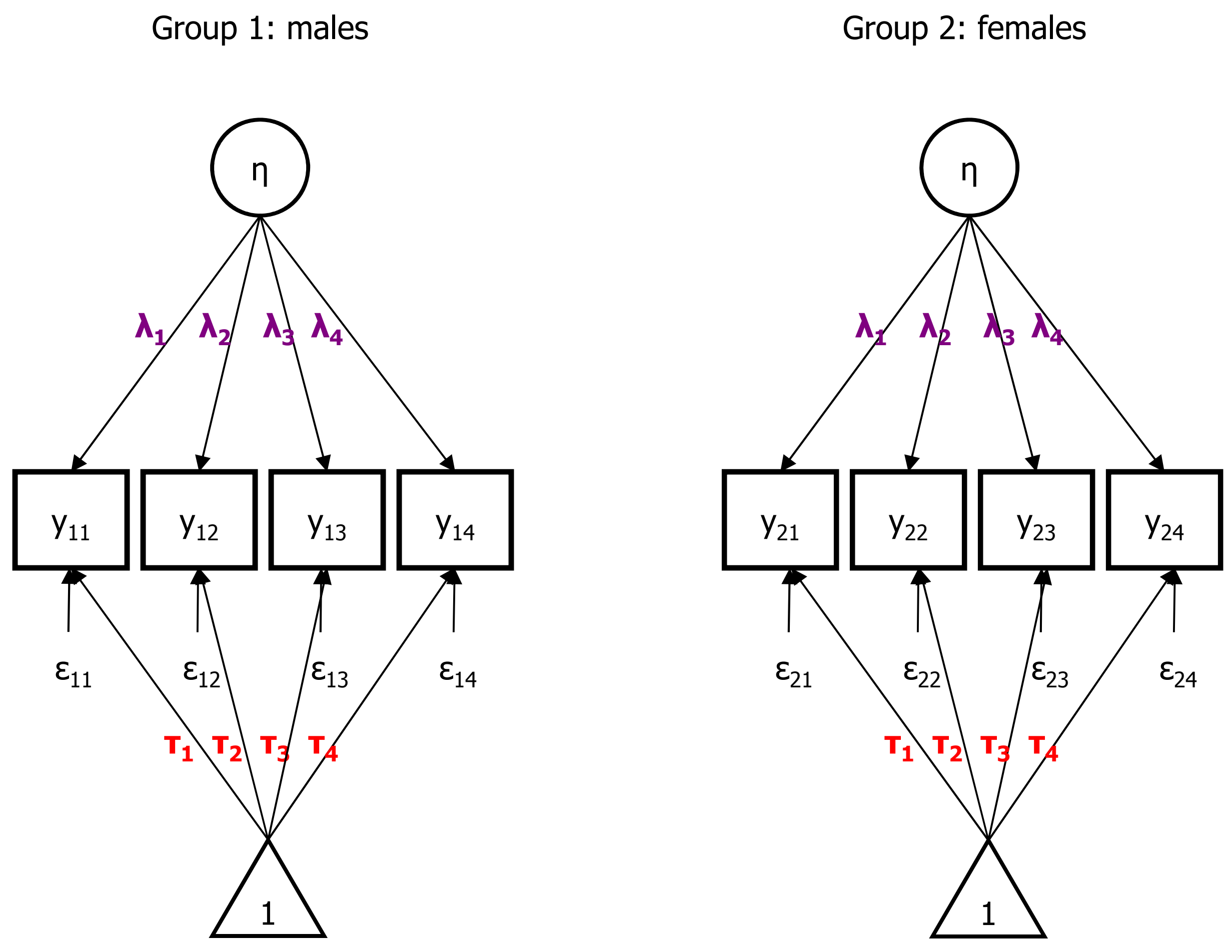
Establishing MI: testing a number of increasingly restrictive models

MODEL 2: Metric invariance -> equal factor loadings over the groups



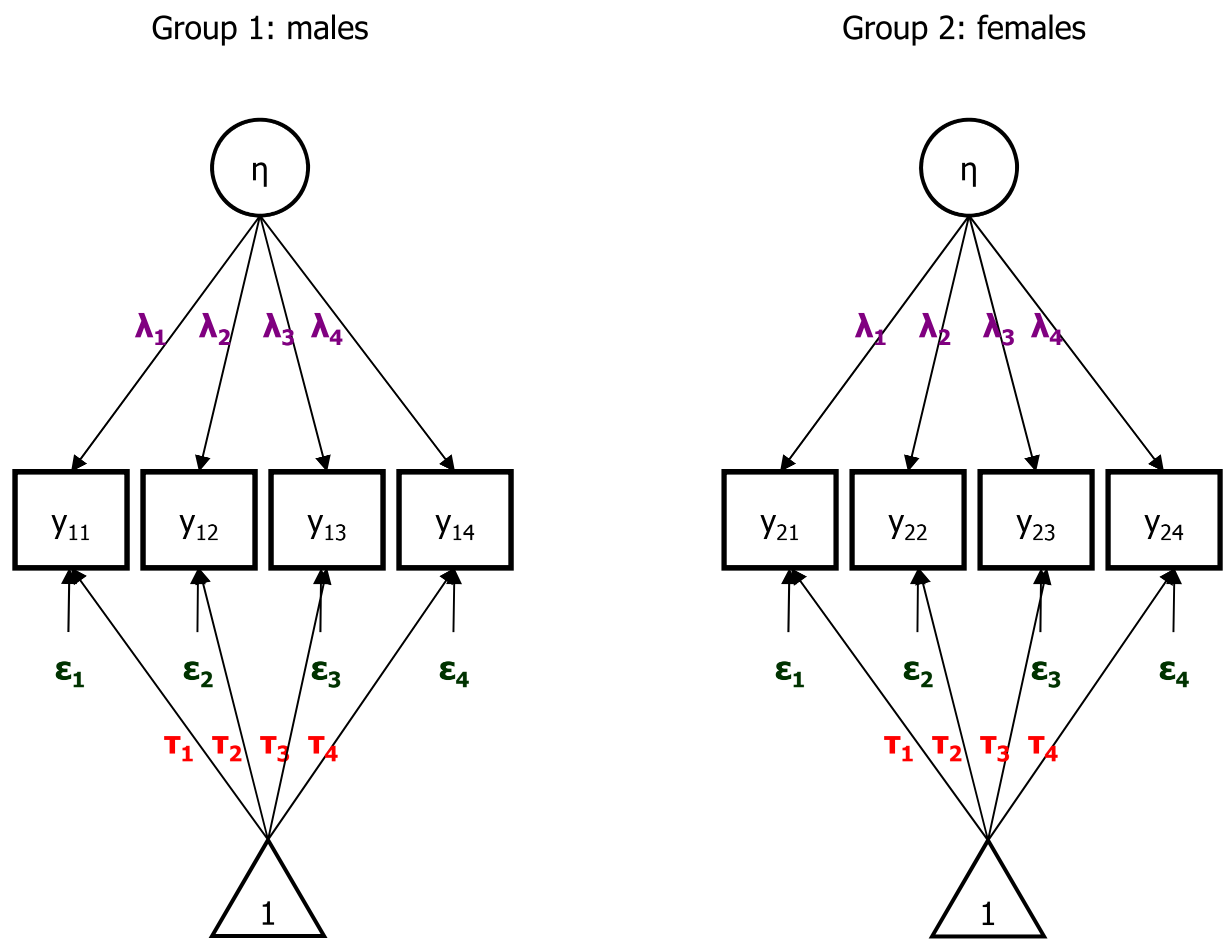
Establishing MI: testing a number of increasingly restrictive models

MODEL 3: Strong factorial invariance -> equal factor loadings and intercepts over the groups



Establishing MI: testing a number of increasingly restrictive models

MODEL 4: Strict factorial invariance -> equal factor loadings, intercepts and residual variances over the groups



Current practical: Are the 4 subscales of the WAIS-III measurement invariant with respect to gender?

Data:

	vci	poi	wmi	psi
gender	**scale1**	**scale2**	**scale3**	**scale4**
2	11	9.33	10.33	13.5
2	10.67	9	10.33	15
2	9.67	7.67	9.33	8.5
2	13	10	8.67	9
2	11	11	13.67	17
2	10	12	9.33	11

...

N = 180 individuals (80 male, 100 female)

Subscales:

vci -- Verbal Comprehension Index
poi -- Perceptual Organization Index
wmi -- Working Memory Index
psi -- Processing Speed Index

Current practical: Are the 4 subscales of the WAIS-III measurement invariant with respect to gender?

OpenMx code:

```
#==========================================================================#
#   PREPARE DATA                                                           #
#==========================================================================#

nv <- 4 # number of phenotype variables to be analyzed
nf <- 1 # number of common factors in the model
selVars <- paste("scale",1:nv,sep="") # phenotype variables to be analyzed
grVars <- c('gender') # grouping variable

data <- read.table(paste(getwd(),"/Measurement_invariance_data.dat",sep=""),header=TRUE)
mData <- round(data[data$gender==1, selVars],2)
fData <- round(data[data$gender==2, selVars],2)

# Generate descriptive statistics
colMeans(mData,na.rm=TRUE)
colMeans(fData,na.rm=TRUE)
cov(mData,use="complete")
cov(fData,use="complete")

# Test for a mean difference between males and females (MANOVA)
summary(manova(cbind(scale1,scale2,scale3,scale4) ~ gender, data = data), test = "Pillai")
```

Current practical: Are the 4 subscales of the WAIS-III measurement invariant with respect to gender?

OpenMx code:

```
#========================================================================#
#   PREPARE MODEL                                                        #
#========================================================================#

# Matrices to store factor loadings of the WAIS subscales on g
loadings1 <- mxMatrix( type="Full", nrow=nv, ncol=nf, free=c(F, rep(T,nv-1)),
values=1, label=paste("l_1", 1:nv, sep=""), name="load1" )
loadings2 <- mxMatrix( type="Full", nrow=nv, ncol=nf, free=c(F, rep(T,nv-1)),
values=1, label=paste("l_2", 1:nv, sep=""), name="load2" )

# Matrices to store the residual variances of the WAIS subscales
residuals1 <- mxMatrix( type="Diag", nrow=nv, free=T, values=2,
label=paste("res_1", 1:nv, sep=""), name="res1" )
residuals2 <- mxMatrix( type="Diag", nrow=nv, free=T, values=2,
label=paste("res_2", 1:nv, sep=""), name="res2" )

# Matrices to store the mean and variance of g (variance estimated, mean set to 0)
latVariance1 <- mxMatrix( type="Symm", nrow=nf, ncol=nf, free=T, values=4,
label=paste("lVar_1", 1:nf, sep=""), name="latVar1" )
latVariance2 <- mxMatrix( type="Symm", nrow=nf, ncol=nf, free=T, values=4,
label=paste("lVar_2", 1:nf, sep=""), name="latVar2" )

latMean1 <- mxMatrix( type="Full", nrow=1, ncol=nf, free=F, values=0,
label=paste("lMean_1",1:nf, sep=""), name="latM1" )
latMean2 <- mxMatrix( type="Full", nrow=1, ncol=nf, free=F, values=0,
label=paste("lMean_2",1:nf, sep=""), name="latM2" )
```

Current practical: Are the 4 subscales of the WAIS-III measurement invariant with respect to gender?

OpenMx code:

```
# Vectors to store intercepts of the WAIS subscales
intercepts1 <- mxMatrix( type="Full", nrow=nv, ncol=1, free=T, values=8,
label=paste("int_1",1:nv,sep=""), name="int1" )
intercepts2 <- mxMatrix( type="Full", nrow=nv, ncol=1, free=T, values=8,
label=paste("int_2",1:nv,sep=""), name="int2" )

# Algebra for the expected means and covariances of the WAIS scores
means1 <- mxAlgebra( expression=t(int1 + load1%*%latM1), name="m1" )
means2 <- mxAlgebra( expression=t(int2 + load2%*%latM2), name="m2" )
variances1 <- mxAlgebra( expression=load1 %*% latVar1 %*% t(load1) + res1, name="v1" )
variances2 <- mxAlgebra( expression=load2 %*% latVar2 %*% t(load2) + res2, name="v2" )

# Data objects for the two groups
data1 <- mxData( observed=mData, type="raw" )
data2 <- mxData( observed=fData, type="raw" )

# Objective objects for the two groups
obj1 <- mxFIMLObjective( covariance="v1", means="m1", dimnames=selVars )
obj2 <- mxFIMLObjective( covariance="v2", means="m2", dimnames=selVars )

# Combine Groups
modelMales  <- mxModel( loadings1, residuals1, latVariance1, latMean1,
intercepts1, means1, variances1, data1, obj1, name="males")
modelFemales  <- mxModel( loadings2, residuals2, latVariance2, latMean2,
intercepts2, means2, variances2, data2, obj2, name="females")
minus2ll <- mxAlgebra( expression=males.objective + females.objective, name="m2LL" )
obj      <- mxAlgebraObjective( "m2LL" )
CImodel  <- mxModel( "CI", modelMales, modelFemales, minus2ll, obj )
```

Current practical: Are the 4 subscales of the WAIS-III measurement invariant with respect to gender?

OpenMx code:

```
#==================================================================#
#   RUN MODEL: CONFIGURAL INVARIANCE                                #
#           - equal configuration of factor loadings over the groups #
#==================================================================#

CImodelFit    <- mxRun(CImodel)
CImodelSumm   <- summary(CImodelFit)
CImodelSumm
```

Current practical: Are the 4 subscales of the WAIS-III measurement invariant with respect to gender?

OpenMx code:

```
#==========================================================================#
#   RUN MODEL: METRIC INVARIANCE                                            #
#              - equal configuration of factor loadings over the groups     #
#              - equal factor loadings over the groups                      #
#==========================================================================#

# Matrices to store factor loadings of the WAIS subscales on g
loadings1 <- mxMatrix( type="Full", nrow=nv, ncol=nf, free=c(F, rep(T,nv-1)),
values=1, label=paste("l_", 1:nv, sep=""), name="load1" )
loadings2 <- mxMatrix( type="Full", nrow=nv, ncol=nf, free=c(F, rep(T,nv-1)),
values=1, label=paste("l_", 1:nv, sep=""), name="load2" )

# Combine Groups
modelMales  <- mxModel( loadings1, residuals1, latVariance1, latMean1,
intercepts1, means1, variances1, data1, obj1, name="males")
modelFemales  <- mxModel( loadings2, residuals2, latVariance2, latMean2,
intercepts2, means2, variances2, data2, obj2, name="females")
minus2ll <- mxAlgebra( expression=males.objective + females.objective, name="m2LL" )
obj      <- mxAlgebraObjective( "m2LL" )
MImodel  <- mxModel( "MI", modelMales, modelFemales, minus2ll, obj )

MImodelFit    <- mxRun(MImodel)
MImodelSumm   <- summary(MImodelFit)
MImodelSumm
```

Current practical: Are the 4 subscales of the WAIS-III measurement invariant with respect to gender?

OpenMx code:

```
#==========================================================================#
#   RUN MODEL: STRONG FACTORIAL INVARIANCE - YOUR TASK                     #
#            - equal configuration of factor loadings over the groups      #
#            - equal factor loadings over the groups                       #
#            - equal intercepts over the groups                            #
#==========================================================================#

# Vectors to store intercepts of the WAIS subscales
???
???
???
???

# Combine Groups
modelMales  <- mxModel( loadings1, residuals1, latVariance1, latMean1,
intercepts1, means1, variances1, data1, obj1, name="males")
modelFemales  <- mxModel( loadings2, residuals2, latVariance2, latMean2,
intercepts2, means2, variances2, data2, obj2, name="females")
minus2ll <- mxAlgebra( expression=males.objective + females.objective, name="m2LL" )
obj      <- mxAlgebraObjective( "m2LL" )
SFImodel  <- mxModel( "SFI", modelMales, modelFemales, minus2ll, obj )

SFImodelFit    <- mxRun(SFImodel)
SFImodelSumm   <- summary(SFImodelFit)
SFImodelSumm
```

Current practical: Are the 4 subscales of the WAIS-III measurement invariant with respect to gender?

OpenMx code:

```
#======================================================================#
#   RUN MODEL: STRONG FACTORIAL INVARIANCE - YOUR TASK                 #
#           - equal configuration of factor loadings over the groups   #
#           - equal factor loadings over the groups                    #
#           - equal intercepts over the groups                         #
#======================================================================#

# Vectors to store intercepts of the WAIS subscales
intercepts1 <- mxMatrix( type="Full", nrow=nv, ncol=1, free=T, values=8,
label=paste("int_",1:nv,sep=""), name="int1" )
intercepts2 <- mxMatrix( type="Full", nrow=nv, ncol=1, free=T, values=8,
label=paste("int_",1:nv,sep=""), name="int2" )

# Combine Groups
modelMales  <- mxModel( loadings1, residuals1, latVariance1, latMean1,
intercepts1, means1, variances1, data1, obj1, name="males")
modelFemales  <- mxModel( loadings2, residuals2, latVariance2, latMean2,
intercepts2, means2, variances2, data2, obj2, name="females")
minus2ll <- mxAlgebra( expression=males.objective + females.objective, name="m2LL" )
obj      <- mxAlgebraObjective( "m2LL" )
SFImodel  <- mxModel( "SFI", modelMales, modelFemales, minus2ll, obj )

SFImodelFit    <- mxRun(SFImodel)
SFImodelSumm   <- summary(SFImodelFit)
SFImodelSumm
```

Current practical: Are the 4 subscales of the WAIS-III measurement invariant with respect to gender?

OpenMx code:

```
#==================================================================#
#   RUN MODEL: STRICT FACTORIAL INVARIANCE - YOUR TASK              #
#            - equal configuration of factor loadings over the groups #
#            - equal factor loadings over the groups                 #
#            - equal intercepts over the groups                      #
#            - equal residuals over the groups                       #
#==================================================================#

???
???
???
???
???

???
???
???
???
???
???
???
???

STFImodelFit    <- mxRun(STFImodel)
STFImodelSumm   <- summary(STFImodelFit)
STFImodelSumm
```

Current practical: Are the 4 subscales of the WAIS-III measurement invariant with respect to gender?

OpenMx code:

```
#====================================================================#
#   RUN MODEL: STRICT FACTORIAL INVARIANCE - YOUR TASK               #
#           - equal configuration of factor loadings over the groups #
#           - equal factor loadings over the groups                  #
#           - equal intercepts over the groups                       #
#           - equal residuals over the groups                        #
#====================================================================#

# Matrices to store the residual variances of the WAIS subscales
residuals1 <- mxMatrix( type="Diag", nrow=nv, free=T, values=2,
label=paste("res_", 1:nv, sep=""), name="res1" )
residuals2 <- mxMatrix( type="Diag", nrow=nv, free=T, values=2,
label=paste("res_", 1:nv, sep=""), name="res2" )

# Combine Groups
modelMales  <- mxModel( loadings1, residuals1, latVariance1, latMean1,
intercepts1, means1, variances1, data1, obj1, name="males")
modelFemales  <- mxModel( loadings2, residuals2, latVariance2, latMean2,
intercepts2, means2, variances2, data2, obj2, name="females")
minus2ll <- mxAlgebra( expression=males.objective + females.objective, name="m2LL" )
obj      <- mxAlgebraObjective( "m2LL" )
STFImodel  <- mxModel( "STFI", modelMales, modelFemales, minus2ll, obj )

STFImodelFit    <- mxRun(STFImodel)
STFImodelSumm   <- summary(STFImodelFit)
STFImodelSumm
```

Current practical: Are the 4 subscales of the WAIS-III measurement invariant with respect to gender?

OpenMx code:

```
#==========================================================================#
#   RUN BASELINE MODEL: 2-GROUP SATURATED MODEL                             #
#==========================================================================#

# Matrix to store variances/covariances
startCov=cov(data[,selVars])
covariances1 <- mxMatrix( type="Symm", nrow=nv, ncol=nv, free=T,
values=startCov, name="covs1" )
covariances2 <- mxMatrix( type="Symm", nrow=nv, ncol=nv, free=T,
values=startCov, name="covs2" )

# Vector to store the means
means1 <- mxMatrix( type="Full", nrow=1, ncol=4, free=T, values=8,
labels=paste("mean_2",1:nv,sep=""), name="m1" )
means2 <- mxMatrix( type="Full", nrow=1, ncol=4, free=T, values=8,
labels=paste("mean_1",1:nv,sep=""), name="m2" )

# Data object
Data1 <- mxData( observed=mData[,selVars], type="raw" )
Data2 <- mxData( observed=fData[,selVars], type="raw" )

# Objective object
obj1 <- mxFIMLObjective( covariance="covs1", means="m1", dimnames=selVars )
obj2 <- mxFIMLObjective( covariance="covs2", means="m2", dimnames=selVars )
```

Current practical: Are the 4 subscales of the WAIS-III measurement invariant with respect to gender?

OpenMx code:

```
# Combine the groups
satModelMales  <- mxModel( covariances1, means1, Data1, obj1, name="satMales")
satModelFemales  <- mxModel( covariances2, means2, Data2, obj2, name="satFemales")
minus2ll <- mxAlgebra( expression=satMales.objective + satFemales.objective, name="m2LL" )
obj      <- mxAlgebraObjective( "m2LL" )
satModel  <- mxModel( "CI", satModelMales, satModelFemales, minus2ll, obj )

# Run the model
satFit <- mxRun(satModel)
satSumm <- summary(satFit)
satSumm
```

Current practical: Are the 4 subscales of the WAIS-III measurement invariant with respect to gender?

OpenMx code:

```
#=======================================================================#
#   COMPARE MODEL FIT                                                    #
#=======================================================================#

tableFitStatistics(satFit,CImodelFit) # test of configural invariance
tableFitStatistics(CImodelFit,MImodelFit) # test of metric invariance
tableFitStatistics(MImodelFit,SFImodelFit) # test of strong f. invariance
tableFitStatistics(SFImodelFit,STFImodelFit) # test of strict f. invariance
```

Current practical: Are the 4 subscales of the WAIS-III measurement invariant with respect to gender?

OpenMx code:

```
#==========================================================================#
#   COMPARE MODEL FIT                                                      #
#==========================================================================#

tableFitStatistics(satFit,CImodelFit) # test of configural invariance
tableFitStatistics(CImodelFit,MImodelFit) # test of metric invariance
tableFitStatistics(MImodelFit,SFImodelFit) # test of strong f. invariance
tableFitStatistics(SFImodelFit,STFImodelFit) # test of strict f. invariance
```

Conclusion...?