Multivariate Twin Analysis

OpenMx Tc24 2010

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Copy Files

- DHBQ_bs.dat
- MultivariateTwinAnalysis_MatrixRawConNL500.R

Multivariate

- Saturated Model
  - equality of means/variances
- Genetic Models (ACE)
  - multivariate -> Cholesky Decomposition
  - Independent Pathway
  - Common Pathway

Practical Example

- general family functioning
- happiness
- life satisfaction
- anxiety-depression
- somatic complaints
- social problems

Scientific Questions

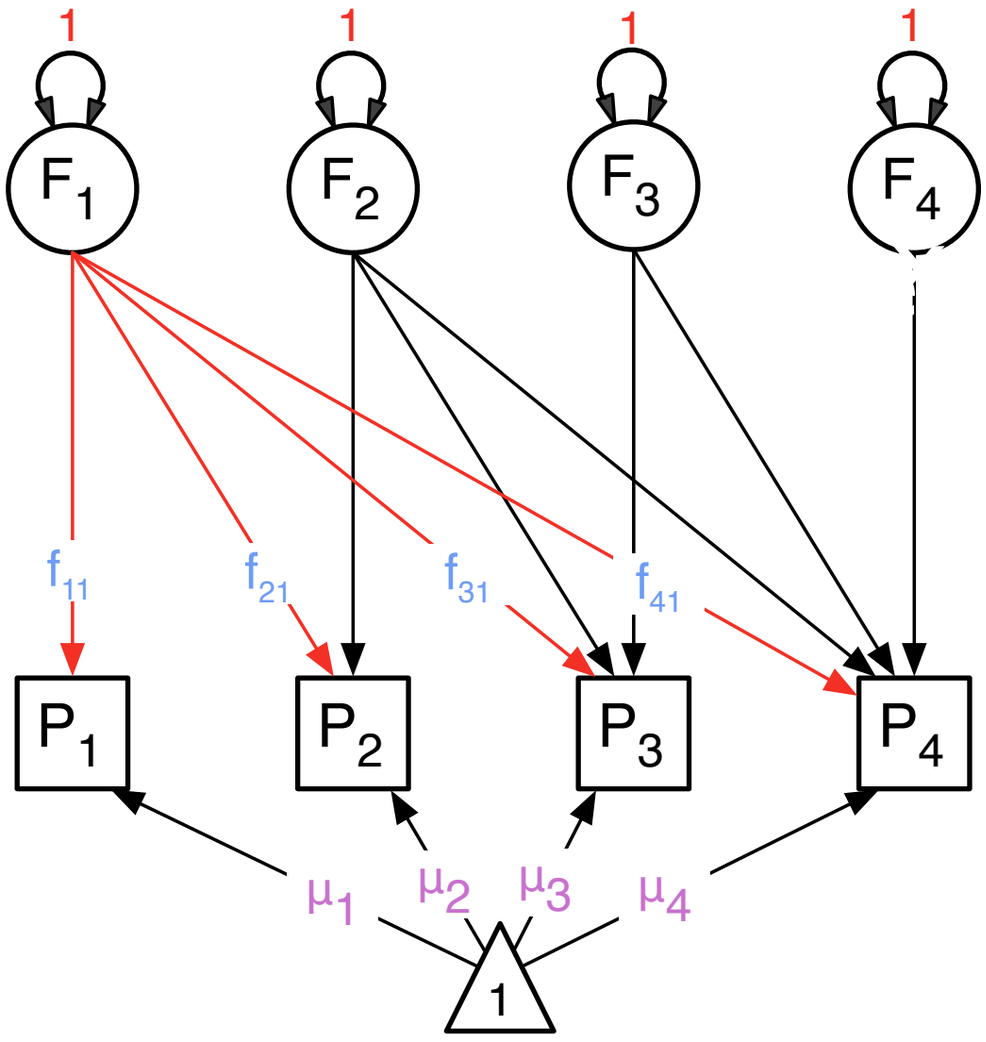
- Are these measures influenced by the same genes (single common factor)?
- Is there more than one factor (happy factor - sad factor)?
- What is the structure of C and E?
- Contribution of A, C, E factors to covariance between traits

Data

- Dutch DHBQ questionnaire
- N=3185, we use first 500 pairs

Fit Statistics

	ep	-2LL	df	chi	df	AIC
Saturated	180	14182.17	5283	-	-	-
EqMV	132	14216.14	5331	33.96	48	0.94
ACE Chol						
AE Chol						
IAICIE IP						
3AIE IP						
ILP CP						



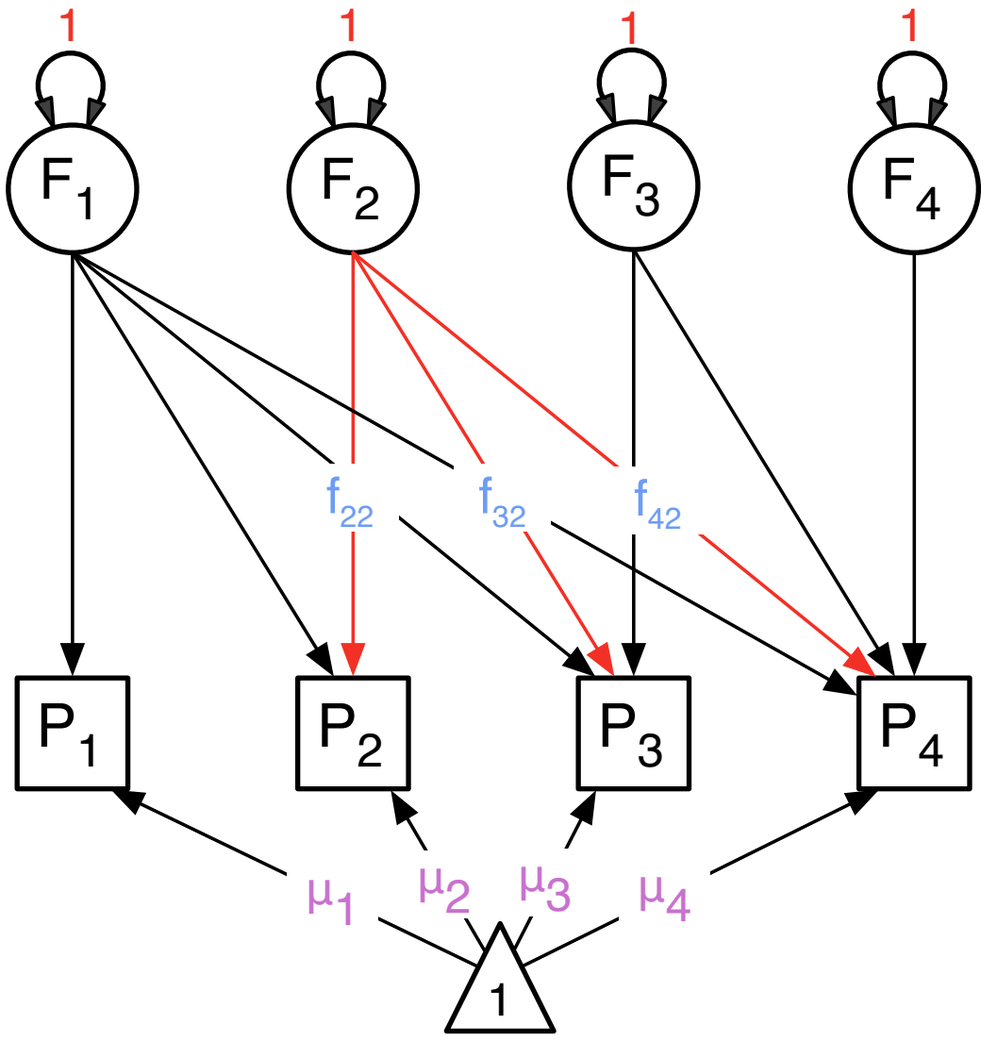
F1 F2 F3 F4

P1 f_{11}

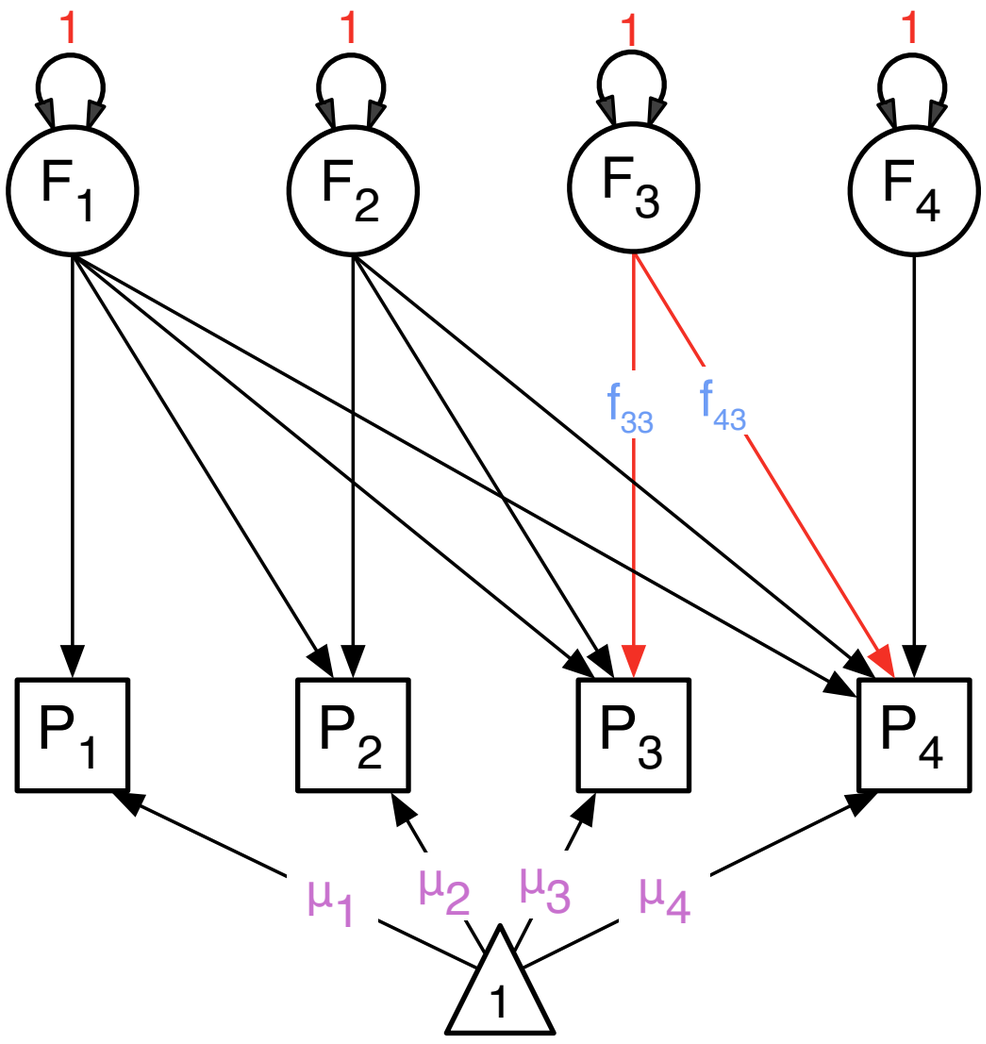
P2 f_{21}

P3 f_{31}

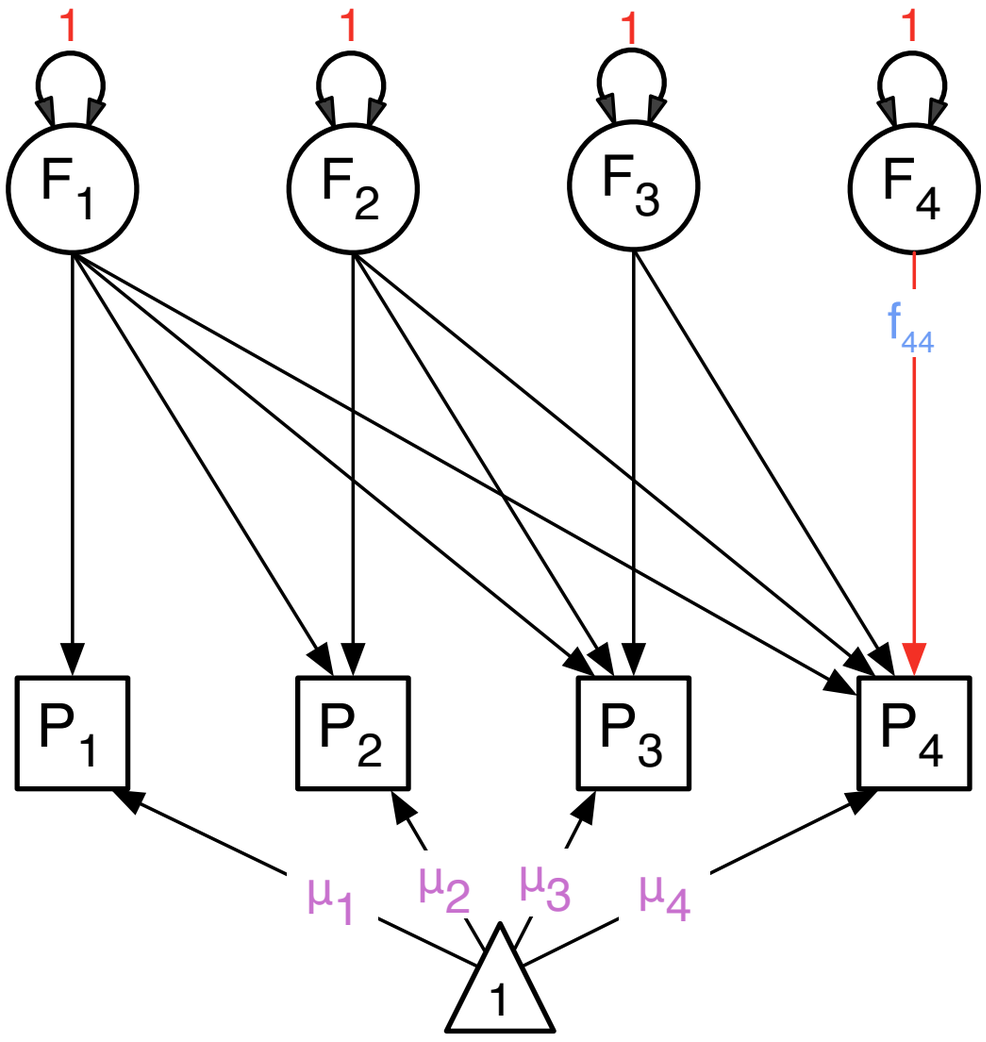
P4 f_{41}



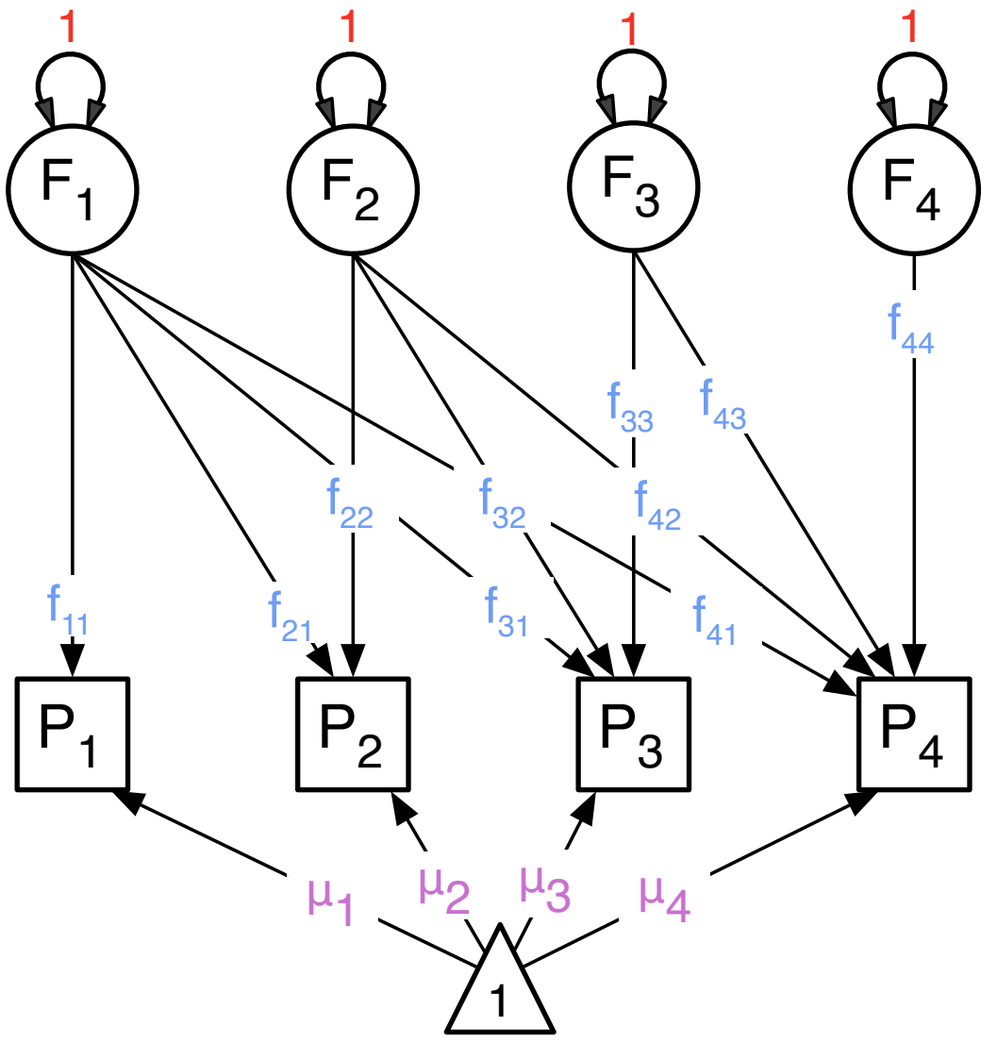
	F1	F2	F3	F4
P1	f_{11}	0		
P2	f_{21}	f_{22}		
P3	f_{31}	f_{32}		
P4	f_{41}	f_{42}		



	F1	F2	F3	F4
P1	f_{11}	0	0	
P2	f_{21}	f_{22}	0	
P3	f_{31}	f_{32}	f_{33}	
P4	f_{41}	f_{42}	f_{43}	



	F1	F2	F3	F4
P1	f_{11}	0	0	0
P2	f_{21}	f_{22}	0	0
P3	f_{31}	f_{32}	f_{33}	0
P4	f_{41}	f_{42}	f_{43}	f_{44}

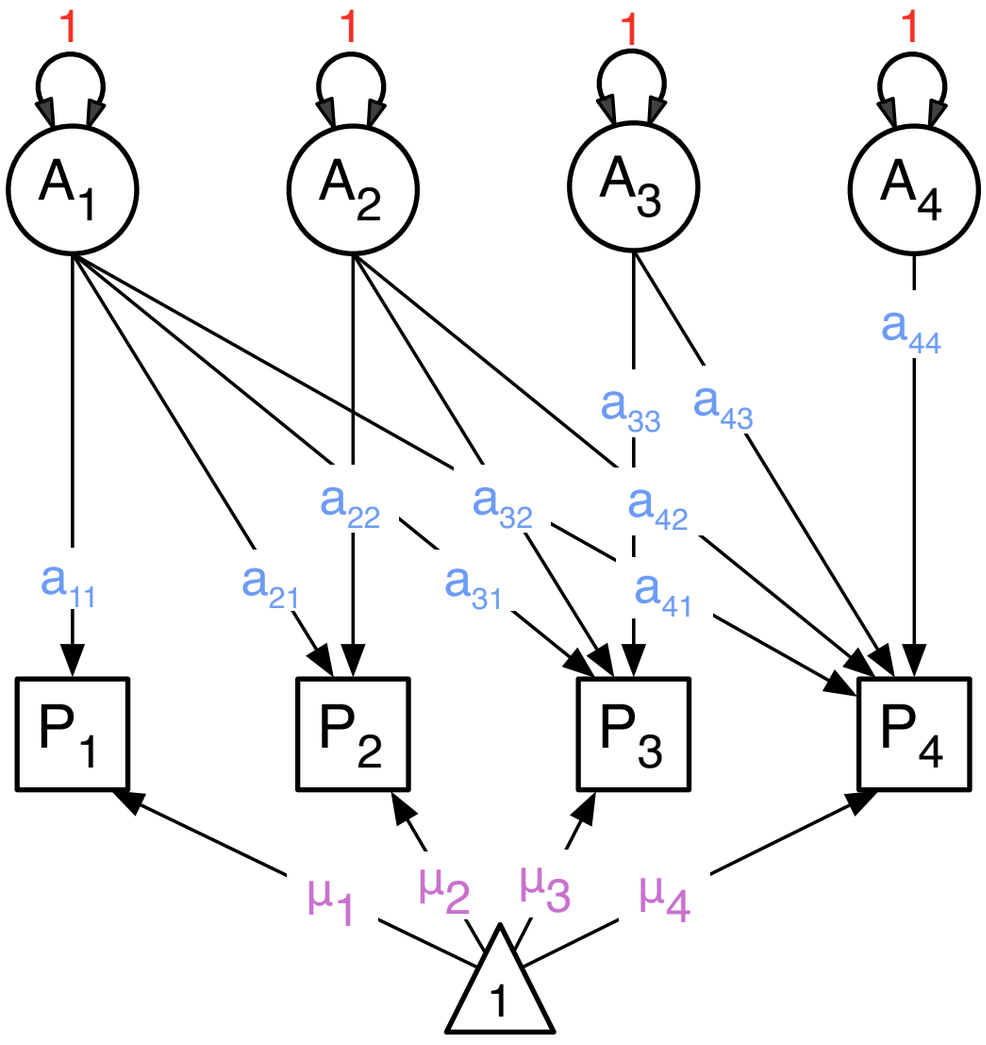

1
1
1
1
F1
F2
F3
F4
f44
f33
f43
f22
f32
f42
f11
f21
f31
f41
P1
P2
P3
P4
μ1
μ2
μ3
μ4
1

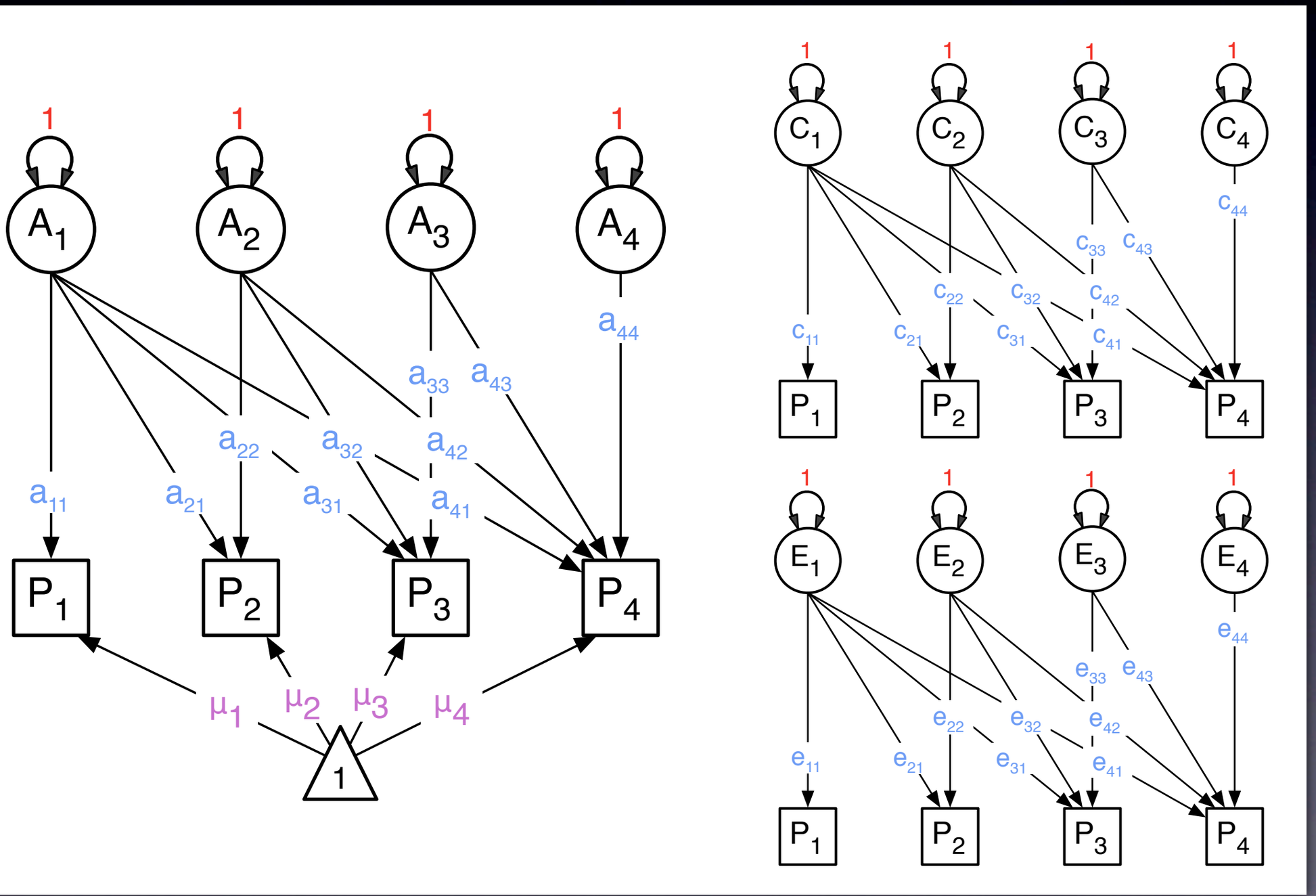
Cholesky Decomposition

Estimate covariance matrix, fully saturated

	F1	F2	F3	F4			F1	F2	F3	F4
P1	f_{11}	0	0	0		P1	f_{11}	f_{21}	f_{31}	f_{41}
P2	f_{21}	f_{22}	0	0	%*%	P2	0	f_{22}	f_{32}	f_{42}
P3	f_{31}	f_{32}	f_{33}	0		P3	0	0	f_{33}	f_{43}
P4	f_{41}	f_{42}	f_{43}	f_{44}		P4	0	0	0	f_{44}

F %*% t(F)


1
1
1
1
A1
A2
A3
A4
a44
a33
a43
a22
a32
a42
a11
a21
a31
a41
P1
P2
P3
P4
μ1
μ2
μ3
μ4
1


1
A1
A2
A3
A4
a44
a33
a43
a22
a32
a42
a11
a21
a31
a41
P1
P2
P3
P4
μ1
μ2
μ3
μ4
1
C1
C2
C3
C4
C44
C33
C43
c22
c32
c42
c11
c21
c31
c41
E1
E2
E3
E4
e44
e33
e43
e22
e32
e42
e11
e21
e31
e41

```
> parameterSpecifications(multiCholACEFit)
model:ACE, matrix:a
     [,1]  [,2]  [,3]  [,4]  [,5]  [,6]
[1,] [a11] 0     0     0     0     0
[2,] [a21] [a22] 0     0     0     0
[3,] [a31] [a32] [a33] 0     0     0
[4,] [a41] [a42] [a43] [a44] 0     0
[5,] [a51] [a52] [a53] [a54] [a55] 0
[6,] [a61] [a62] [a63] [a64] [a65] [a66]

model:ACE, matrix:c
     [,1]  [,2]  [,3]  [,4]  [,5]  [,6]
[1,] [c11] 0     0     0     0     0
[2,] [c21] [c22] 0     0     0     0
[3,] [c31] [c32] [c33] 0     0     0
[4,] [c41] [c42] [c43] [c44] 0     0
[5,] [c51] [c52] [c53] [c54] [c55] 0
[6,] [c61] [c62] [c63] [c64] [c65] [c66]

model:ACE, matrix:e
     [,1]  [,2]  [,3]  [,4]  [,5]  [,6]
[1,] [e11] 0     0     0     0     0
[2,] [e21] [e22] 0     0     0     0
[3,] [e31] [e32] [e33] 0     0     0
[4,] [e41] [e42] [e43] [e44] 0     0
[5,] [e51] [e52] [e53] [e54] [e55] 0
[6,] [e61] [e62] [e63] [e64] [e65] [e66]

model:ACE, matrix:Mean
     [,1] [,2] [,3] [,4] [,5] [,6]
[1,] [NA] [NA] [NA] [NA] [NA] [NA]
```

Fit Statistics

	ep	-2LL	df	chi	df	AIC
Saturated	180	14182.17	5283	-	-	-
EqMV	132	14216.14	5331	33.96	48	0.94
ACE Chol						
AE Chol						
IAICIE IP						
3AIE IP						
ILP CP						

Fit Statistics

	ep	-2LL	df	chi	df	AIC
Saturated	180	14182.17	5283	-	-	-
EqMV	132	14216.14	5331	33.96	48	0.94
ACE Chol	69	14362.14	5394	179.97	111	0
AE Chol						
1A1C1E IP						
3A1E IP						
1LP CP						

```
> formatOutputMatrices(multiCholACEFit,ACEpathMatrices,ACEpathLabels,Vars,4)

1] "Matrix ACE.iSD %*% ACE.a"
        stPathEst_a1 stPathEst_a2 stPathEst_a3 stPathEst_a4 stPathEst_a5 stPathEst_a6
family  0.6126       0.0000       0.0000       0.0000       0.0000       0.0000
happy   0.3796       0.4008       0.0000       0.0000       0.0000       0.0000
life    0.3438       0.2414       0.1798       0.0000       0.0000       0.0000
anxdep  -0.2460      -0.4120      -0.0278      0.1307       0.0000       0.0000
somatic -0.2515      -0.2657      -0.1270      0.3126       0.1110       0.0000
social  -0.1885      -0.2243      0.4444       -0.0055      0.1166       0.0000

[1] "Matrix ACE.iSD %*% ACE.c"
        stPathEst_c1 stPathEst_c2 stPathEst_c3 stPathEst_c4 stPathEst_c5 stPathEst_c6
family  0.4236       0.0000       0.0000       0.0000       0.0000       0.0000
happy   0.2580       -0.1519      0.0000       0.0000       0.0000       0.0000
life    0.3508       -0.1150      0.1505       0.0000       0.0000       0.0000
anxdep  -0.3039      0.0634       -0.1614      0.3641       0.0000       0.0000
somatic -0.1478      -0.0508      -0.4463      0.0581       0.0000       0.0000
social  -0.2685      0.0325       -0.2261      0.2427       0.0000       0.0000

[1] "Matrix ACE.iSD %*% ACE.e"
        stPathEst_e1 stPathEst_e2 stPathEst_e3 stPathEst_e4 stPathEst_e5 stPathEst_e6
family  0.6673       0.0000       0.0000       0.0000       0.0000       0.0000
happy   0.1296       0.7673       0.0000       0.0000       0.0000       0.0000
life    0.0879       0.5241       0.5915       0.0000       0.0000       0.0000
anxdep  -0.0419      -0.2804      -0.0644      0.6421       0.0000       0.0000
somatic -0.0067      -0.0448      0.0253       0.2766       0.6587       0.0000
social  -0.0410      -0.0672      -0.0944      0.1590       0.0450       0.6909
```

Fit Statistics

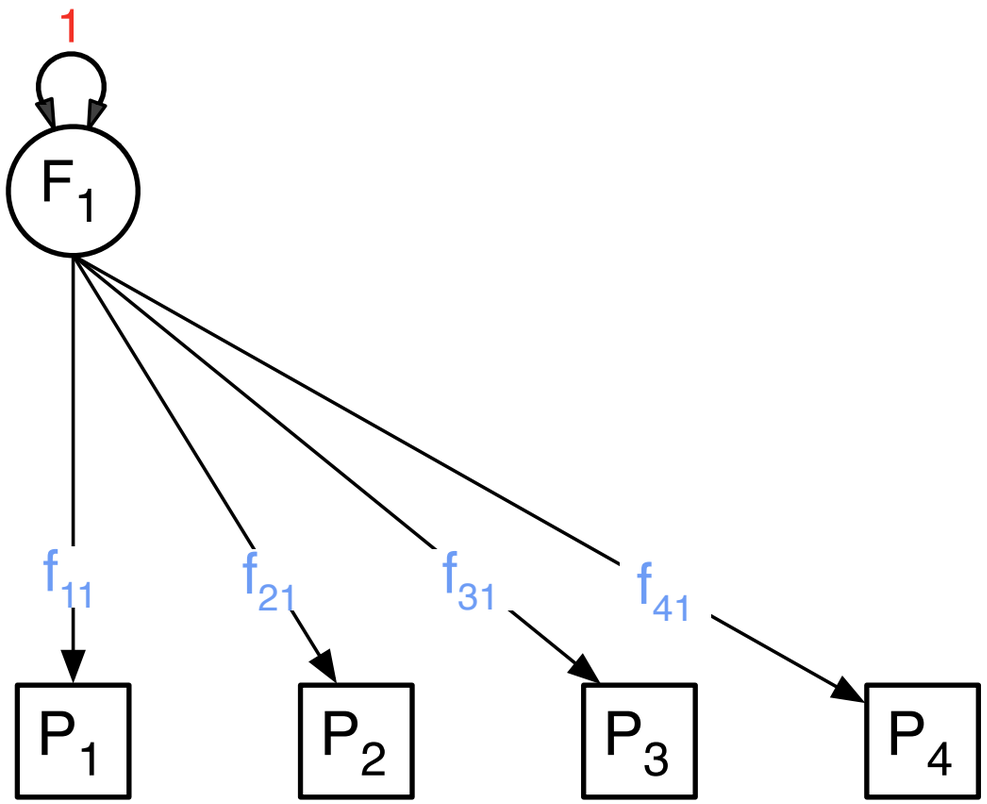
	ep	-2LL	df	chi	df	AIC
Saturated	180	14182.17	5283	-	-	-
EqMV	132	14216.14	5331	33.96	48	0.94
ACE Chol	69	14362.14	5394	-	-	-
AE Chol						
1A1C1E IP						
3A1E IP						
1LP CP						

Fit Statistics

	ep	-2LL	df	chi	df	AIC
Saturated	180	14182.17	5283	-	-	-
EqMV	132	14216.14	5331	33.96	48	0.94
ACE Chol	69	14362.14	5394	-	-	-
AE Chol	48	14379.03	5415	16.89	21	0.72
1A1C1E IP						
3A1E IP						
1LP CP						

```
> formatOutputMatrices(multiCholAEFit,ACEpathMatrices,ACEpathLabels,Vars,4)

[1] "Matrix ACE.iSD %*% ACE.a"
         stPathEst_a1 stPathEst_a2 stPathEst_a3 stPathEst_a4 stPathEst_a5 stPathEst_a6
family   0.7536       0.0000       0.0000       0.0000       0.0000       0.0000
happy    0.4627       0.4341       0.0000       0.0000       0.0000       0.0000
life     0.4857       0.2797       0.2526       0.0000       0.0000       0.0000
anxdep  -0.3650      -0.4166      -0.1363       0.4423       0.0000       0.0000
somatic -0.2655      -0.2207      -0.3754       0.2623      -0.4333       0.0000
social  -0.2952      -0.2230       0.1401       0.3666      -0.1349       0.4285

[1] "Matrix ACE.iSD %*% ACE.c"
         stPathEst_c1 stPathEst_c2 stPathEst_c3 stPathEst_c4 stPathEst_c5 stPathEst_c6
family   0.0000       0.0000       0.0000       0.0000       0.0000       0.0000
happy    0.0000       0.0000       0.0000       0.0000       0.0000       0.0000
life     0.0000       0.0000       0.0000       0.0000       0.0000       0.0000
anxdep   0.0000       0.0000       0.0000       0.0000       0.0000       0.0000
somatic  0.0000       0.0000       0.0000       0.0000       0.0000       0.0000
social   0.0000       0.0000       0.0000       0.0000       0.0000       0.0000

[1] "Matrix ACE.iSD %*% ACE.e"
         stPathEst_e1 stPathEst_e2 stPathEst_e3 stPathEst_e4 stPathEst_e5 stPathEst_e6
family   0.6574       0.0000       0.0000       0.0000       0.0000       0.0000
happy    0.1118       0.7649       0.0000       0.0000       0.0000       0.0000
life     0.0680       0.5140       0.5943       0.0000       0.0000       0.0000
anxdep  -0.0344      -0.2740      -0.0667       0.6311       0.0000       0.0000
somatic -0.0167      -0.0524       0.0439       0.2647       0.6390       0.0000
social  -0.0373      -0.0635      -0.0883       0.1386       0.0261       0.6886
```

- IP


1
F1
f11
f21
f31
f41
P1
P2
P3
P4

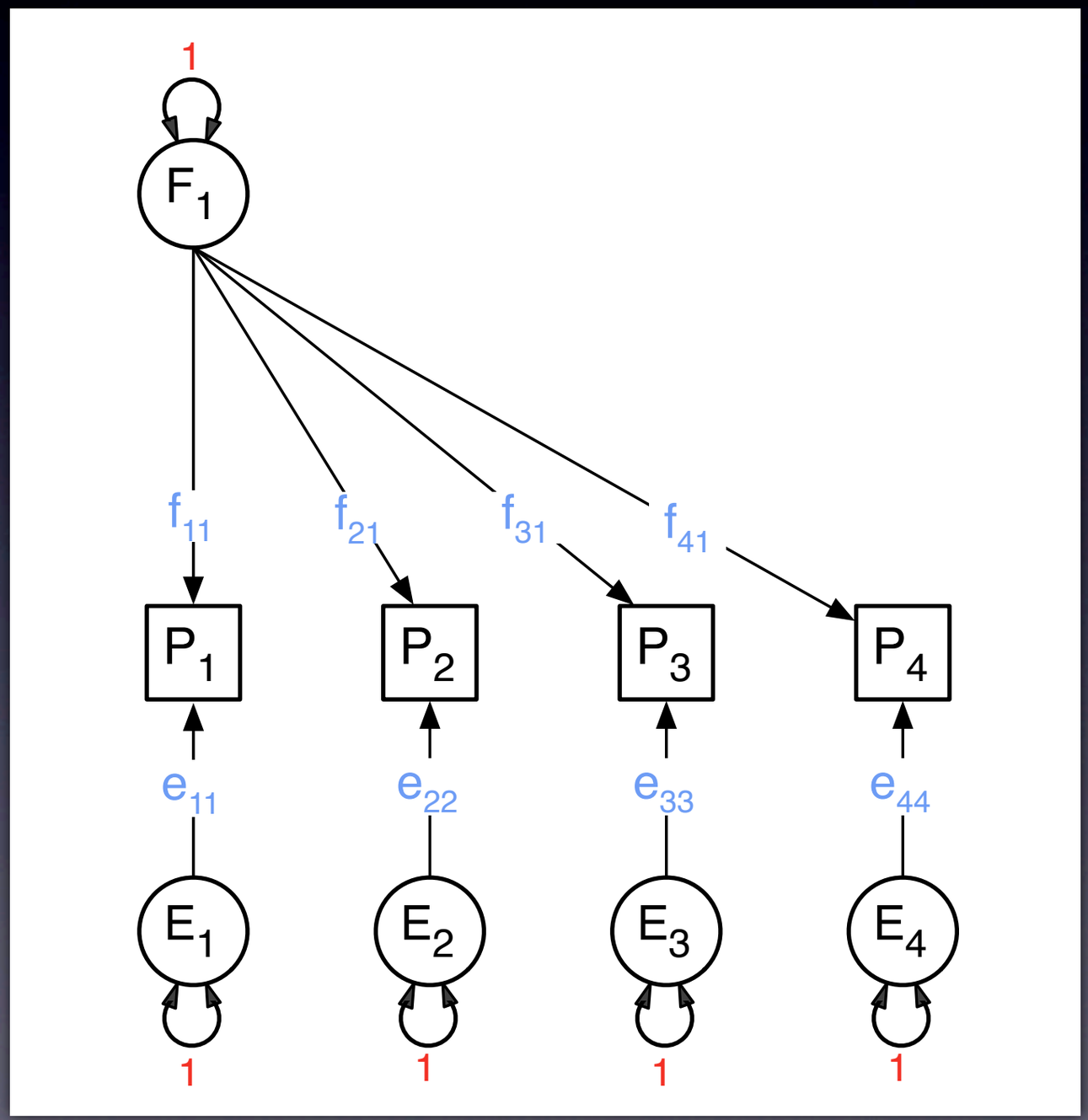
FI

P1 f_{11}
P2 f_{21}
P3 f_{31}
P4 f_{41}

%*%

F1 F2 F3 F4
P1 f_{11} f_{21} f_{31} f_{41}

F %*% t(F)


1
F1
f11
f21
f31
f41
P1
P2
P3
P4
e11
e22
e33
e44
E1
E2
E3
E4
1
1
1
1

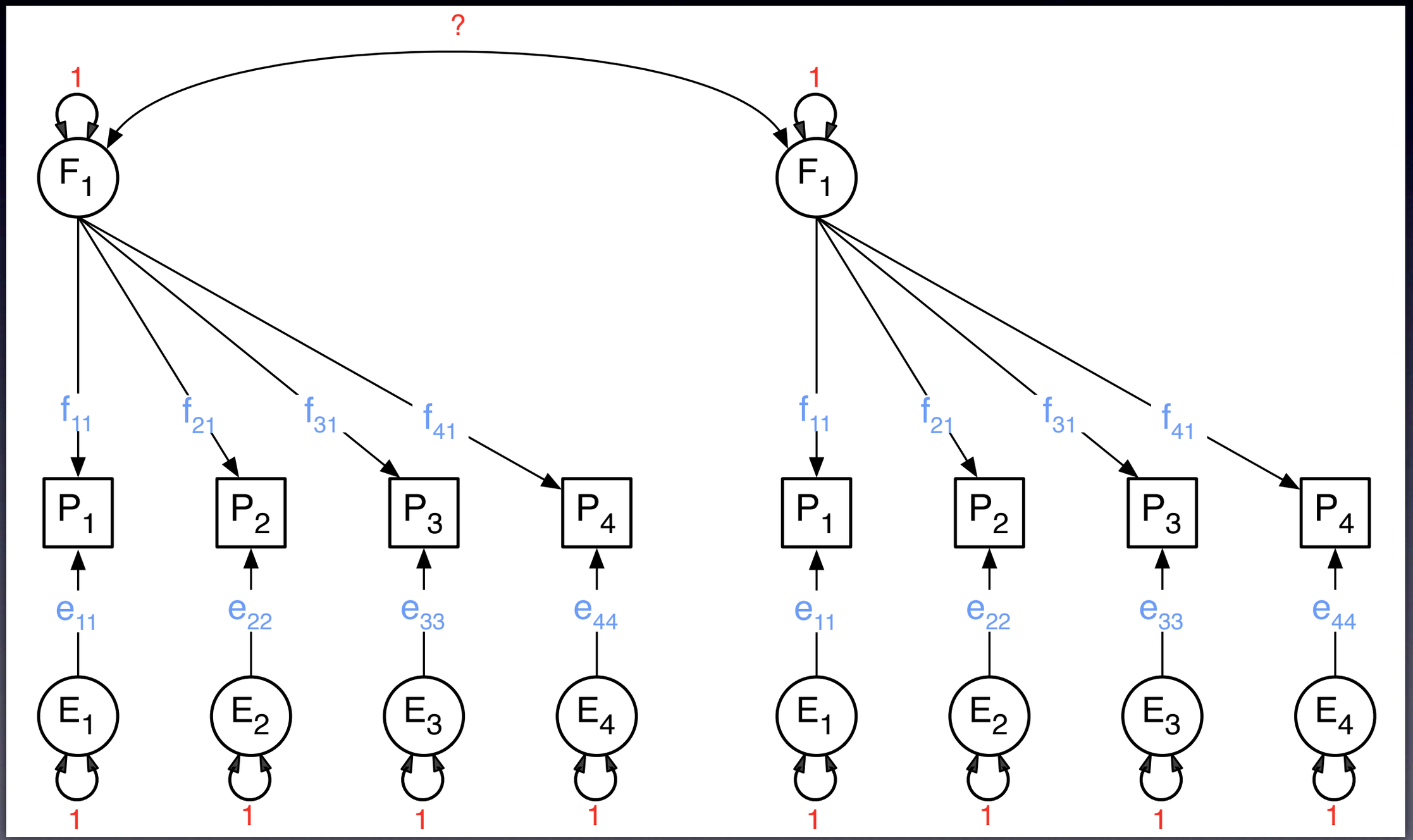
	E1	E2	E3	E4
P1	f_{11}	0	0	0
P2	0	f_{22}	0	0
P3	0	0	f_{33}	0
P4	0	0	0	f_{44}

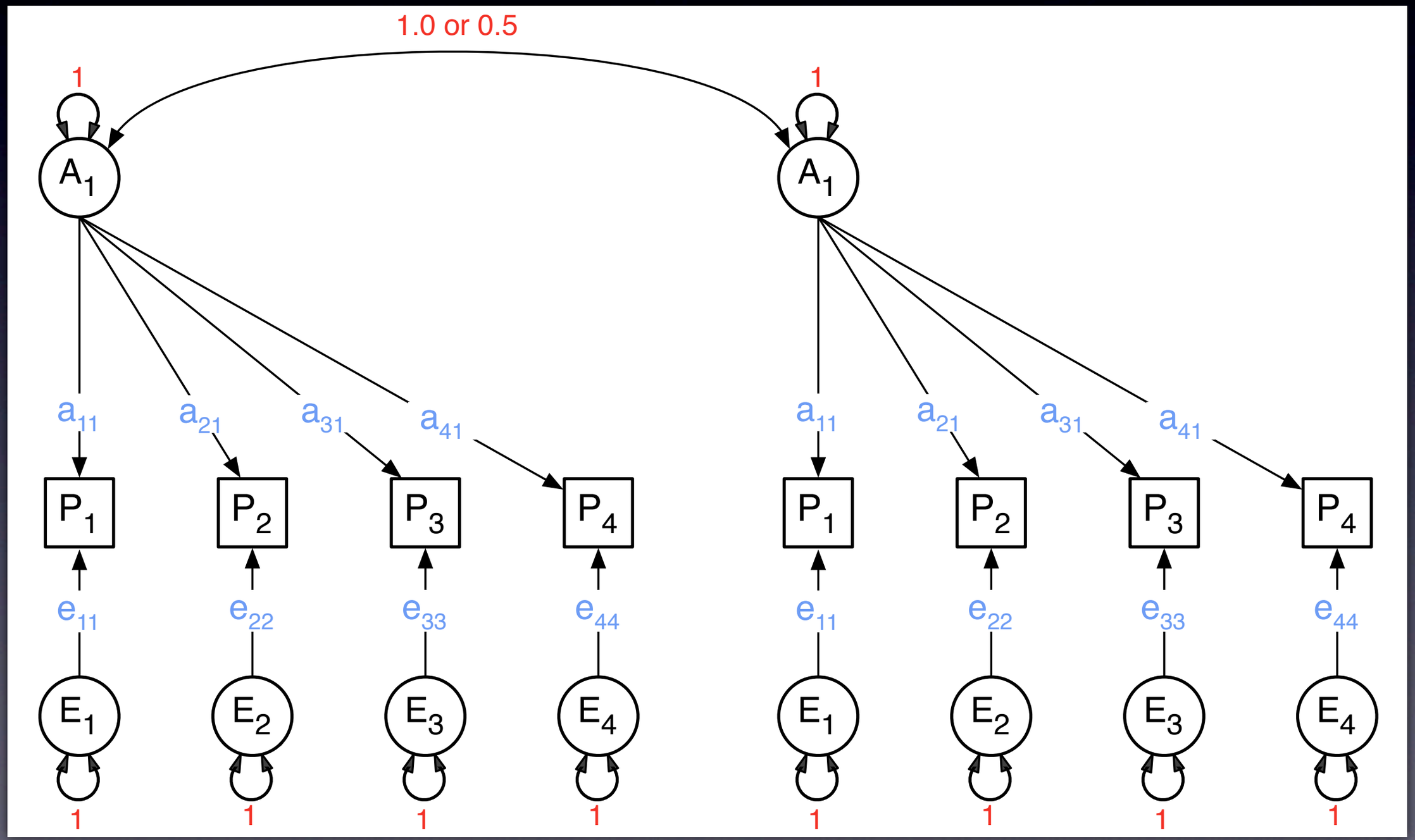
%*%

	E1	E2	E3	E4
P1	f_{11}	0	0	0
P2	0	f_{22}	0	0
P3	0	0	f_{33}	0
P4	0	0	0	f_{44}

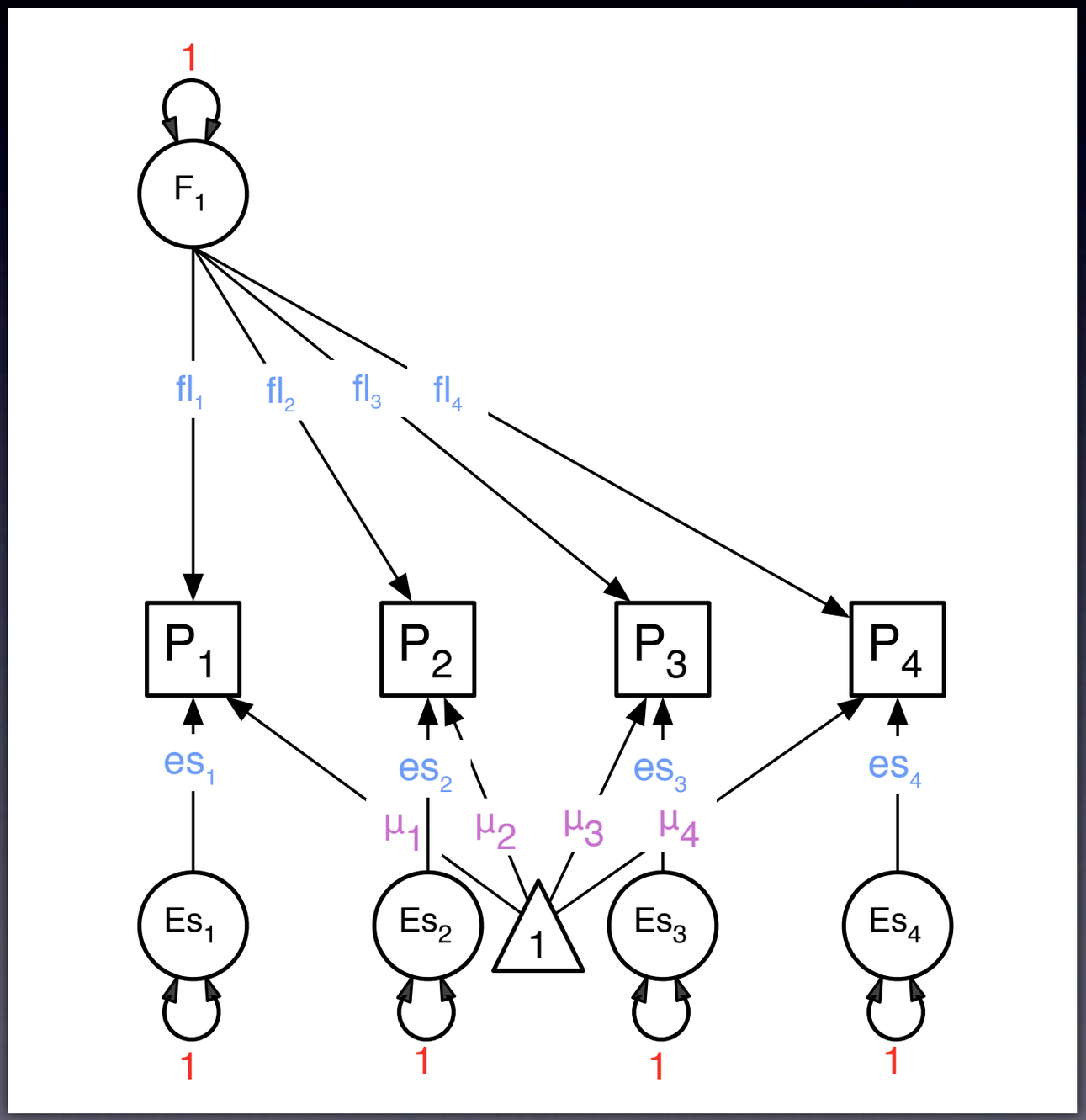
E %*% t(E)

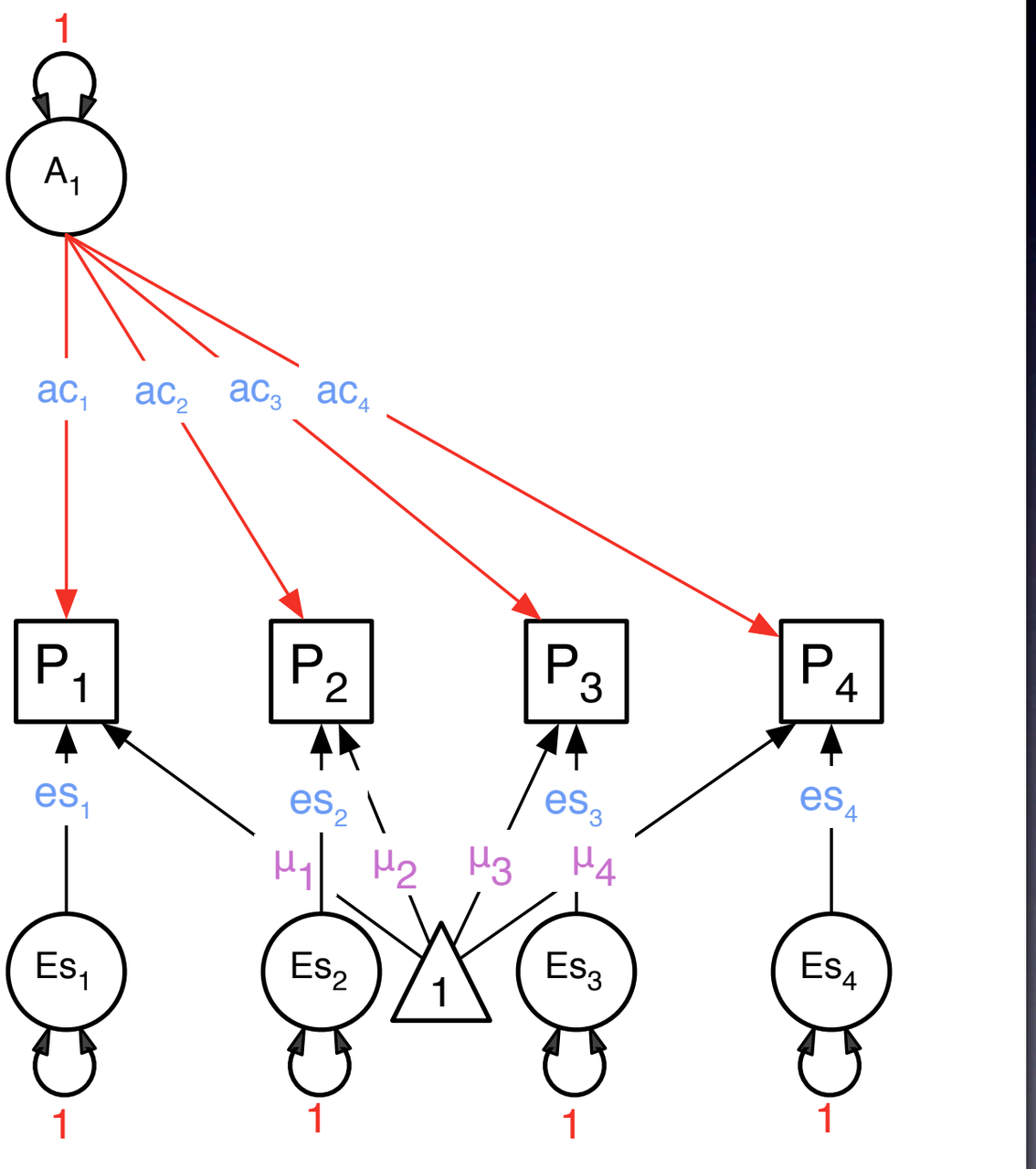
F %*% t(F) + E %*% t(E)

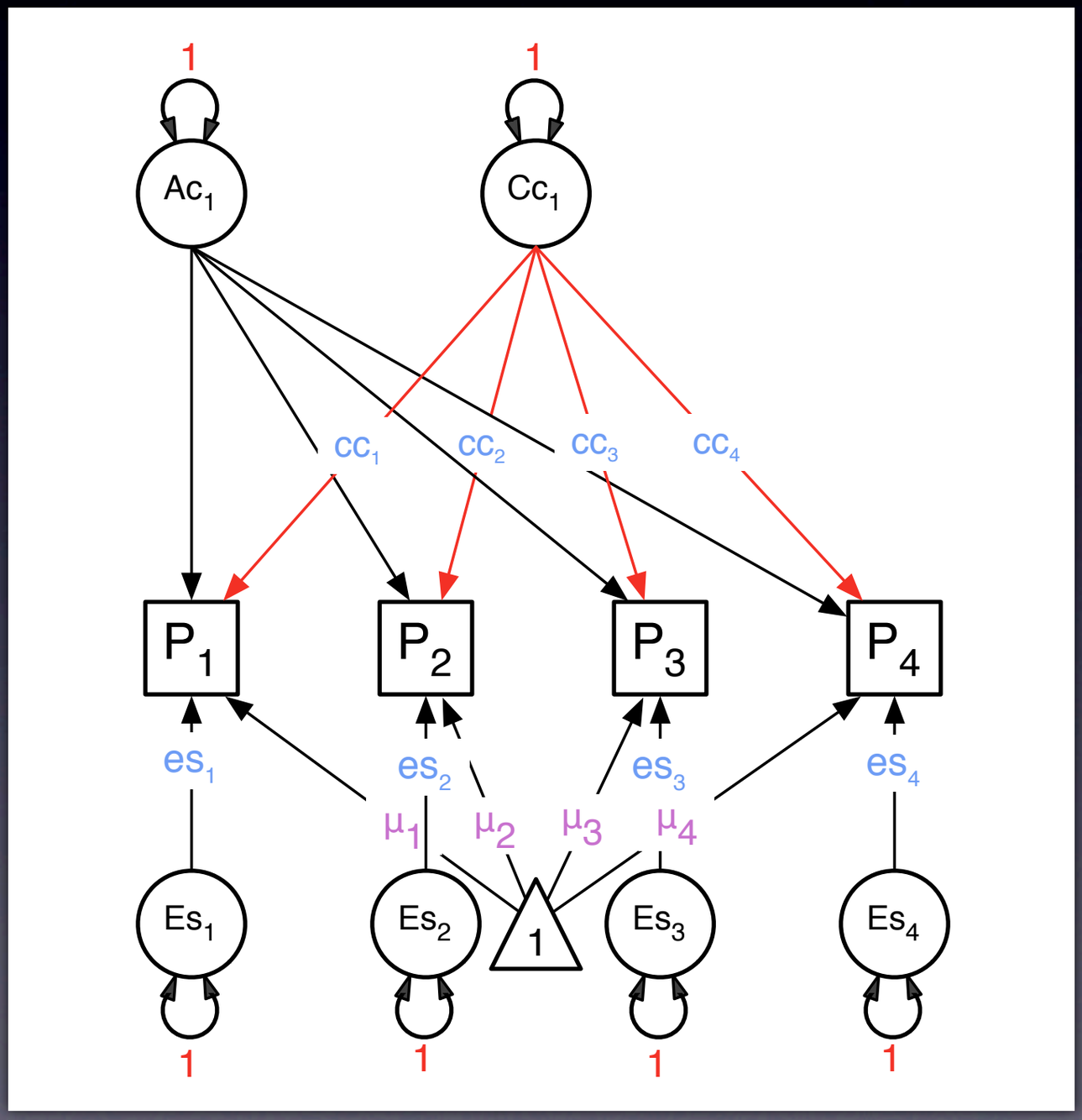

?
1
1
F1
F1
f11
f21
f31
f41
f11
f21
f31
f41
P1
P2
P3
P4
P1
P2
P3
P4
e11
e22
e33
e44
e11
e22
e33
e44
E1
E2
E3
E4
E1
E2
E3
E4
1
1
1
1
1
1
1
1

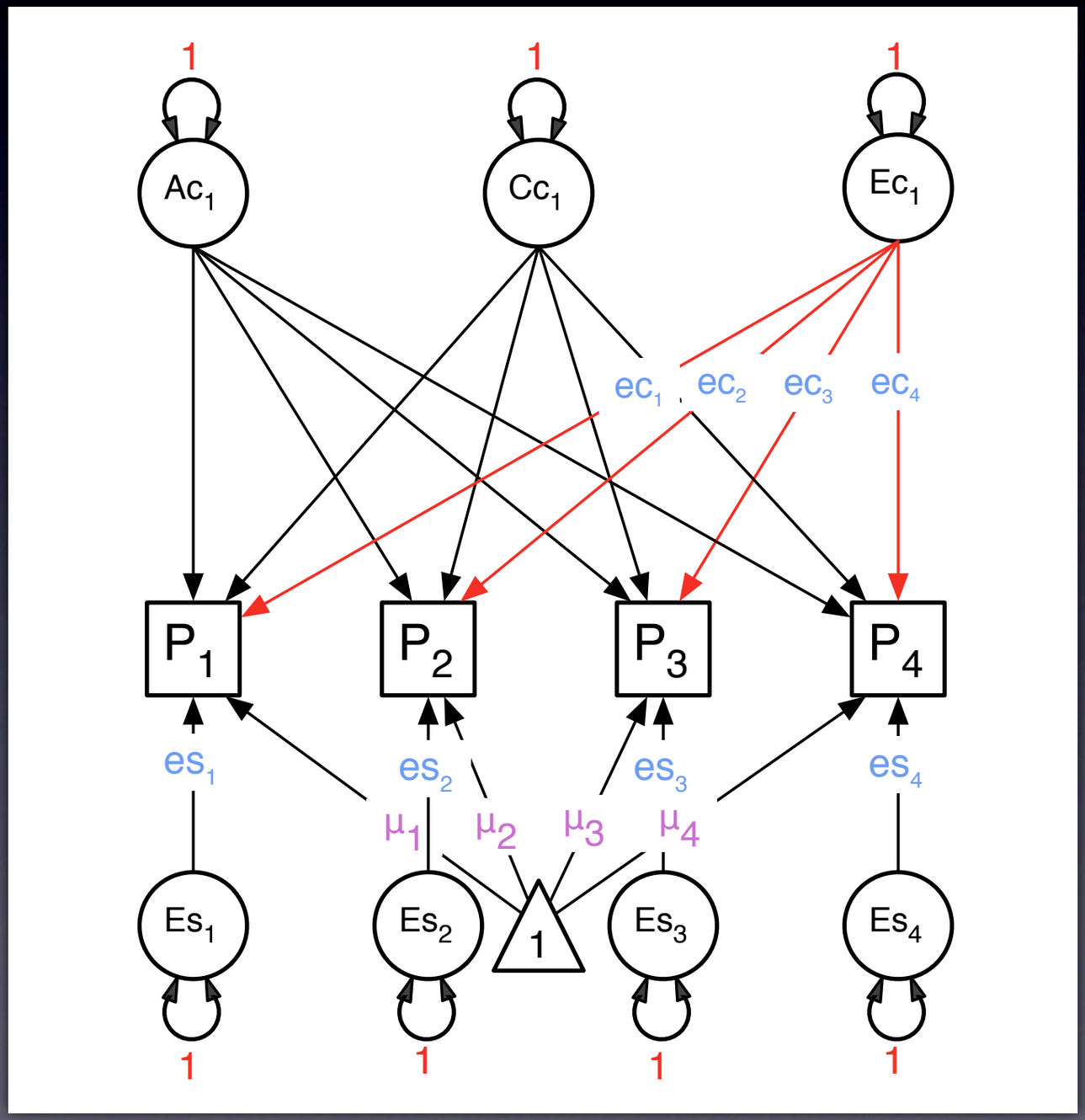

1.0 or 0.5
1
1
A1
A1
a11
a21
a31
a41
a11
a21
a31
a41
P1
P2
P3
P4
P1
P2
P3
P4
e11
e22
e33
e44
e11
e22
e33
e44
E1
E2
E3
E4
E1
E2
E3
E4
1
1
1
1
1
1
1
1

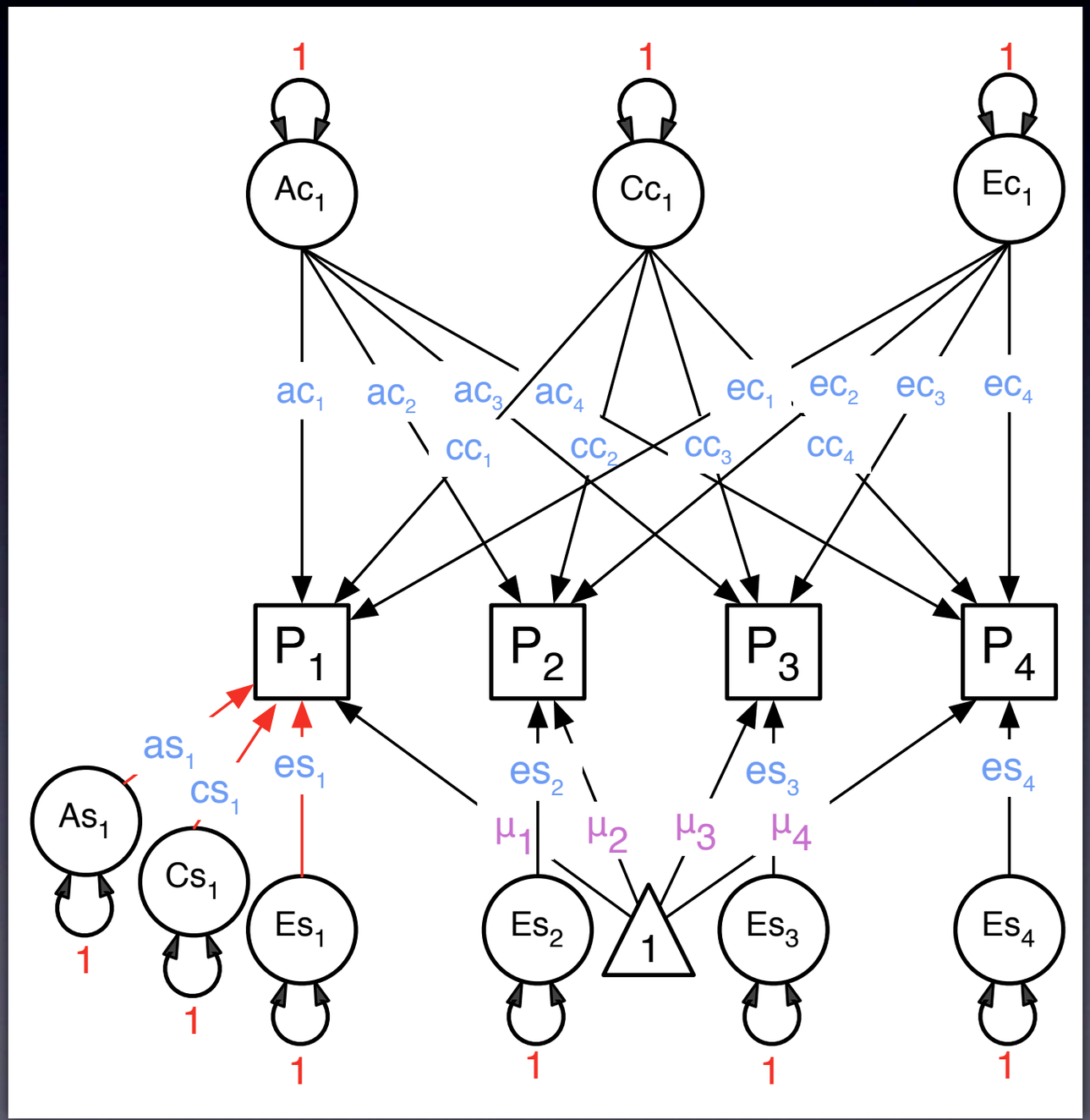
twin 1


1
F1
fl1
fl2
fl3
fl4
P1
P2
P3
P4
es1
es2
es3
es4
μ1
μ2
μ3
μ4
Es1
Es2
1
Es3
Es4
1
1
1
1


1
A1
ac1
ac2
ac3
ac4
P1
P2
P3
P4
es1
es2
es3
es4
μ1
μ2
μ3
μ4
Es1
Es2
1
Es3
Es4
1
1
1
1


1
1
Ac1
Cc1
cc1
cc2
cc3
cc4
P1
P2
P3
P4
es1
es2
es3
es4
μ1
μ2
μ3
μ4
Es1
Es2
1
Es3
Es4
1
1
1
1


1
1
1
Ac1
Cc1
Ec1
ec1
ec2
ec3
ec4
P1
P2
P3
P4
es1
es2
es3
es4
μ1
μ2
μ3
μ4
Es1
Es2
1
Es3
Es4
1
1
1
1


1
1
1
Ac1
Cc1
Ec1
ac1
ac2
ac3
ac4
cc1
cc2
cc3
cc4
ec1
ec2
ec3
ec4
P1
P2
P3
P4
as1
cs1
es1
es2
es3
es4
μ1
μ2
μ3
μ4
As1
Cs1
Es1
Es2
Es3
Es4
1
1
1
1
1
1
1
1

AI

P1 a_{11}
P2 a_{21}
P3 a_{31}
P4 a_{41}

%*%

P1 P2 P3 P4
AI a_{11} a_{21} a_{31} a_{41}

ac %*% **t(ac)**

	A1	A2	A3	A4
P1	a_{11}	0	0	0
P2	0	a_{22}	0	0
P3	0	0	a_{33}	0
P4	0	0	0	a_{44}

%*%

	P1	P2	P3	P4
A1	a_{11}	0	0	0
A2	0	a_{22}	0	0
A3	0	0	a_{33}	0
A4	0	0	0	a_{44}

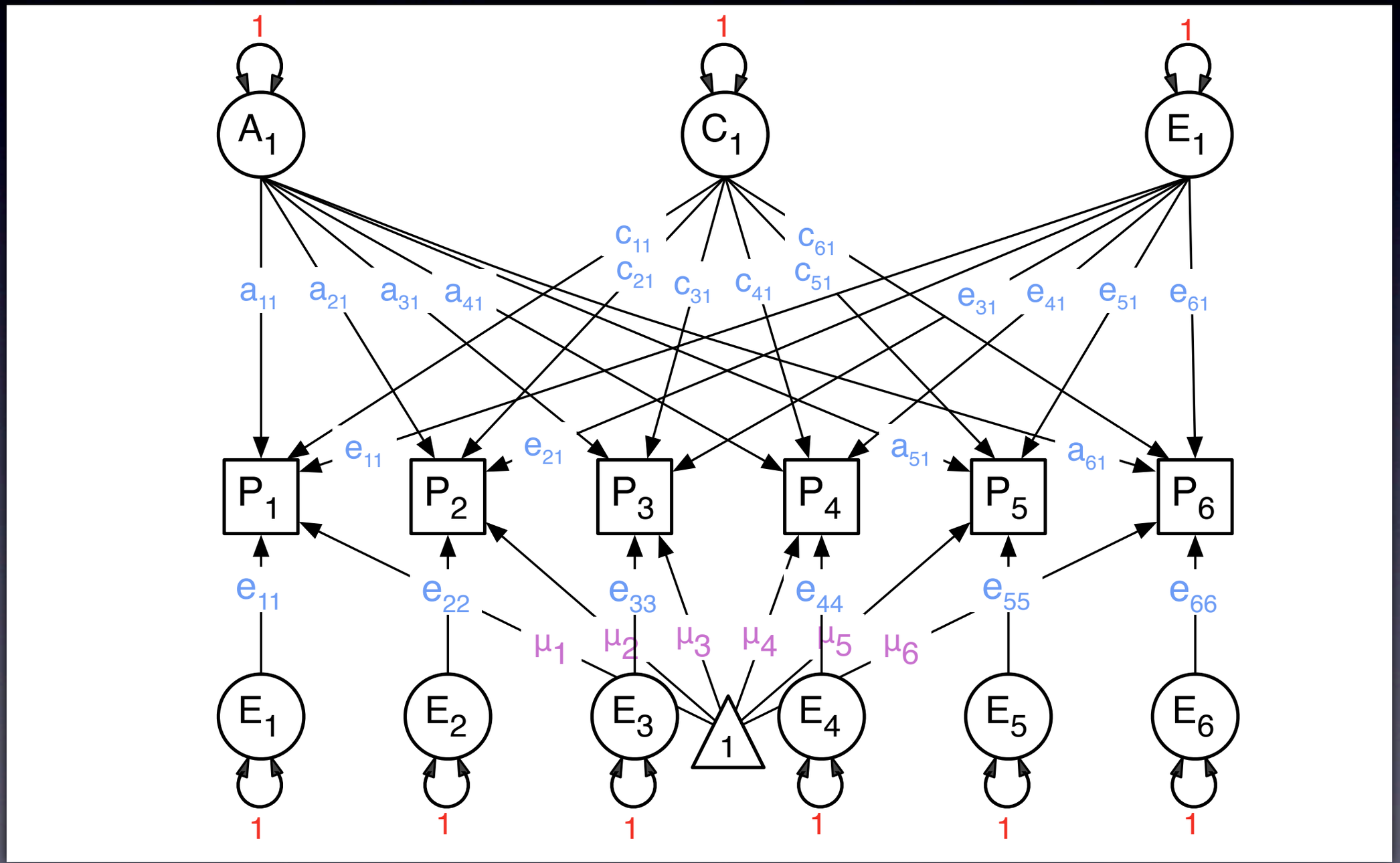
as %*% t(as)

```
> parameterSpecifications(multiIndPathACEFit)
model:ACE, matrix:ac
     [,1]
[1,] [ac11]
[2,] [ac21]
[3,] [ac31]
[4,] [ac41]
[5,] [ac51]
[6,] [ac61]

model:ACE, matrix:cc
     [,1]
[1,] [cc11]
[2,] [cc21]
[3,] [cc31]
[4,] [cc41]
[5,] [cc51]
[6,] [cc61]

model:ACE, matrix:ec
     [,1]
[1,] [ec11]
[2,] [ec21]
[3,] [ec31]
[4,] [ec41]
[5,] [ec51]
[6,] [ec61]
```

```
model:ACE, matrix:as
     [,1]   [,2]   [,3]   [,4]   [,5]   [,6]
[1,] [as11] 0      0      0      0      0
[2,] 0      [as21] 0      0      0      0
[3,] 0      0      [as31] 0      0      0
[4,] 0      0      0      [as41] 0      0
[5,] 0      0      0      0      [as51] 0
[6,] 0      0      0      0      0      [as61]

model:ACE, matrix:cs
     [,1]   [,2]   [,3]   [,4]   [,5]   [,6]
[1,] [cs11] 0      0      0      0      0
[2,] 0      [cs21] 0      0      0      0
[3,] 0      0      [cs31] 0      0      0
[4,] 0      0      0      [cs41] 0      0
[5,] 0      0      0      0      [cs51] 0
[6,] 0      0      0      0      0      [cs61]

model:ACE, matrix:es
     [,1]   [,2]   [,3]   [,4]   [,5]   [,6]
[1,] [es11] 0      0      0      0      0
[2,] 0      [es21] 0      0      0      0
[3,] 0      0      [es31] 0      0      0
[4,] 0      0      0      [es41] 0      0
[5,] 0      0      0      0      [es51] 0
[6,] 0      0      0      0      0      [es61]

model:ACE, matrix:Mean
     [,1] [,2] [,3] [,4] [,5] [,6]
[1,] [NA] [NA] [NA] [NA] [NA] [NA]
```

- Independent Pathway Model
- Biometric model
- Different covariance structure for A, C and E

IP Model


1
A1
C1
E1
a11
a21
a31
a41
c11
c21
c31
c41
c51
c61
e31
e41
e51
e61
e11
e21
a51
a61
P1
P2
P3
P4
P5
P6
e11
e22
e33
e44
e55
e66
μ1
μ2
μ3
μ4
μ5
μ6
E1
E2
E3
E4
E5
E6
1

Variance Component	a^2	c^2	e^2
Common Factors	ac nv x 1	cc nv x 1	ec nv x 1
Residual Factors	as nv x nv	cs nv x nv	es nv x nv

Fit Statistics

	ep	-2LL	df	chi	df	AIC
Saturated	180	14182.17	5283	-	-	-
EqMV	132	14216.14	5331	33.96	48	0.94
ACE Chol	69	14362.14	5394	-	-	-
AE Chol	48	14379.03	5415	16.89	21	0.72
1A1C1E IP						
3A1E IP						
1LP CP						

Fit Statistics

	ep	-2LL	df	chi	df	AIC
Saturated	180	14182.17	5283	-	-	-
EqMV	132	14216.14	5331	33.96	48	0.94
ACE Chol	69	14362.14	5394	-	-	-
AE Chol	48	14379.03	5415	16.89	21	0.72
1A1C1E IP	42	14449.87	5421	87.73	27	0.00
3A1E IP						
1LP CP						

```
formatOutputMatrices
(multiIndPathACEFit,ACEpathMatricesC,ACEpathLabelsC,Vars,4)

[1] "Matrix ACE.iSD %*% ACE.ac"
         stPathEst_ac1
family  0.1454
happy   0.2471
life    0.2306
anxdep  -0.7073
somatic -0.5972
social  -0.4640

[1] "Matrix ACE.iSD %*% ACE.cc"
         stPathEst_cc1
family  -0.5533
happy   -0.4472
life    -0.4829
anxdep  0.2779
somatic 0.2024
social  0.2418

[1] "Matrix ACE.iSD %*% ACE.ec"
         stPathEst_ec1
family  -0.1863
happy   -0.7594
life    -0.6251
anxdep  0.3512
somatic 0.0696
social  0.1003
```

```
[1] "Matrix ACE.iSD %*% ACE.as"
         stPaths_as1 stPaths_as2 stPaths_as3 stPaths_as4 stPaths_as5 stPaths_as6
family   0.4327      0.0000      0.0000      0.0000      0.0000      0.0000
happy    0.0000      0.1339      0.0000      0.0000      0.0000      0.0000
life     0.0000      0.0000      0.2051      0.0000      0.0000      0.0000
anxdep   0.0000      0.0000      0.0000      0.0000      0.0000      0.0000
somatic  0.0000      0.0000      0.0000      0.0000      0.1742      0.0000
social   0.0000      0.0000      0.0000      0.0000      0.0000      -0.4839

[1] "Matrix ACE.iSD %*% ACE.cs"
         stPaths_cs1 stPaths_cs2 stPaths_cs3 stPaths_cs4 stPaths_cs5 stPaths_cs6
family   0.0000      0.0000      0.0000      0.0000      0.0000      0.0000
happy    0.0000      -0.1275     0.0000      0.0000      0.0000      0.0000
life     0.0000      0.0000      0.0000      0.0000      0.0000      0.0000
anxdep   0.0000      0.0000      0.0000      0.0000      0.0000      0.0000
somatic  0.0000      0.0000      0.0000      0.0000      0.3518      0.0000
social   0.0000      0.0000      0.0000      0.0000      0.0000      0.0000

[1] "Matrix ACE.iSD %*% ACE.es"
         stPaths_es1 stPaths_es2 stPaths_es3 stPaths_es4 stPaths_es5 stPaths_es6
family   0.6714      0.0000      0.0000      0.0000      0.0000      0.0000
happy    0.0000      0.3578      0.0000      0.0000      0.0000      0.0000
life     0.0000      0.0000      0.5300      0.0000      0.0000      0.0000
anxdep   0.0000      0.0000      0.0000      0.5470      0.0000      0.0000
somatic  0.0000      0.0000      0.0000      0.0000      0.6659      0.0000
social   0.0000      0.0000      0.0000      0.0000      0.0000      0.6943
```

Theory

- Happy Factor
- Sad Factor
- General Mood?

```
> parameterSpecifications(multiIndPathACEFit2)
model:ACE, matrix:ac
     [,1]  [,2]  [,3]
[1,] [a11] [a12] 0
[2,] [a21] [a22] 0
[3,] [a31] [a32] 0
[4,] [a41] 0     [a43]
[5,] [a51] 0     [a53]
[6,] [a61] 0     [a63]

model:ACE, matrix:cc
     [,1] [,2] [,3]
[1,]    0    0    0
[2,]    0    0    0
[3,]    0    0    0
[4,]    0    0    0
[5,]    0    0    0
[6,]    0    0    0

model:ACE, matrix:ec
     [,1]  [,2]  [,3]
[1,] [e11] [e12] 0
[2,] [e21] [e22] 0
[3,] [e31] [e32] 0
[4,] [e41] 0     [e43]
[5,] [e51] 0     [e53]
[6,] [e61] 0     [e63]
```

```
model:ACE, matrix:as
     [,1]   [,2]   [,3]   [,4]   [,5]   [,6]
[1,] [as11] 0      0      0      0      0
[2,] 0      [as21] 0      0      0      0
[3,] 0      0      [as31] 0      0      0
[4,] 0      0      0      [as41] 0      0
[5,] 0      0      0      0      [as51] 0
[6,] 0      0      0      0      0      [as61]

model:ACE, matrix:cs
     [,1]   [,2]   [,3]   [,4]   [,5]   [,6]
[1,] [cs11] 0      0      0      0      0
[2,] 0      [cs21] 0      0      0      0
[3,] 0      0      [cs31] 0      0      0
[4,] 0      0      0      [cs41] 0      0
[5,] 0      0      0      0      [cs51] 0
[6,] 0      0      0      0      0      [cs61]

model:ACE, matrix:es
     [,1]   [,2]   [,3]   [,4]   [,5]   [,6]
[1,] [es11] 0      0      0      0      0
[2,] 0      [es21] 0      0      0      0
[3,] 0      0      [es31] 0      0      0
[4,] 0      0      0      [es41] 0      0
[5,] 0      0      0      0      [es51] 0
[6,] 0      0      0      0      0      [es61]

model:ACE, matrix:Mean
     [,1] [,2] [,3] [,4] [,5] [,6]
[1,] [NA] [NA] [NA] [NA] [NA] [NA]
```

Fit Statistics

	ep	-2LL	df	chi	df	AIC
Saturated	180	14182.17	5283	-	-	-
EqMV	132	14216.14	5331	33.96	48	0.94
ACE Chol	69	14362.14	5394	-	-	-
AE Chol	48	14379.03	5415	16.89	21	0.72
1A1C1E IP	42	14449.87	5421	87.73	27	0.00
3A1E IP						
1LP CP						

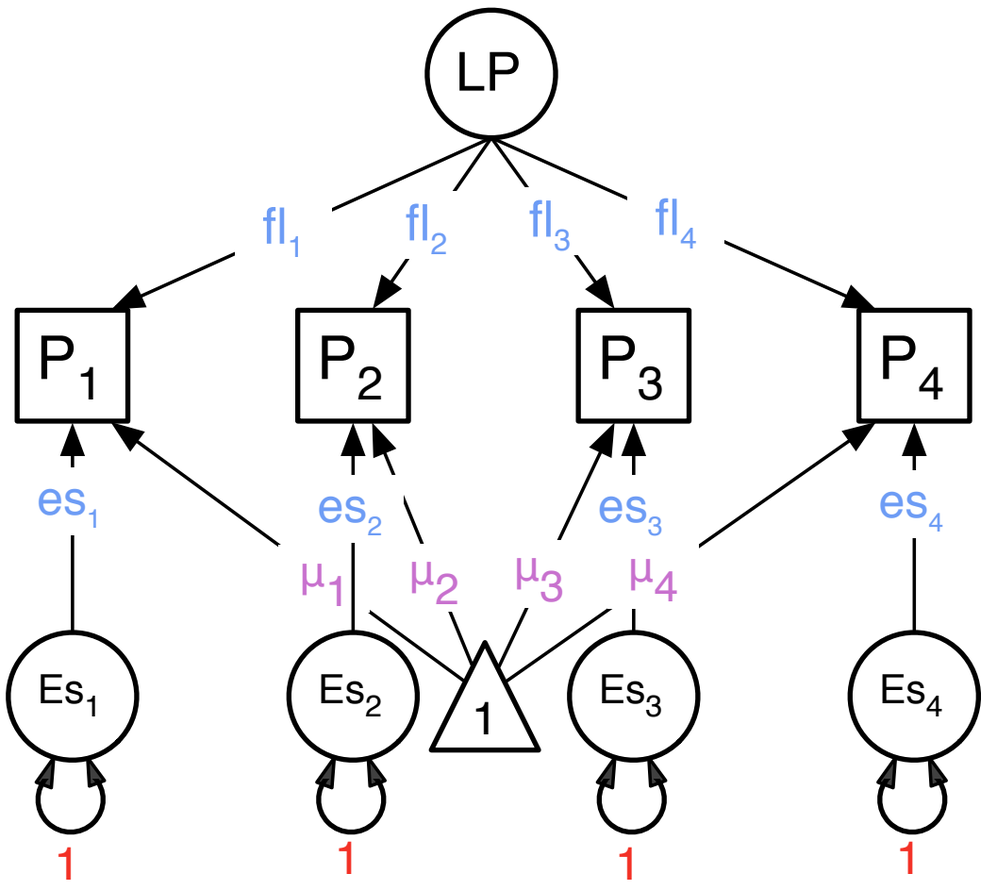
Fit Statistics

	ep	-2LL	df	chi	df	AIC
Saturated	180	14182.17	5283	-	-	-
EqMV	132	14216.14	5331	33.96	48	0.94
ACE Chol	69	14362.14	5394	-	-	-
AE Chol	48	14379.03	5415	16.89	21	0.72
1A1C1E IP	42	14449.87	5421	87.73	27	0.00
3A1E IP	42	14397.45	5421	35.32	27	0.13
1LP CP						

```
> formatOutputMatrices(
multiIndPathACEFit2,ACEstPathMatricesC,ACEstPathLabelsC,Vars,4)

[1] "Matrix ACE.iSD %*% ACE.ac"
        stPathEst_ac1 stPathEst_ac2 stPathEst_ac3
family  -0.5082       0.5592        0.0000
happy   -0.5678       0.1204        0.0000
life    -0.5524       0.1590        0.0000
anxdep  0.5755        0.0000        0.4178
somatic 0.4251        0.0000        0.2580
social  0.4230        0.0000        0.2615

[1] "Matrix ACE.iSD %*% ACE.cc"
        stPathEst_cc1 stPathEst_cc2 stPathEst_cc3
family  0.0000        0.0000        0.0000
happy   0.0000        0.0000        0.0000
life    0.0000        0.0000        0.0000
anxdep  0.0000        0.0000        0.0000
somatic 0.0000        0.0000        0.0000
social  0.0000        0.0000        0.0000

[1] "Matrix ACE.iSD %*% ACE.ec"
        stPathEst_ec1 stPathEst_ec2 stPathEst_ec3
family  0.0251        0.1144        0.0000
happy   0.6048        0.4585        0.0000
life    0.4891        0.2570        0.0000
anxdep  -0.3899       0.0000        0.5696
somatic -0.0561       0.0000        0.2831
social  -0.0677       0.0000        0.1370
```

```
> formatOutputMatrices(multiIndPathACEFit2,ACEstPathMatricesS,ACEstPathLabelsS,Vars,4
[1] "Matrix ACE.iSD %*% ACE.as"
         stPathEst_as1 stPathEst_as2 stPathEst_as3 stPathEst_as4 stPathEst_as5 stPathE
family   -0.0002       0.0000        0.0000        0.0000        0.0000        0.0000
happy    0.0000        0.1966        0.0000        0.0000        0.0000        0.0000
life     0.0000        0.0000        0.2150        0.0000        0.0000        0.0000
anxdep   0.0000        0.0000        0.0000        0.0000        0.0000        0.0000
somatic  0.0000        0.0000        0.0000        0.0000        0.5199        0.0000
social   0.0000        0.0000        0.0000        0.0000        0.0000        0.4992

[1] "Matrix ACE.iSD %*% ACE.cs"
         stPathEst_cs1 stPathEst_cs2 stPathEst_cs3 stPathEst_cs4 stPathEst_cs5 stPathE
family   0.0000        0.0000        0.0000        0.0000        0.0000        0.0000
happy    0.0000        0.0000        0.0000        0.0000        0.0000        0.0000
life     0.0000        0.0000        0.0000        0.0000        0.0000        0.0000
anxdep   0.0000        0.0000        0.0000        0.0000        0.0000        0.0000
somatic  0.0000        0.0000        0.0000        0.0000        0.0000        0.0000
social   0.0000        0.0000        0.0000        0.0000        0.0000        0.0000

[1] "Matrix ACE.iSD %*% ACE.es"
         stPathEst_es1 stPathEst_es2 stPathEst_es3 stPathEst_es4 stPathEst_es5 stPathE
family   0.6444        0.0000        0.0000        0.0000        0.0000        0.0000
happy    0.0000        0.2202        0.0000        0.0000        0.0000        0.0000
life     0.0000        0.0000        0.5640        0.0000        0.0000        0.0000
anxdep   0.0000        0.0000        0.0000        0.1334        0.0000        0.0000
somatic  0.0000        0.0000        0.0000        0.0000        0.6318        0.0000
social   0.0000        0.0000        0.0000        0.0000        0.0000        0.6929
```

- CP


LP
fl1
fl2
fl3
fl4
P1
P2
P3
P4
es1
es2
es3
es4
μ1
μ2
μ3
μ4
Es1
Es2
1
Es3
Es4
1
1
1
1

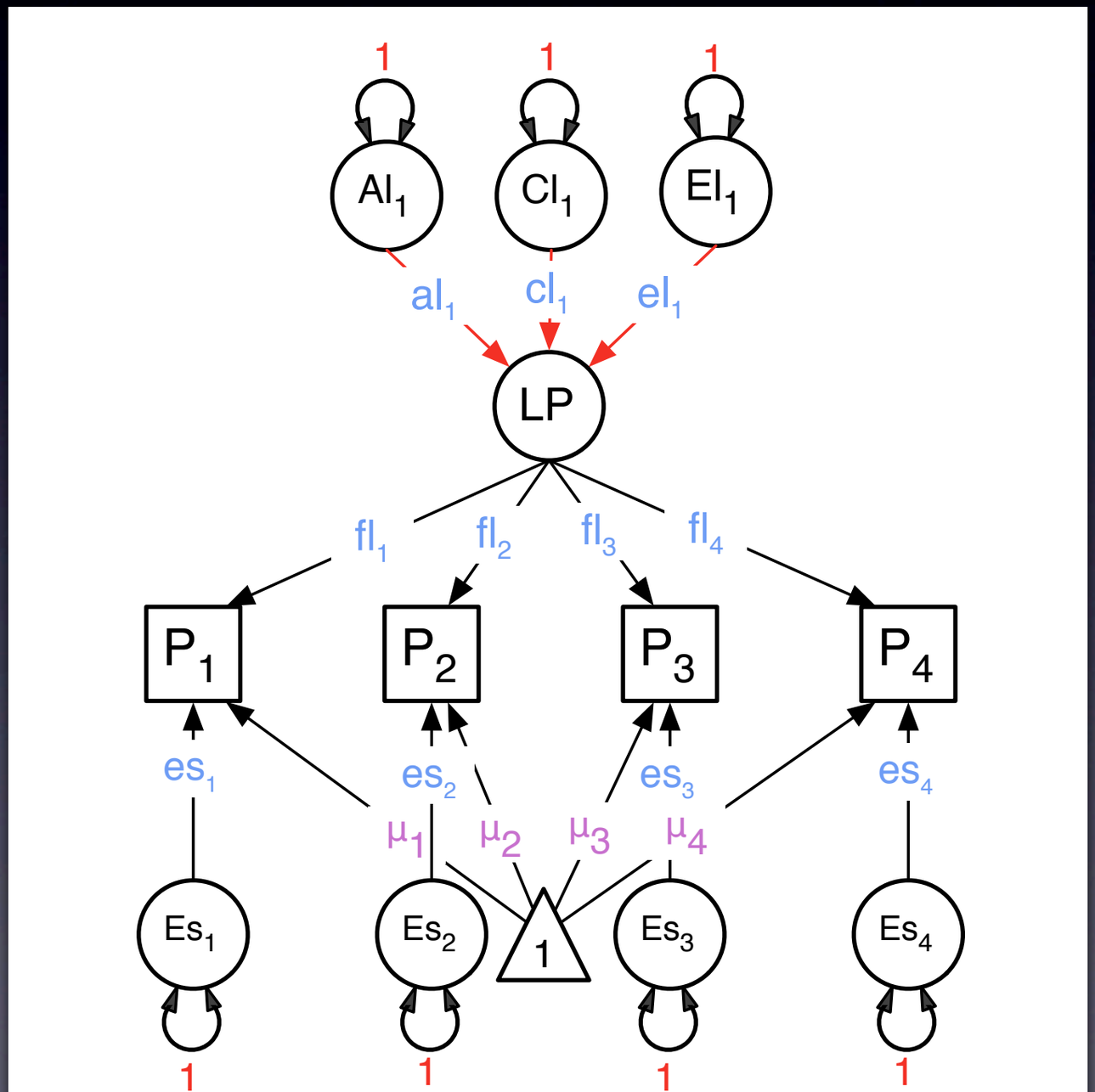
FI

P1 f_{11}
P2 f_{21}
P3 f_{31}
P4 f_{41}

%*%

F1 F2 F3 F4
P1 f_{11} f_{21} f_{31} f_{41}

F %*% t(F)


1
1
1
AI_1
CI_1
EI_1
al_1
cl_1
el_1
LP
fl_1
fl_2
fl_3
fl_4
P_1
P_2
P_3
P_4
es_1
es_2
es_3
es_4
μ_1
μ_2
μ_3
μ_4
Es_1
Es_2
1
Es_3
Es_4
1
1
1
1

F1

P1 f_{11}

P2 f_{21}

P3 f_{31}

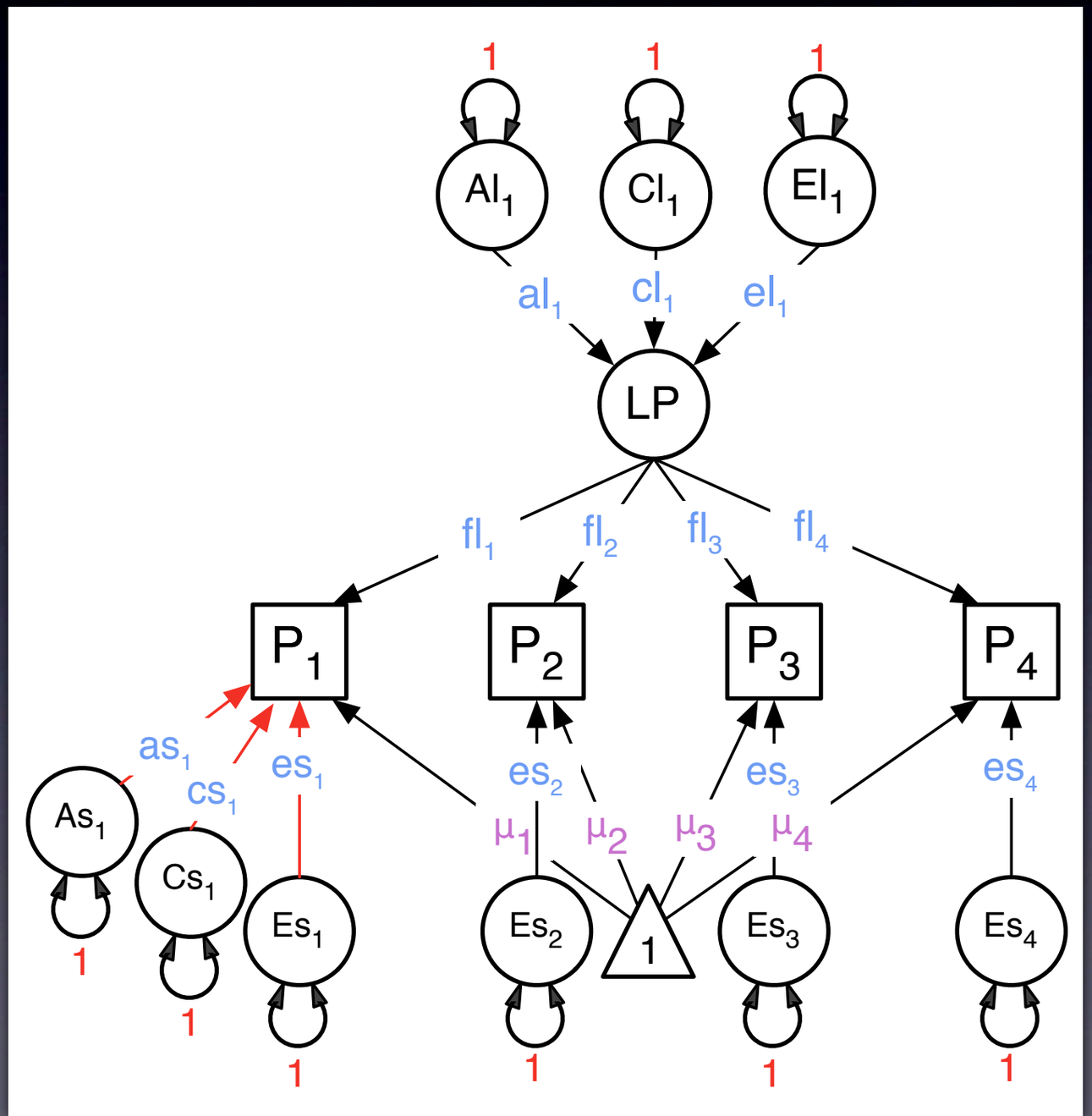
P4 f_{41}

%*% al_{11} %*% t(al_{11}) %*%

	F1	F2	F3	F4
P1	f_{11}	f_{21}	f_{31}	f_{41}

fl %*% al %*% t(al) %*% t(fl)

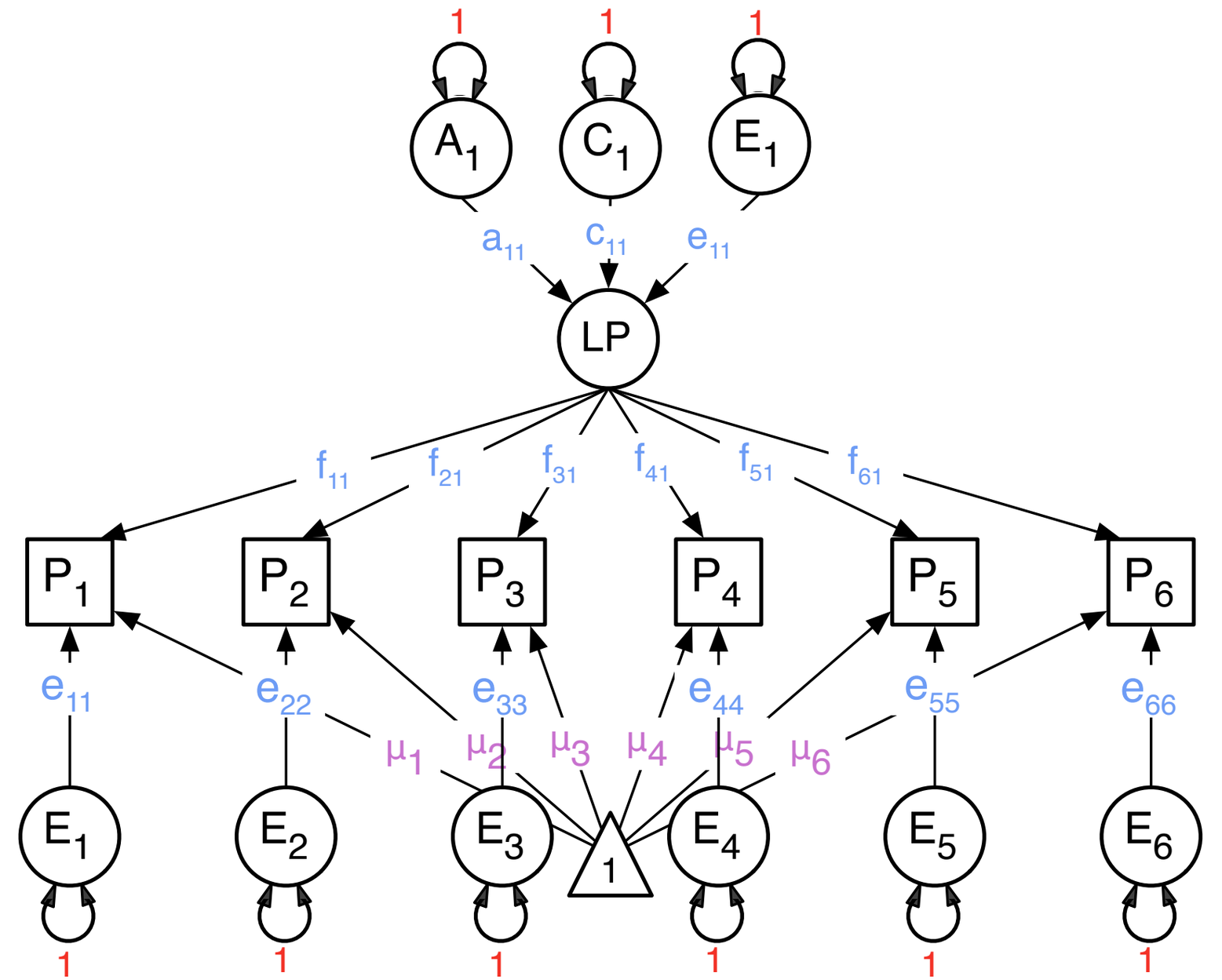
fl %x% al %*% t(al)


1
1
1
Al_1
Cl_1
El_1
al_1
cl_1
el_1
LP
fl_1
fl_2
fl_3
fl_4
P_1
P_2
P_3
P_4
as_1
cs_1
es_1
es_2
es_3
es_4
μ_1
μ_2
μ_3
μ_4
As_1
Cs_1
Es_1
Es_2
Es_3
Es_4
1
1
1
1
1
1
1

```
> parameterSpecifications(multiComPathACEFit)
model:ACE, matrix:al
     [,1]
[1,] [al11]

model:ACE, matrix:cl
     [,1]
[1,] [cl11]

model:ACE, matrix:el
     [,1]
[1,] [el11]

model:ACE, matrix:fl
     [,1]
[1,] [fl11]
[2,] [fl21]
[3,] [fl31]
[4,] [fl41]
[5,] [fl51]
[6,] [fl61]
```

```
model:ACE, matrix:as
     [,1]   [,2]   [,3]   [,4]   [,5]   [,6]
[1,] [as11] 0      0      0      0      0
[2,] 0      [as21] 0      0      0      0
[3,] 0      0      [as31] 0      0      0
[4,] 0      0      0      [as41] 0      0
[5,] 0      0      0      0      [as51] 0
[6,] 0      0      0      0      0      [as61]

model:ACE, matrix:cs
     [,1]   [,2]   [,3]   [,4]   [,5]   [,6]
[1,] [cs11] 0      0      0      0      0
[2,] 0      [cs21] 0      0      0      0
[3,] 0      0      [cs31] 0      0      0
[4,] 0      0      0      [cs41] 0      0
[5,] 0      0      0      0      [cs51] 0
[6,] 0      0      0      0      0      [cs61]

model:ACE, matrix:es
     [,1]   [,2]   [,3]   [,4]   [,5]   [,6]
[1,] [es11] 0      0      0      0      0
[2,] 0      [es21] 0      0      0      0
[3,] 0      0      [es31] 0      0      0
[4,] 0      0      0      [es41] 0      0
[5,] 0      0      0      0      [es51] 0
[6,] 0      0      0      0      0      [es61]

model:ACE, matrix:Mean
     [,1] [,2] [,3] [,4] [,5] [,6]
[1,] [NA] [NA] [NA] [NA] [NA] [NA]
```

- Common Pathway Model
- Psychometric model
- Same covariance structure for A, C and E


1
1
1
A1
C1
E1
a11
c11
e11
LP
f11
f21
f31
f41
f51
f61
P1
P2
P3
P4
P5
P6
e11
e22
e33
e44
e55
e66
μ1
μ2
μ3
μ4
μ5
μ6
E1
E2
E3
E4
E5
E6
1
1
1
1
1
1
1

Variance Component	a^2	c^2	e^2	
Common Factors	al l x l	cl l x l	el l x l	fl nv x l
Residual Factors	as nv x nv	cs nv x nv	es nv x nv	

Fit Statistics

	ep	-2LL	df	chi	df	AIC
Saturated	180	14182.17	5283	-	-	-
EqMV	132	14216.14	5331	33.96	48	0.94
ACE Chol	69	14362.14	5394	-	-	-
AE Chol	48	14379.03	5415	16.89	21	0.72
1A1C1E IP	42	14449.87	5421	87.73	27	0.00
3A1E IP	42	14397.45	5421	35.32	27	0.13
1LP CP						

Fit Statistics

	ep	-2LL	df	chi	df	AIC
Saturated	180	14182.17	5283	-	-	-
EqMV	132	14216.14	5331	33.96	48	0.94
ACE Chol	69	14362.14	5394	-	-	-
AE Chol	48	14379.03	5415	16.89	21	0.72
1A1C1E IP	42	14449.87	5421	87.73	27	0.00
3A1E IP	42	14397.45	5421	35.32	27	0.13
1LP CP	33	14713.00	5431	350.87	37	0.00